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Kimball's
Commercial Arithmetic

Prepared for Use in Normal, Commercial and High Schools and for the Higher Grades of the Common Schools

By

Gustavus S. Kimball

Author of "Business English," "Word Book," "Business Speller," etc.

G. P. Putnam's Sons
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PREFACE

The principles of arithmetic remain unchanged. It is only the introduction of new ideas and objects to which numbers are applied and the working out of these in business customs and usages that stamp the value of the modern commercial arithmetic. With an experience of over thirty years as a teacher of mathematics in some of the representative schools of this country, and after a careful study of the needs of business and the counting-room, the author feels that he is competent to present the subject of commercial arithmetic in a manner well calculated to arouse and hold the interest of the student.

The book has been written with the thought that it should be helpful for students pursuing a commercial course in high schools, colleges, normal schools, and business schools. The work embodies every necessary form of illustration and exercise, with due emphasis throughout upon *oral* exercise. With this in view, the purpose has been to make it intensely practical as well as logical throughout; simple and concise in definitions; clear and brief in solutions, with ample explanations; and withal sensible in arrangement and classification of subjects.

Attention is especially directed to that part of the book which treats of the fundamental processes of arithmetic—addition, subtraction, multiplication, division, and fractions. As these are the only operations to which a boy is at all likely to be set on his introduction into a business office, too much stress cannot be laid upon these processes. The fact is that pupils are not sufficiently drilled in the elementary principles; hence, considerable attention is given to this most important part of the text. Work suitable for rapid calculation drills does not, as a rule, involve complex principles, but rather emphasizes skill and

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dexterity of mind and hand in the simpler processes of arithmetic, and it is these simpler operations and processes which are most often called into use by accountants and business men.

Much care has been exercised in the preparation of the sections devoted to bills and accounts, denominate numbers, and practical measurements. Many business men and practical mechanics have been consulted in the treatment of these subjects in order that they may conform to present usages and customs. Due emphasis is placed upon percentage and its applications, including commercial discounts, marking goods, commission and brokerage, interest, and commercial paper. Special attention has been devoted to the subjects of banking and exchange, stocks and bonds, and insurance. In the treatment of these subjects, obsolete terms and discarded usages have been studiously avoided; the instruction is therefore trustworthy and in accordance with present usage. Some subjects, such as the *value of signs*, *ratio and proportion*, omitted by some authors on commercial arithmetic, have been retained, with the belief that these properly belong to every well-balanced course, and because of their educational value.

Studious care has been taken to make the work educative throughout, and to this end, a large amount of valuable information is embodied in a concise form and presented in a manner easily understood. Numerous business forms are introduced in order to assist the student to a thorough understanding of business customs and usages. These forms and methods have been procured from reliable sources and from persons who are specialists in their several departments. This has necessitated numerous interviews with many different persons, all of whom cannot be mentioned, but the author must acknowledge his indebtedness to Mr. James A. Harris of John A. Roebling's Sons Company; to Mr. Charles K. Hammitt of the Northwestern Life Insurance Company; to Taylor, Smith & Evans, New York; to Mr. Arthur H. Wood, cashier of the First National Bank, Trenton; to the Cleveland Trust Company, Cleveland, Ohio, for valuable information on the subject of Banking and Exchange; and to the many students under his instruction who from time to time have so willingly aided in working out the mechanical details and answers to the numerous problems.

Finally, the book must speak for itself, but it is hoped that known truths have been brought together in such new relations as to have all the force of newness and originality. If it shall prove that this has been done in the present treatise, the writer's ambition will be satisfied.

G. S. K.

NEW HAVEN, CONN.,
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Kimball's Commercial Arithmetic

KIMBALL'S COMMERCIAL ARITHMETIC

INTRODUCTION

1. **Arithmetic** is the science of numbers and the art of computation by figures.

2. A **number** is a unit, or a collection of units; as, one book, ten flags, twelve boys.

3. A **unit** is one, or a single thing; as, *one*, *one* horse, *one* dozen.

NOTE 1. Numbers are classified as *concrete* when applied to a particular kind of object, or quantity; as, five *gallons*, ten *books*, and as *abstract* when not applied to any object; as, *five*, *eight*, *nine*.

NOTE 2. Numbers are said to be *like* when they have the same kind of unit, or express the same kind of quantity; as, six *books*, ten *books*, twelve *books*, and *unlike* when they have different kinds of units, or express different kinds of quantity; as, five *books*, three *horses*, one *boy*.

NOTE 3. An *even number* is one that can be exactly divided by 2; as, 10, 14, 26, and an *odd number* is one that cannot be exactly divided by 2; as, 5, 9, 27.

4. An **integer** is a whole number; as, *nine*, *four* days.

NOTATION AND NUMERATION

5. **Notation** is a method of representing numbers and quantities by marks or signs.

The invention of arithmetical notation must have been cœval with the earliest use of writing, whether hieroglyphic or otherwise.

6. **Numeration** is the act or art of reading numbers when expressed by means of numerals.

7. Numbers may be expressed by three methods:

1. By *figures;* called the *Arabic* method.
2. By *letters;* as, I, V, X, etc.; called the *Roman* method.
3. By *words;* as, five, ten, twelve, etc.

NOTE. The *Arabic* method is so-called because it was first used by the Arabs. The *Roman* is supposed to have been invented by the Romans, and employs seven capital letters to express numbers.

8. A **figure** is a character standing for a number. Ten different figures are used; they are:

0	1	2	3	4	5	6	7	8	9
cipher	one	two	three	four	five	six	seven	eight	nine

NOTE. The figure o denotes nothing, or the absence of number when standing by itself, but when placed at the right hand of a whole number increases its value tenfold. The other nine figures express the number of units shown by their names.

9. Numbers larger than one hundred are divided into divisions of three figures, commencing at the *right* of the number and counting to the *left.* These divisions are called periods and each period is divided into units, tens, and hundreds.

NOTE. This method of grouping is called the *Decimal System* from the Latin word *decem,* which signifies *ten.*

10. A point called a *decimal* point is understood to be placed to the *right* of every number equal to or greater than one, if not already there; as, 254.

11. The names of *periods* to left of decimal point are as follows:

864,	371,	254,	307,	985,	432,	637,	401,	045,	875,	923,	450.
decillions	nonillions	octillions	septillions	sextillions	quintillions	quadrillions	trillions	billions	millions	thousands	units

Rule for Notation

Write the figure expressing the number of hundreds, tens, and ones of each period in the order in which they are named, and fill the places of all orders and periods not named by ciphers.

Rule for Numeration

1. Begin with the lowest order of units, and separate the figures of the given number into periods of three figures each.

2. Begin with the highest period; read the figures expressing the number of hundreds, tens, and ones of each period, and give the name of the period after reading the ones of that period, omitting the name of the ones' period.

WRITTEN EXERCISE

Express by figures:

1. Seven thousand three hundred four.
2. Eight thousand ninety.
3. One million two hundred thousand ninety.
4. Fifty-eight billion six million twenty-five.
5. Forty million forty thousand forty.
6. One million one thousand one hundred one.
7. Eight trillion two hundred sixty million three hundred thousand fifty-six.
8. Fifty million fifty thousand fifty-one.
9. Forty-six billion fifty-five million five thousand five.

Point off into periods and write the following numbers in words:

1. 1019
2. 4008
3. 9090
4. 8008
5. 300025
6. 56075
7. 704000
8. 100000
9. 40840005
10. 20020022
11. 99000990
12. 10000100
13. 230067500
14. 100000001
15. 123004560
16. 700700077

NOTE. In reading numbers the word *and* should not be used, and they should always be expressed in the shortest way possible. Thus, fifteen hundred ninety-six, not one thousand five hundred *and* ninety-six. In decimal fractions *and* has an office to perform which is treated later in this book.

Roman Notation

12. The **Roman notation** employs seven capital letters to express numbers, as follows:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

13. Roman Values. The value of these characters is twofold, as (1) each when standing alone has a definite value as above indicated; and (2) a varying value when written in different positions in combination with other numerals.

14. General Principles. 1. Repeating a letter repeats its value; as, XX, twenty; CCC, three hundred.

2. When a letter is placed before one of greater value, the difference of their values is the number represented; as, XL, forty; CD, four hundred.

3. When a letter is placed after one of greater value, the sum of their values is the number represented; as, XIII, thirteen; CXVII, one hundred seventeen.

4. A bar placed over a letter increases its value a thousand fold; as, $\overline{X}$, ten thousand; $\overline{D}$, five hundred thousand.

5. A bar is never placed over the letter I.

Roman Table

I	1	XIV	14	CC	200
II	2	XIX	19	CD	400
III	3	XL	40	D	500
IV	4	IL	49	DXC	590
V	5	L	50	CM	900
VI	6	LX	60	M	1000
VII	7	LXXX	80	MCMIX	1909
VIII	8	XC	90	MM	2000
IX	9	VC	95	$\overline{X}$	10000
X	10	C	100	$\overline{C}$	100000
XII	12	CL	150	$\overline{M}$	1000000

NOTE. Computation by the Roman symbols is too cumbersome, yet they are still used in numbering volumes, chapters, sections, and other important divisions of books, and on clock faces, etc.

WRITTEN EXERCISE

Read the following numbers:

1. XVIII
2. XXVII
3. XXXIX
4. XLVI
5. XC
6. CIX
7. MCDXCII
8. MMCCXXII
9. CDLVI
10. DCIX
11. IC
12. IL

Express the following numbers by letters:

1. 101
2. 163
3. 999
4. 10200
5. 7896
6. 4372
7. 26753
8. 10000
9. 10099
10. 5693
11. 9004
12. 1909

REVIEW EXERCISE

1. Define a *concrete* number; an *abstract* number.
2. Name two *like* numbers; two *unlike* numbers.
3. How many different figures are used to express numbers?
4. Why is the Arabic system better than the Roman system?
5. Give the name of the first five periods of notation.
6. How many figures are required to write millions? trillions?
7. Why should the word *and* be omitted in reading integers?
8. What is the value of a cipher in any number? Why is it used? Explain the ciphers in 10,006,005.
9. Read aloud the following:

(a) In a certain year the acreage of flax seed was as follows: Minnesota, 431,048; Missouri, 35,894; California, 1,042; North Dakota, 1,465,745; South Dakota, 397,000.

(b) In the same year the acreage of wheat as indicated by the Government report was as follows: Minnesota, 5,119,412; Missouri, 2,144,250; California, 1,572,144; North Dakota, 5,992,000; South Dakota, 3,131,000.

ADDITION

15. **Addition** is the process of finding the sum of two or more numbers.

16. The **amount** or **sum** is the result obtained by addition.

17. The **sign** of **addition** is + and is read *plus*.

18. The **sign** of **equality** is = and is read *equals*; as, $7+2=9$, is read 7 plus 2 equals 9.

19. Principles. Only like numbers can be added. The sum is a number similar to the numbers added.

20. The subject of addition is generally considered an easy one and not unfrequently receives only a portion of the attention it should. It is the basis of all mathematical processes and easy as it may seem there are more errors made in the business world in simple addition than in all other operations combined. Frequent drills and practice will be found very beneficial. The great aim should be rapidity and accuracy.

21. Rapid addition is accomplished through combining figures in such groups that the column may be read up very rapidly. We should learn to *read* up a column of figures just as rapidly as we can learn to read the printed page. But it requires practice. The drawling adder who goes up the column, figure by figure, placing a marked emphasis on the word *and* before each succeeding result, needs only more practice in correct methods. In adding do not name the figures in the column or use the word *and.* There is a time to quit spelling out each word and a time to begin reading. Allow results only to be given and you will have laid the correct foundation for rapid adding.

ORAL EXERCISE

Repeat rapidly the sum of each combination at sight. The list contains all possible groups of two figures each.

1	2	3	2	1	2	3	3	4	1	2	3	1	2	1
1	2	5	3	3	4	4	3	4	4	5	6	9	6	2
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	4	1	3	4	7	5	4	1	3	6	6	1	2	5
9	5	5	7	8	7	6	6	6	8	9	6	7	7	8
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1	2	3	8	8	4	7	6	5	7	5	5	7	6	9
8	8	9	8	9	9	9	8	9	4	7	5	8	7	9
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

A three-minute-a-day, live exercise in simple addition means a solid hour's work every month, and a skill that is invaluable.

ORAL EXERCISE

Combinations of three or more figures. Repeat the sums at sight.

1.	4	9	5	1	2	3	3	5	3	3	1	4	8	6	2
	6	1	7	2	1	6	6	5	4	2	1	3	5	8	7
	4	9	5	1	2	3	6	6	7	3	8	9	7	2	3
2.	5	4	3	5	7	6	4	1	2	2	7	8	3	7	6
	1	2	3	5	4	2	2	2	1	4	3	3	5	9	0
	9	9	9	9	8	8	8	7	7	7	9	6	8	5	4
3.	2	1	2	1	2	1	4	3	1	3	2	2	3	3	3
	8	1	4	2	4	7	5	7	2	3	4	7	6	5	6
	2	2	2	3	4	1	5	3	1	6	3	1	6	3	2
4.	3	9	4	8	6	7	8	9	2	3	4	7	8	9	8
	1	2	3	4	5	6	7	8	9	7	5	6	8	2	3
	7	6	5	4	3	9	8	7	6	5	4	4	5	7	8
5.	2	3	8	9	2	6	8	6	7	7	7	5	8	6	3
	2	7	6	5	9	6	6	6	8	3	2	9	9	4	1
	9	9	7	5	3	2	9	0	4	5	6	3	8	7	5
6.	5	3	1	1	4	1	3	4	2	1	1	5	8	2	3
	5	3	8	6	6	5	5	4	5	3	9	5	6	2	3
	4	8	5	7	9	4	1	5	3	2	3	4	1	2	1
7.	2	3	4	1	2	1	3	4	2	1	5	1	1	1	2
	7	5	5	4	8	5	9	5	5	2	1	1	2	3	8
	6	1	3	2	6	3	4	6	8	9	0	8	7	4	0
	7	9	5	9	4	8	3	5	6	4	6	9	4	8	7

8.	9	9	7	5	4	3	6	2	6	8	9	6	4	3	1
	4	9	5	1	2	3	5	2	3	6	5	4	3	2	7
	6	1	7	2	3	1	8	5	2	4	7	1	2	3	1
	7	8	9	7	6	5	4	6	7	0	0	2	3	4	5
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

9.	2	3	6	5	4	7	5	3	2	8	6	4	2	6	5
	5	2	1	3	4	1	5	6	4	7	8	9	7	4	8
	5	2	8	1	1	2	7	4	8	9	5	4	3	6	5
	3	2	5	6	2	3	4	1	7	5	4	6	5	6	8
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

10.	2	2	3	4	1	5	3	1	6	3	1	6	3	2	5
	6	2	3	4	1	2	1	4	2	1	8	1	8	2	3
	4	8	5	7	9	4	1	5	3	5	1	2	2	7	5
	4	8	5	9	5	2	6	2	4	5	2	1	1	7	4
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

COUNTING EXERCISE

In the lowest grades much of this work should have been done, but as it was not, it is well to give practice in even the simplest exercises. This counting should always be accurate and rapid.

1. Count to 100.
2. Count backward from 100.
3. Count by 2's from 2 to 100 and backward.
4. Count by 3's from 3 to 99 and backward.
5. Count by 4's from 4 to 100 and backward.
6. Count by 5's from 5 to 100 and backward.
7. Count by 6's from 6 to 96 and backward.
8. Count by 7's from 7 to 98 and backward.
9. Count by 8's from 8 to 96 and backward.
10. Count by 9's from 9 to 99 and backward.
1 . Count by 11's from 11 to 99 and backward.
12. Count by 12's from 12 to 96 and backward.

NOTE. There should be practice on all these combinations until rapidity is secured, and even the occasional practice for fixing the rhythmic power.

22. After these exercises, which mean really the multiplication and division tables, pupils should write the combination for seat work, always in a column rather than in a row. After this add these columns often until combinations come naturally.

1	9	2	1	3	2
2	8	3	9	4	1
3	7	4	8	5	9
4	6	5	7	6	8
5	5	6	6	7	7
6	4	7	5	8	6
7	3	8	4	9	5
8	2	9	3	1	4
9	1	1	2	2	3
45	45				

1. Count by 2's from 1 to 99 and backward.
2. Count by 3's from 1 to 100 and backward.
3. Count by 3's from 2 to 98 and backward.
4. Count by 4's from 1, from 2, from 3.
5. Count by 5's from 1, from 2, from 3, from 4.
6. Count by 6's from 1, from 2, from 3, from 4, from 5.
7. Count by 7's from 1, 2, 3, 4, 5, 6.
8. Count by 8's from 1, 2, 3, 4, 5, 6, 7.
9. Count by 9's from 1, 2, 3, 4, 5, 6, 7, 8.

Do not dwell upon these until rhythmic power is secured, but have a reasonable amount of practice, a little practice frequently.

23. Do not stop with counting by 12's, but use the following exercises:

1. Count by 13's from 13 to 156.
2. Count by 14's to 168.
3. Count by 15's to 180.
4. Count by 16's to 192.
5. Count by 17's to 204.
6. Count by 18's to 216.

It is important that the adding be slow at first and that the pupil think his own way along. There is to be no attempt to secure rhythmic power. After this work has been done orally

~~until the habit of thinking the~~ way is formed, then have it done with the pencil, thus:

13	14	15	16	17	18
26	28	30	32	34	36
39	42	45	48	51	54
52	56	60	64	68	72
65	70	75	80	85	90
78	84	90	96	102	108
91	98	105	112	119	126
104	112	120	128	136	144
117	126	135	144	153	162

24. **Example.** Find the sum of 549, 626, 429, and 627.

$$\begin{array}{r} 549 \\ 626 \\ 429 \\ 627 \\ \hline 2231 \end{array}$$

Solution. The sum of the units 7, 9, 6, and 9 is 31 units, or 3 tens and 1 unit. Write the one unit under the column of units and add 3 tens to the column of tens. The sum of the tens column with the 3 tens added is 13; write the 3 in the tens column and add the 1 hundred to the hundreds column. The sum of the hundreds column with the 1 added is 22 which written under the hundreds column completes the work and gives 2231.

Suggestion. In adding, name results only. Instead of saying 7 and 9 are 16 and 6 are 22, name the sum only, as 16, 22, etc. A good way to test the correctness of addition is by adding the columns a second time in reversed order.

Grouping

25. To teach grouping use something like the following to show the process, but do not use this form in actual work. The grouping must be done mentally. Make the groups as large as possibly convenient.

$$\begin{array}{l} 9 \\ 6 \\ \left.\begin{array}{l}2\\8\end{array}\right\}10 \\ \left.\begin{array}{l}4\\2\\4\end{array}\right\}10 \\ 8 \\ \left.\begin{array}{l}6\\3\end{array}\right\}9 \\ \hline 56 \end{array} \qquad \begin{array}{l} 4 \\ \left.\begin{array}{l}2\\8\end{array}\right\}10 \\ \left.\begin{array}{l}3\\5\end{array}\right\}8 \\ \left.\begin{array}{l}1\\2\\5\end{array}\right\}8 \\ \left.\begin{array}{l}2\\7\end{array}\right\}9 \\ \hline 3 \end{array} \qquad \begin{array}{l} \left.\begin{array}{l}5\\4\end{array}\right\}9 \\ \left.\begin{array}{l}6\\2\end{array}\right\}8 \\ \left.\begin{array}{l}3\\5\\1\end{array}\right\}9 \\ 8 \\ \left.\begin{array}{l}3\\4\end{array}\right\}7 \\ \hline 1 \end{array}$$

Suggestion. Have the first column read as follows: 7, 15, 24, 32, 41. Second column: 13, 21, 29, 39, 43. Third column: 13, 21, 31, 41, 47, 56. Sum, 5631.

It will be found a good practice to take a column and allow so many seconds for its addition, remembering that persistent drill is the price of success.

It is helpful to know combinations, or groups that form certain numbers. Thus:

5	4	3	2	1
5	6	7	8	9
—	—	—	—	—

and

4	5	5	6	6	7	8
3	3	4	3	2	2	1
3	2	1	1	2	1	1
—	—	—	—	—	—	—

etc., are groups that form 10, and

7	8	8	8	9	9	9	9
7	6	7	8	6	7	8	9
6	6	5	4	5	4	3	2
—	—	—	—	—	—	—	—

are groups that form 20.

ORAL EXERCISE

Find the sum of the following problems. Name results only and make the groups as large as possibly convenient.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	2	4	8	1	4	6	2	7	1	8	5
6	5	2	4	6	4	8	2	3	3	3	5
4	5	8	6	3	3	7	4	5	8	4	2
4	7	7	2	6	2	5	2	7	9	6	7
7	3	3	7	2	4	5	5	3	6	9	7
2	5	3	2	2	5	9	7	2	3	4	9
2	4	5	7	2	5	4	5	4	4	2	0
9	6	5	3	4	6	6	5	5	7	3	1
4	9	4	5	1	4	7	4	8	8	5	1
3	1	6	5	5	7	3	3	9	2	7	2
4	3	7	8	5	8	2	6	0	6	9	1
1	3	5	2	9	5	4	2	1	2	0	8
5	7	5	8	1	2	5	8	8	1	1	2
5	8	3	2	2	3	3	5	2	4	2	7

WRITTEN EXERCISE

Copy and find the sum of:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
687	594	743	934	821	314	674	567
548	678	926	672	456	201	543	987
735	903	432	210	652	856	294	152
829	162	315	357	975	312	963	852
567	123	435	674	689	406	468	234
689	672	357	962	457	239	357	237
230	732	312	560	349	321	385	793
651	396	469	456	753	975	864	753
548	357	468	579	246	579	790	246
428	135	246	468	579	681	791	802
562	135	246	963	852	185	941	368
396	213	697	432	956	736	489	832
354	138	965	231	972	389	236	734
809	898	564	557	156	689	877	497
234	998	514	342	866	691	235	838

(9)	(10)	(11)	(12)	(13)
68375	92687	37246	83469	68759
43624	31264	62796	72143	45836
76879	21752	43217	38967	73549
43260	49214	78364	43621	82965
57893	35638	52183	14385	56784
62475	82196	96714	56732	68935
89738	65438	21365	21369	54896
96214	82795	52189	87342	42874
87682	21326	96543	46536	56283
23496	62145	21397	64978	39657
67542	34782	76843	83265	35482
94219	13896	43295	73648	86997

Civil Service Method

26. This method may be new to many who have not had a varied experience. The example shown consists in writing the footings of the columns, as illustrated, and finding their sums.

46927	Proof Method
39218	29
75670	24
14843	40
57913	35
74698	27
309269	309269

Begin at the right and add each column separately. This method is used by clerks whose time is often occupied by answering questions to many who are in waiting, and if an error occurs he can quite easily detect it by this method, nor will it be necessary to re-add any portions already completed.

WRITTEN EXERCISE

Add the following using the Civil Service Method:

(1)	(2)	(3)	(4)	(5)
45928	98765	74329	36295	12345
65385	68372	57628	86769	67891
74768	42351	25671	35348	23456
93756	45678	42372	76452	78912
38549	91234	65915	38568	34567
84753	56789	89753	67829	89123

Horizontal Addition

27. In commercial operations it is sometimes convenient to add numbers written in a line across the page. The ability to add horizontally, and with rapidity, often saves much time in making out bills, etc. Care must be exercised in combining only units of the same order.

89 + 94 + 53 + 79 = 315
97 + 53 + 92 + 38 = 280
23 + 42 + 41 + 52 = 158
66 + 47 + 39 + 28 = 180
52 + 43 + 69 + 80 = 244

327 + 279 + 294 + 277 = 1177

Illustration. Check by adding the rows horizontally, as shown in the adjoined illustration, and then adding the column of results. Add from left to right and verify results by adding from right to left.

WRITTEN EXERCISE

Copy and add the following numbers horizontally. Verify the work by adding in a reversed order:

1. 6, 6, 4, 8, 1, 5, 3, 2, 7, 3, 9, 1.
2. 4, 5, 8, 2, 4, 3, 8, 6, 4, 5, 5, 8.
3. 2, 2, 6, 5, 5, 3, 7, 8, 9, 1, 8, 2, 5, 3, 7.
4. 63, 97, 60, 79, 23, 45, 89, 91, 84, 67.
5. 48, 24, 19, 56, 34, 82, 18, 91, 56, 78, 32, 53, 56.
6. 28, 51, 35, 47, 22, 44, 31, 82, 17, 18, 71, 49, 73, 58.
7. 46, 85, 13, 53, 72, 24, 45, 18, 21, 74, 71, 29, 24, 22.
8. 35, 84, 62, 51, 35, 34, 22, 44, 56, 82, 17, 45, 71, 29.
9. 24, 58, 46, 27, 64, 72, 87, 72, 87, 33, 87, 33, 46, 27.
10. 62, 87, 76, 28, 75, 87, 56, 97, 75, 69, 78, 78, 34, 57.
11. 289, 432, 807, 543, 696, 235, 637, 790, 451, 123, 345, 284.
12. 947, 478, 302, 987, 117, 671, 596, 720, 847, 987, 446, 222.
13. 896, 329, 706, 432, 584, 289, 567, 538, 952, 211, 634, 501.
14. 476, 787, 201, 871, 117, 674, 964, 207, 476, 874, 464, 469.

28. *Careful attention* should be given to the figure processes. It is my conviction that the so-called seat-work in arithmetic, done by the pupils of the lower and middle grades, should be mainly mechanical, and that nothing short of absolute accuracy should be commended. The teacher should make himself certain that each pupil knows how to do that which is required. The pupil's main effort should be to do it accurately. The work done, the papers should be divided into two classes: the *correct* and the *incorrect*. The boy who has one figure wrong should be classed with the one who has five or six figures wrong. Both are inaccurate, and inaccurate work in figures has no value whatever.

WRITTEN EXERCISE

1. A good time test. Add horizontally and vertically.

$$\begin{array}{rcrcrcrc}
357433&+&552729&+&526063&+&169962&=\\
578834&+&218272&+&378831&+&827946&=\\
269365&+&173228&+&547328&+&143478&=\\
121268&+&739561&+&266721&+&434747&=\\
676277&+&395587&+&821185&+&382427&=\\
810468&+&404432&+&415073&+&678913&=\\
\hline
&+&&+&&+&&=
\end{array}$$

2. Another good plan for a long test is to make a monthly and yearly statement, and then require the numbers to be added both horizontally and perpendicularly, thus:

Monthly and Yearly Statement

MONTH	1910	1911	1912	1913	1914	TOTALS
Jan.	124832	345325	784395	528349	112233	
Feb.	728941	167832	286948	956873	668734	
Mar.	325768	589435	758586	424632	297865	
April	924876	654321	321476	594657	192843	
May	543768	234567	592763	294632	765345	
June	928328	891234	198725	528647	294673	
July	764732	567895	539876	328943	348694	
Aug.	654924	438927	247632	294742	728643	
Sept.	628328	624932	586529	486532	397544	
Oct.	784623	864743	738564	674384	867347	
Nov.	398472	157635	328924	586432	247689	
Dec.	153287	594765	895855	975683	348789	
Totals						

Another Addition Test

43576
132464
1021352
9910240
18799128
etc., etc.

Commencing with the line, say 43576, add a number having the same figure throughout; for example, the figure 8, as in the illustrative example. To this number add the number composed of five 8's. Then continue adding to this and each successive sum numbers composed of as many 8's as there are digits in each sum respectively. I know of no better drill on addition, as you can practice with all the digits. There is no limit to the number of examples, nor to the various ways or combinations of taking the nine digits. It not only opens up an immense field for variety in addition, but equally so in subtraction, in division, and particularly so in multiplication.

Keep the class adding, subtracting, multiplying, or dividing, as the case may be, for two minutes, then compare the answers of some particular line and also the number of lines done. Class work slowly and accurately at first, then speed with accuracy will soon follow.

Rapid Addition

29. What the practical man wants, next to accuracy, is speed in arithmetical operations; and it is right that school children should be drilled in rapid processes. One of many cases where rapid addition may be applied is given on the following page.

52
78
34
45
72
281

Process. Begin with the number at the bottom and add as follows: 72, 77, 117, 121, 151, 159, 229, 231, 281—thus reaching the result without adding the columns separately. The same process may be used in adding three or more columns of figures. While it may not be considered practical, it will strengthen the memory.

WRITTEN EXERCISE

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
76	89	43	14	67	82	15	45
68	31	72	89	13	67	41	83
34	56	74	56	78	89	98	12
39	12	34	82	50	23	45	76
92	38	74	65	51	23	45	92
34	56	17	22	42	35	64	65
92	43	52	76	43	72	15	29
12	45	47	91	23	62	47	34
54	54	22	33	44	92	34	62

30. When numbers are in arithmetical order, as 3, 4, 5, 6, 7, or 2, 4, 6, 8, 10, or 5, 10, 15, 20, 25, there being an odd number of them, the total may be found by multiplying the middle figure by the number of consecutive numbers; or, add the first and last, take half and multiply by the number of numbers.

Thus: $2+3+4+5+6=4\times5=20.$
$5+10+15+20+25+30+35=20\times7=140.$

ORAL EXERCISE

By inspection find the sum of:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
5	15	25	35	45	2	7	12	17	22
7	17	27	37	47	3	8	13	18	23
9	19	29	39	49	4	9	14	19	24
11	21	31	41	51	5	10	15	20	25
13	23	33	43	53	6	11	16	21	26

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
32	41	1	12	23	34	3	5	3	5
33	42	2	13	24	35	6	10	5	15
34	43	3	14	25	36	9	15	7	25
35	44	4	15	26	37	12	20	9	35
36	45	5	16	27	38	15	25	11	45
37	46	6	17	28	39	18	30	13	55
38	47	7	18	29	40	21	35	15	65
39	48	8	19	30	41	24	40	17	75
40	49	9	20	31	42	27	45	19	85
41	50	10	21	32	43	30	50	21	95
42	51	11	22	33	44	33	55	23	105

NOTE. When a number is repeated several times the sum may be found by mutiplication.

21. Find the sum of 3, 7, 6, 6, 6, 6, 6, 6.

22. Find the sum of 7, 7, 7, 7, 4, 4, 4, 4.

23. If four loads of coal weigh respectively 1892, 1754, 2134, and 2087 pounds, what is the entire weight?

24. North America has an area of 8,825,537 square miles, South America, 6,954,131 square miles, and the West Indies, 93,810 square miles. What is the area of the entire American Continent?

Use of Addition

31. The principal use of addition in business is in adding columns of figures numbering dollars and cents.

The **sign of dollars** is **$**. It is read *dollars*. Thus, $42 is read 42 *dollars;* $16 is read 16 *dollars*. When dollars and cents are written, a point (.) is placed before the cents, or between the dollars and cents. Thus $18.25 is read 18 dollars and 25 cents. Cents always occupy *two* places, and never more than two, and if the number of cents is less than 10 and expressed by a single figure, a *cipher* must occupy the first place at the right of the point. Thus 9 dollars 5 cents are written $9.05; 8 cents are written $ .08.

WRITTEN EXERCISE

1. Find the sum of eighteen hundred dollars, 564 dollars

23 cents, 64 dollars thirty-five cents, $600, and 342 dollars 7 cents.

2. A man owns horses valued at $1542, hogs $325.50, sheep $175, cows $642, and other property to the amount of $678.90. What is the total value of his property?

3. What is the sum of ninety thousand, ninety-six; eight thousand, sixty-nine; nineteen; seven hundred, nine; four hundred thousand, eight; forty-six thousand; two thousand one hundred twenty; eighty-two; five million?

4. Find the sum of ten million, ten thousand, ten; forty-seven million, nine thousand, five; six hundred thousand, six hundred six; eight thousand, four hundred; six hundred ninety-seven; eighty-six; ninety thousand, forty-seven; one million, one thousand, one hundred one.

5. An agent bought a house for $7450. He paid $543.65 for repairs, and $134.25 for painting it. For how much must he sell it to gain $650.50?

6. How many years elapsed from the battle of Marathon, 490 B.C., to the discovery of America, A.D. 1492?

7. Find the sum of five units of the third order and three of the first; seven of the fifth, two of the third, and eight of the second; nine of the sixth, five of the fourth; nine of the fourth, seven of the second; six of the ninth, seven of the seventh, one of the fifth, and two of the first; four of the seventh, five of the fifth, and six of the second; one of the fourth, and two of the first.

8. Add together all the numbers ending in "*teen*" and "*ty*" from 1 to 50.

9. Find the sum of all the *odd* numbers from 1 to 50.

10. Find the sum of all the *even* numbers from 1 to 40.

11. Make the vertical and horizontal additions in the following Postal Table:

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOTAL
Ordin'y lett'rs	9243	8564	9204	7695	7368	9986	
Regis'd lett'rs	659	827	655	238	1346	399	
Postal cards..	2129	2214	1986	1873	989	1655	
Book packets.	843	296	231	655	495	378	
Parcels.......	257	328	201	199	276	301	
Newspapers..	11698	12325	14276	16883	17319	20205	
Total......							

12. The following table shows the receipts of cattle, hogs, and sheep at the places named during a recent year. Find the total of each:

	CATTLE	HOGS	SHEEP
Chicago	3,329,250	7,799,450	4,805,449
Kansas City	2,295,979	3,272,335	1,616,788
South Omaha	1,079,373	2,393,551	2,165,116
St. Louis	1,121,380	1,923,209	578,652
St. Joseph	606,187	1,908,207	826,764
Sioux City	385,040	1,158,180	64,167
St. Paul	426,941	860,167	734,759
Indianapolis	350,016	1,869,353	76,570
Ft. Worth	603,592	550,661	97,614
Louisville	131,699	698,106	324,474
Denver	328,751	192,720	825,766
Total			

13. The following may be given for speed and accuracy tests:

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
56789	34567	98657	85976	67958
12345	89123	86579	59758	79586
67891	45678	65798	97685	95867
23456	91234	57986	76859	58679
78912	56789	79865	68597	86795
34567	12345	98675	85976	67958
89123	67891	86579	59768	79586
45678	23456	65789	97685	95867
91234	78912	57986	76859	58679
56789	34567	79865	68597	86795

14. The following shows the acreage and production of corn as indicated by the final Government report. Find the total of each column:

States and Territories	Acreage 1906	Bushels 1906	Bushels 1905
Maine	12,350	456,950	445,900
New Hampshire	26,234	983,775	1,000,665
Vermont	56,491	2,005,430	2,020,859
Massachusetts	44,799	1,778,520	1,679,962
Rhode Island	10,011	331,364	325,358
Connecticut	55,595	2,223,800	2,373,906
New York	650,000	22,685,000	19,312,744
New Jersey	277,749	10,082,289	9,943,414
Pennsylvania	1,441,797	57,960,239	56,085,903
Delaware	196,472	5,894,160	5,972,749
Maryland	628,795	22,007,825	23,202,536
Virginia	1,859,610	45,188,523	43,514,874
North Carolina	2,731,820	41,796,846	37,596,331
South Carolina	1,935,347	23,611,233	20,480,860
Georgia	4,338,883	52,066,596	47,255,164
Florida	625,000	6,875,000	6,518,702
Alabama	2,990,587	47,849,392	42,971,548
Mississippi	2,204,822	40,789,207	30,027,569
Louisiana	1,524,281	26,217,633	19,516,499
Texas	6,924,657	155,804,782	139,146,404
Arkansas	2,237,397	52,802,569	38,323,738
Tennessee	3,075,762	86,428,912	77,207,912
West Virginia	750,000	22,725,000	22,813,122
Kentucky	3,195,072	105,437,376	94,893,638
Ohio	3,325,000	141,645,000	112,399,396
Michigan	1,475,000	54,575,000	41,775,936
Indiana	4,643,782	183,893,767	187,130,623
Illinois	9,616,886	347,169,585	382,752,063
Wisconsin	1,458,877	60,105,732	55,407,849
Minnesota	1,492,538	50,149,277	48,997,455
Iowa	9,450,000	373,275,000	305,112,376
Missouri	7,075,000	228,522,500	203,294,798
Kansas	6,750,000	195,075,000	193,275,836
Nebraska	7,325,000	249,782,500	263,551,772
South Dakota	1,875,000	62,812,500	51,614,739
North Dakota	150,000	4,170,000	2,458,638
Montana	3,980	93,132	76,455
Wyoming	2,528	68,256	56,678
Colorado	113,159	3,157,136	2,776,484
New Mexico	40,211	1,182,203	997,402
Arizona	7,462	220,129	205,578
Utah	11,126	356,032	410,979
Idaho	5,231	148,037	149,763
Washington	11,444	288,389	261,263
Oregon	18,083	499,091	403,788
California	57,158	1,994,814	1,810,944
Oklahoma	1,998,095	65,737,326	48,144,584
Indian Ter.	2,038,490	68,493,264	62,297,784
Total			

SUBTRACTION

32. **Subtraction** is the process of finding the difference between two numbers.

33. The **subtrahend** is the smaller of the two numbers. The **minuend** is the larger.

34. The **difference** or **remainder** is the result obtained by subtracting.

35. The **sign of subtraction** is —. It is read *minus*, and means *less*.

When the sign of subtraction is placed between two numbers it shows that the number placed after it is to be taken from the number before it. Thus, 14—6=8.

36. A **parenthesis** ( ) is used to indicate within it such numbers as are to be considered together. A **vinculum** —— has the same signification.

Thus, $36-(11+6)$, or $36-\overline{11+6}$ signifies that from 36 the sum of 11 and 6 is to be subtracted.

ORAL EXERCISE

State the number which added to the smaller produces the larger in each of the following:

1.	6	6	6	6	7	7	3	5	4	5	8	8	9	6	7	8
	3	2	4	5	6	1	2	2	3	4	2	4	3	2	5	3
2.	10	15	20	11	16	19	12	17	13	14	18	15	17	16	18	19
	5	9	8	7	6	8	8	5	7	9	8	4	8	7	9	9
3.	20	21	21	24	24	22	22	23	21	22	26	24	21	23	25	24
	14	15	12	10	16	17	14	15	19	18	19	17	16	15	14	17

4.	26	27	25	28	26	29	25	27	30	29	31	32	30	33	34	35
	17	19	18	19	17	21	17	16	21	22	29	27	24	29	29	26

5.	36	28	27	37	39	38	31	40	44	42	45	43	46	45	47
	28	25	22	29	21	31	24	33	39	37	38	36	37	38	39

6.	61	48	60	47	59	48	58	49	57	50	56	61	53	55	54
	53	38	51	38	51	39	48	42	48	41	51	41	44	46	45

In the work of subtraction it is important that the pupil should be able to see at once what number added to the smaller of the two will produce the larger.

37. The complete process of subtraction is shown in the following exercise:

Example. Find the difference between 7906 and 5943.

7906
5943
————
1963

Solution. Since only like orders of units can be taken one from another, write the less number under the greater so that units of the same order shall stand in the same column; and draw a line under all. Begin at the right hand side and say, 3 from 6 leaves 3. 4 cannot be subtracted from 0, but 4 from 10 leaves 6. Since 1 of the 9 hundreds has been taken, there are but 8 hundreds remaining. 9 cannot be subtracted from 8, but 9 from 18 leaves 9. Since 1 of the 7 thousands has been taken, there are but 6 thousands remaining. 5 from 6 leaves 1.

38. **Check.** To check or prove the work, add the remainder and the subtrahend. The sum should equal the minuend.

Making Change

39. The common method of *making change* is to add to the amount of the purchase until a sum is obtained equal to that presented in payment. For practical computation in subtraction work this method is best, as it is easily understood, and much more rapid when once learned.

Find the difference between 6194 and 4323.

Solution. Begin at the right. Think "3+1=4," and write 1; "2+7=9," and write 7; "3+8=11," and write 8; "1 and 4+1=6," and write 1.

ORAL EXERCISE

1. 15+10+?=50?
2. 17+16+?=56?
3. 16+14+10+?=100?
4. 15+19+16+14+?=80?
5. 11+24+16+29+?=90?
6. 23+16+11+?=75?
7. 27+14+16+?=70?
8. 22+23+9+?=75?
9. 18+29+12+11+?=90?
10. 14+12+16+12+?=75?

ORAL EXERCISE

You have on hand these denominations of U. S. money: coins, 1¢, 5¢, 10¢, 25¢, 50¢; bills, $1, $2, $5, $10, $20.

Make change for the following purchases:

	Amount of Purchase	Amount Paid		Amount of Purchase	Amount Paid
1.	$4.45	$5.00	9.	$10.62	$20.00
2.	3.56	5.00	10.	13.45	50.00
3.	3.78	5.00	11.	18.91	20.00
4.	9.11	20.00	12.	3.56	10.00
5.	8.33	10.00	13.	1.67	5.00
6.	7.21	50.00	14.	.32	1.00
7.	.67	1.00	15.	5.02	10.00
8.	2.34	20.00	16.	3.09	20.00

There are a number of ways of making change, but it is usually best to make it in the simplest way possible; that is, with the largest possible denominations. On receiving money for a purchase, state at once the value of the bill given you, as this will prevent any later disagreement between the purchaser and salesman by calling attention of both to the actual amount of the bill or coin.

WRITTEN EXERCISE

1. If I have $5, and buy a book for $1.25, a hat for $2.20, and a tie for 35¢, how much have I left?

2. A boy who had $1.50 spent 25¢ for a knife, 18¢ for a ball club, and 47¢ for a fishing rod; how much had he left?

3. A merchant gave $52 for a box of goods, and paid $5 freight; for how much must he sell them to gain $15?

4. A had $725.40, B had $178.37 more than A, and C had as much as A and B together minus $214. How much had C?

5. A had in bank July 1, $756.25. He drew out, from July 1 to August 1, $134.56, $54.34, $73.25, $63.23. What is his balance in bank August 1?

6. A man owns property valued at $75,860, of which $45,640 is invested in real estate, $25,175.75 in personal property, and the remainder he has in bank. How much has he in bank?

Short Methods

40. Arithmetical Complement. The complement of a number is the difference between the number and a unit of the next higher order.

Thus, the arithmetical complement of 6 is 4, since 6+4=10, and 27 is the complement of 73. 154 and 846, 375 and 625 are complementary numbers.

41. In finding the complement of a number, begin at the left to subtract. Take 1 from the highest order in the minuend, and regard each of the lower orders as 9's, except the last, which regard as 10.

$10.00
6.34
$ 3.66

Thus, begin at the left. 6 from 9 leaves 3; 3 from 9 leaves 6; 4 from 10 leaves 6. Or the following may be used: 6 and 3 are 9; 3 and 6 are 9; 4 and 6 are 10. $3.66.

This method is of especial interest to the student as it is the method of *making change* used in stores. It is also very useful in long division.

ORAL EXERCISE

Name rapidly the difference between the following:

1.	$1.00 .27	$1.00 .46	$1.00 .78	$1.00 .34	$1.00 .91	$1.00 .64	$1.00 .55
2.	$1.00 .38	$2.00 1.63	$5.00 2.46	$5.00 3.87	$10.00 6.31	$20.00 12.87	$5.00 1.23
3.	$10.00 7.65	$20.00 13.56	$50.00 23.43	$50.00 33.33	$20.00 14.32	$10.00 6.87	$50.00 26.88
4.	$5.00 2.33	$10.00 6.45	$15.00 11.49	$20.00 13.59	$25.00 22.34	$30.00 27.54	$40.00 33.76

5. Name rapidly the complements of the following numbers: 25, 46, 23, 21, 90, 33, 25, 125, 0.25, 0.18, 400, 1250, 5000.

6. Name the amount of change a clerk must return if he receives a five-dollar bill in payment of each of the following amounts: $1.40, $2.75, $2.88, $0.25, $0.88, $3.91, $1.75, $0.56, $4.23.

ORAL EXERCISE

Name quickly the amount of change returned in each of the following purchases:

	Amount of Purchase	Amount Paid		Amount of Purchase	Amount Paid
1.	17¢, 18¢, 25¢.	$2	**9.**	$ 1.00, $ 1.25, $ 3.75.	$ 10.00
2.	16¢, 19¢, 40¢.	$2	**10.**	$ 1.35, $ 1.65, $ 5.50.	$ 10.00
3.	27¢, 48¢, 25¢.	$2	**11.**	$ 0.45, $ 1.30, $ 3.75.	$ 10.00
4.	43¢, 21¢, 11¢.	$2	**12.**	$ 3.25, $ 1.75, $ 8.34.	$ 20.00
5.	33¢, 22¢, 45¢.	$2	**13.**	$ 1.44, $ 1.66, $12.90.	$ 20.00
6.	24¢, 26¢, 32¢.	$2	**14.**	$16.45, $ 4.55, $23.50.	$ 50.00
7.	25¢, 75¢, $1.25.	$5	**15.**	$23.91, $12.09, $ 4.59.	$ 50.00
8.	72¢, 88¢, $1.85.	$5	**16.**	$25.50, $56.25, $ 8.25.	$100.00

Addition and Subtraction

42. The method of combining two operations in one, that of addition and subtraction, is often very convenient, especially to the bookkeeper. For instance, the sum of several numbers may be taken from the sum of several others without transferring the figures to separate paper.

43. Example. From 62,834 take the sum of 4692, 5684, 9204.

$$\begin{array}{r} 62834 \\ \hline 4692 \\ 5684 \\ 9204 \\ \hline 43254 \end{array}$$

Solution. As illustrated, separate the minuend from the numbers to be subtracted. Beginning at the right, add and subtract at the same time. Thus:

4, 8, 10 and *4*; 14.
1, 9, 18 and *5*; 23.
4, 10, 16 and *2*; 18.
10, 15, 19 and *3*; 22.
2 and *4*; 6.

WRITTEN EXERCISE

1. From 63,497 take the sum of 345, 6527, 41234 and 652.
2. From 77,563 take the sum of 3121, 3246, and 7623.
3. From 887,675 take the sum of 63545, 84376, 125860, and 321075.
4. A clerk receives a fifty-dollar bill in payment of the following items: $6.36, $11.34, $0.88, $23.43, $7.89. How much change does he return?
5. A deposited in a bank $965, and checked out at different times $234, $56, $34, $1.56, $65, $305. How much remained in bank?

NOTE. Additional problems for practice should be supplied by the teacher until fundamental operations are sunk into the automatic. Give all drills as much variety as possible.

44. **Ledger Balances.** The method as explained in 43 is particularly helpful in balancing ledger accounts as illustrated by the following example:

Example. What number will balance the following account?

Dr. Cash Cr.

1909 Aug	1	Mdse		500		1909 Aug	12	Salaries		75	
	10	J. E. Gill		240	40		31	Rent of Store		100	
	16	R. L. Fisk		175			31	Balance		740	40
				915	40					915	40
Sept	1	Balance		740	40						

It will be seen in the above example that the debit side is greater; therefore add that side first and place the sum under the foot-line of the column added. Then find the balance between the sides by a single operation according to the method as previously explained.

WRITTEN EXERCISE

By the short methods explained find the balance of each of the following accounts:

(1)

Dr THE STATE BANK in account with F. B. Moore Cr

1909 mar 10	Cash	1275	95	1909 mar 15		364	
16	Checks	496	25	26		721	25
25	Cash	1703	05	apr 1		1050	
				18		872	50
				25		250	

(2)

Dr THE STATE BANK in account with J. E. Caldwell & Co. Cr

1910 Sept 1	Cash	1796		1910 Sept 2		1500	
15	Cash	2434		9		1685	50
18	Cash	6285	50	16		1479	30
21	Cash	1720	50	25		1623	20
				Oct. 2		1780	
				9		1567	

(3)

Dr Lord & Taylor Cr

1910 Oct	1		16	375	90	1910 Oct	3		17	150	
	15		19	286	60		10		19	135	70
	20		21	597			17		20	70	30
							23		21	65	90
							30		23	142	30
						Nov.	7		26	176	24
							13		27	34	36
							20		29	72	25

(4)

Dr S. P. Dunham & Co. Cr

1910 June	12		106	472		1910 June	14		109	256	
	15		108	596			21		112	370	
	18		109	1372	50		28		114	460	
	22		110	1827	25	July	5		116	325	40
							10		115	476	80
							15		118	396	50
							20		120	1265	30

ORAL REVIEW

1. Subtract by 4's from 70 back to 2.
2. Subtract by 6's from 87 back to 3.
3. Subtract by 9's from 86 back to 5.
4. Subtract by 7's from 66 back to 3.
5. Read 9010009, 20010005, 1009008898898.
6. From the difference between 376 and 215 take 84.
7. Run this race: begin at 3, add by 3's, as far as 135.
8. In the **State Bank** account, page 27, what are the total receipts?
9. The total gains of a business are $765.40 and the total losses, $432. What are the net gains?
10. From 100 take 23; 27; 15; 87; 42; 63; 88; 37; 67; 43.
11. $10 − $5.88 = ?
12. $20 − $11.65 = ?
13. $20 − $2.75 + $7.35 = ?
14. $25 − $22.56 = ?
15. $50 − $23.67 + $22.33 = ?
16. $50 − $34.08 + $15.92 = ?
17. By inspection, name quickly the difference between the following:

100	1000	2000	900	600	3000	4000
37	234	655	456	89	189	365

WRITTEN REVIEW

1. Without putting the figures down, that is the 7's, subtract a line of 7's from 624479012 as far as the seventh remainder.

2. Write in figures, twenty-five million two hundred fifty-seven thousand six hundred thirty; and in words, 402050407.

3. From seventeen million seventeen take eight thousand eight.

4. The difference between two numbers is 2,001,005; the larger number is 89,009,089. Find the smaller.

5. How many are 871645 − 7986 + 394 − 10089?

(6.)	(7.)	(8.)	(9.)
6544458904	2300456498	5439700112	9844500324
2856944099	1078543267	4569802348	568345023

10. A man's account at a store stands thus:

Dr.	Cr.
$4.75	$2.56
2.63½	1.24
1.27	.62½
.45	3.45
5.28½	1.87½

What is due the merchant?

11. I handed a shop-keeper a $5 bill for the following items: 1 ladies' pocketbook, $1.25; 1 ladies' parasol, $2.15; ½ doz. handkerchiefs, $2.80 a dozen. Make the change.

12. A man owns property valued at $89,506, of which $43,576 is invested in real estate, $23,175.40 in personal property, and the remainder in bank. How much has he in bank?

13. A merchant commenced business with a capital of $10,000. The first year he gained $1756, and the second he lost $2306, and the third he gained $2376.25. What was his capital then, and how much had he gained or lost in all?

14. A's balance in bank March 1, was $534.25. He deposited March 20, $243.65 and checked out from March 1 to

May 1, $211.92, $65.34, $283.45. What is his balance in bank May 1?

15. A section of an individual ledger of a banking house exhibits the following accounts. Find the total checks, deposits, and new balances.

Names	Balances		Checks		Deposits		Balances	
Allen, John.......	1629	56	1233	73	867	43	....	..
Abbott, A. J......	2345	76	456		543	25	....	..
Bacon, C. A......	432	34	876	34	674	80	....	..
Baldwin, R. C....	334	67	239	65	900		....	..
Clancy & Co......	17605	45	8954	72	5045	45	....	..
Dean, W. J. G....	4325	65	1237	92	1325	56	....	..
	....	..	....	..	....	..	....	..

MULTIPLICATION

45. **Multiplication** is the process of repeating one of two numbers as many times as there are units in the other. It is a *short method* of addition.

46. The **multiplicand** is the number to be multiplied. The **multiplier** is the number by which to multiply. The **product** is the result obtained by multiplication.

47. **Factors** are numbers used in obtaining a product.

The multiplication table should be so well known that the factors will at once suggest the product.

48. The **sign of multiplication** is the oblique cross ×, and is read *times*, or *multiplied by*.

1. When the sign of multiplication is placed between two numbers it shows that they are to be multiplied together; as, 6×7 is read 6 times $7 = 42$.

2. The student should be able to see at once what numbers added to the product of two numbers will produce a given number; as, the number added to 7×8 to produce 61 is 5, or $7 \times 8 + 5 = 61$.

3. In examples containing *concrete* numbers, the *concrete* number is the multiplicand as the multiplier is always an abstract quantity, but the multiplicand may be either *abstract* or *concrete*. When the multiplicand is smaller it is often used *abstractly* as the multiplier.

ORAL EXERCISE

1. If a man earns \$4 a day, how many times \$4 does he earn in 5 days? \$4+\$4+\$4+\$4+\$4=\$20.

2. There are 4 quarts in a gallon. How many quarts in 6 gallons? In 7? In 8? In 12?

3. What is the difference between four 6's and six 4's?

4. Add by 2's from 2 to 30.

5. Subtract by 3's from 36 to 3.

Multiplication Square

49. Copy and complete. When finished it forms the Multiplication Table.

1's	2's	3's	4's	5's	6's	7's	8's	9's	10's	11's	12's
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16				
3	6	9	12	15	18	21					
4	8	12	16	20	24						
5	10	15	20	25							
6	12	18	24								
7	14	21									
8	16										
9											
10											
11											
12											

ORAL EXERCISE

1. Name quickly answers to the following:

6 × 6 + 10 = ?   12 × 8 + 4 = ?   15 × 2 − 15 = ?
5 × 3 + 7 = ?   9 × 9 + 19 = ?   11 × 11 − 9 = ?
12 × 4 + 2 = ?   10 × 9 − 14 = ?   12 × 0 + 25 = ?
10 × 7 − 5 = ?   7 × 9 + 17 = ?   12 × 11 − 2 = ?
11 × 5 + 10 = ?   8 × 3 − 7 = ?   7 × 12 + 16 = ?

2. Multiply at sight each of the following numbers by 2; by 3; by 4; by 7; by 9; by 6; by 5.

3 7 5 9 11 12 2 4 8 6 10 20
15 13 14 16 21 22 19 18 17 23 24 25

WRITTEN EXERCISE

50. Examples. 1. How many are 5 × 641?

$$\begin{array}{r} 641 \\ \underline{5} \\ 3205 \end{array}$$

Solution. Write the multiplier 5 under the multiplicand as illustrated in the margin and begin at the right to multiply. 5 × 1 unit = 5 units. Write the 5 in units' place. Next 5 × 4 tens = 20 tens, and write 0 in the place of tens, and reserve 2 to add to the place of hundreds. Then 5 × 6 = 30, and adding the 2 hundreds reserved gives 32 hundreds. Hence the *product* is 3205.

2. How many are 326 × 874?

$$\begin{array}{rl} 874 & \\ \underline{326} & \\ 5244 & = \text{product by } 6 \\ 1748 & = \text{product by } 20 \\ \underline{2622} & = \text{product by } \underline{300} \\ 284924 & = \text{product by } 326 \end{array}$$

Solution. Write the numbers as before. Since the multiplier consists of 6 units, 2 tens, and 3 hundreds, the multiplicand is multiplied by 6, by 20, and by 300. The sum of these partial products is 284,924, the entire product. In practice the ciphers are omitted.

51. The work may be *checked* by changing the order of the factors; that is, by multiplying the multiplier by the multiplicand; if the same product is obtained the work is presumably correct.

NOTE. When two numbers are to be multiplied, it is generally easier to take the smaller number as the multiplier.

3. Multiply 375 by 52.

$$\begin{array}{r} 375 \\ \underline{52} \\ 750 \\ \underline{1875} \\ 19500 \end{array} \qquad \begin{array}{r} 375 \\ \underline{52} \\ 1875 \\ \underline{750} \\ 19500 \end{array}$$

Solution. In multiplying one number by another there is no practical advantage in beginning with the lowest order of units of the multiplier. There are plenty of problems where considerable work is saved, especially in decimals, by beginning with the digit of the highest order. The arrangement of work for both methods is shown in the margin.

WRITTEN EXERCISE

In the following exercises multiply and then add products.

(1)	(2)	(3)
13 × 231 = ?	9 × 1678 = ?	376 × 347 = ?
15 × 471 = ?	27 × 4364 = ?	165 × 572 = ?
16 × 609 = ?	39 × 2569 = ?	472 × 694 = ?
14 × 566 = ?	46 × 1293 = ?	593 × 804 = ?
Total = ?	Total = ?	Total = ?

4. Multiply nine hundred nine million ninety-nine by nine hundred ninety-nine.

5. Multiply 567 by 634, and write in words the number represented by each of the three partial products.

6. Sold 38 cords of wood at $2.565 a cord. What did it bring?

7. Bought 140 barrels of apples for $610, but finding 25 barrels worthless, sold the remainder at $5.30 a barrel. Did I gain or lose, and how much?

8. A farmhouse is worth $2345, the farm is worth 3 times as much plus $875, and the stock is worth twice as much as the house, less $1125. What is the value of the whole?

9. A merchant bought 15 barrels of sugar for $135, and retailed it at $17.25 a barrel. How much did he gain in all?

10. A train by its time-table must run 465 miles in 12 hours. If it runs 38 miles an hour for 11 hours, how far must it run the last hour to arrive on time?

11. A government township contains 36 sections of 640 acres each; how many acres of land in a county containing 25 townships?

12. How many hours in the month of September?

13. How many seconds in the month of February, 1917?

Powers of Numbers

52. A **power** is a product arising from using a number two or more times as a *factor*. Thus, 16 is the second power of 4; 125 is the third power of 5.

53. The **first power** is the number itself. The **second power,** or *square,* is the product obtained by taking the number twice as a factor. The **third power,** or *cube,* is the product obtained by taking the number three times as a factor.

The higher powers are respectively, the *fourth, fifth, sixth, seventh,* etc.

54. The **exponent** of a power is a small number placed at the right of a quantity to indicate the number of times it is to be used as a factor. Thus, $4^3=4\times4\times4=64$.

NOTE. The product of two or more like factors with equal or unequal exponents is equal to the sum of the exponents of these powers. Thus, $4^2\times4^3=4^5$, or 1024.

ORAL EXERCISE

1. What is the square of 8; of 11; of 12; of 15?

2. What is the cube of 3; of 4; of 5; of 6; of 10?

3. What is the fifth power of 1; of 2; of 3?

4. What is the product of $5^2 \times 2^3$?

5. Find the cost of 125 lb. of sugar at 5¢ per pound.

6. How much more than $35 will 8 tons of coal cost, at $6 a ton?

7. At $6 a barrel, how much are 2 loads of flour, each containing 20 barrels, worth? Four loads, each 25 barrels?

8. A man earns $4 a day. At this rate, how much will he earn in 8 weeks by working 5 days each week?

9. I bought 15 yards of cloth at $1.05 per yd. If I gave in payment four five-dollar bills, what change should I receive?

10. If I buy 8 penholders at 11¢ each, and 6 pencils at 5¢ each, and some ink costing 20¢, how much change should I receive from a five-dollar bill?

11. The sum of $8+12+16$ equals the product of $4 \times$ what number?

12. The product of what two factors is equal to the sum of 9, 20, and 11?

Rapid Multiplication

55. Rapid calculation is a valuable art, not only because it is practical, but because it affords an excellent mental drill. It is recommended that the pupil learn the multiplication table thoroughly, for without this at the tongue's end, progress will be very slow.

56. A contraction is useful and valuable to the extent that it applies to cases usually and repeatedly done. The following suggestions will be found very useful.

CASE I

57. To find the product of two numbers when there are ciphers on the right of one or both factors.

NOTE. Annexing ciphers to the multiplicand removes each digit one place to the left; hence the value of the entire number is increased tenfold for each cipher annexed.

RULE 1. *When the multiplier is a unit of any order, annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

Example 1. 263 × 100 = ?

Solution. Omitting the two ciphers from the multiplier, we have 1. Then 263 × 1 = 263, and to this annex the two ciphers omitted, and we have 26300, the product.

RULE 2. *In multiplying numbers in which there are ciphers to the right of both multiplicand and multiplier, the ciphers may be omitted. But after multiplying, annex as many ciphers to the product as have been omitted from both factors.*

Example 2. 78200 × 3600 = ?

$$\begin{array}{r} 78200 \\ \underline{3600} \\ 4692 \\ \underline{2346} \\ 281520000 \end{array}$$

Solution. Arrange the numbers as indicated in the illustration and then multiply 782 by 36; the product thus obtained is 28152. Annexing the four ciphers which have been omitted from the multiplicand and multiplier we have 281520000, as the complete product.

WRITTEN EXERCISE

1. Multiply 564 by 100; by 2300; by 20.

2. What will it cost to build 3500 miles of railroad at $37500 a mile?

3. Multiply two hundred fifty thousand by fifteen thousand.

4. If 1200 pounds of seed cotton can be raised from an acre of land, how many pounds will 350 acres produce?

(5)	(6)	(7)
5600 × 700 = ?	34000 × 500 = ?	640 × 780000 = ?
8200 × 650 = ?	73400 × 220 = ?	34 × 5000 = ?
8050 × 900 = ?	23000 × 333 = ?	6 × 4500600 = ?
4536 × 600 = ?	75060 × 200 = ?	342 × 540000 = ?
Total = ?	Total = ?	Total = ?

CASE II

58. To multiply together two numbers whose tens are alike, and the sum of whose units is ten.

RULE. *Multiply the units together for the two right-hand figures of the product, and the remaining part of the multiplicand by the remaining part of the multiplier increased by 1.*

Example. 64×66 = ?

$$\begin{array}{r} 64 \\ \underline{66} \\ 4224 \end{array}$$

Solution. The 6 units × 4 units = 24 units which we write for the two right-hand figures of the product. Then 6 tens multiplied by 1 more than itself for the remaining figures. Thus, $6\times(6+1)=42$, and we have the complete product 4224.

NOTE. If the first multiplication contains but one figure place a cipher in tens' place.

WRITTEN EXERCISE

1. Multiply 24 by 26.
2. Multiply 78 by 72.
3. Multiply 93 by 97.
4. Multiply 44 by 46.
5. Multiply 99 by 91.
6. Multiply 116 by 114.
7. Multiply 996 by 994.
8. Multiply 295 by 295.

ORAL EXERCISE

1. 25 by 25; 35 by 35; 55 by 55; 75 by 75.
2. 17 by 13; 43 by 47; 56 by 54; 100 by 100.
3. 15 by 15; 22 by 28; 65 by 65; 109 by 101.

CASE III

59. To multiply any number by 11.

RULE. *Set down the first right-hand figure of the multiplicand, then add it to the next figure to the left, set down the units and carry the tens, add the second figure to the third, and so on; finally write the left-hand figure, carrying as usual.*

Example. 547893×11 = ?

$$\begin{array}{r} \underline{547893} \\ 6026823 \end{array}$$

Solution. Write 3 as the first figure in the product. $3+9=12$. Write 2 as the second figure, and carry 1. $9+8+1=18$. Write 8 and carry one. $8+7+1=16$. Write 6 and carry 1. $7+4+1=12$. Write 2 and carry 1. $4+5+1=10$. Write 0 and carry 1. $5+1=6$. Write 6, thus completing the multiplication.

ORAL EXERCISE

By inspection find the product of:

1. 23 × 11.
2. 54 × 11.
3. 76 × 11.
4. 89 × 11.
5. 57 × 11.
6. 11 × 67.
7. 84 × 11.
8. 35 × 11.
9. 28 × 11.
10. 11 × 65.
11. 21 × 11.
12. 24 × 11.
13. 91 × 11.
14. 71 × 11.
15. 93 × 11.
16. 225 × 11.
17. 121 × 11.
18. 11 × 2315.
19. 11 × 164.
20. 151 × 11.

WRITTEN EXERCISE

1. Multiply the following numbers by 11: 23456; 23065; 768; 708065; 4321; 1728; 2154; 112233.

2. Find the total cost of 11 yards of cloth at 32¢; 45¢; 33¢; 67¢; 56¢; 64¢; $1.65; $2.33; $3.22; $1.87; $2.67.

CASE IV

60. To multiply by any number of 9's.

RULE. *Annex to the multiplicand as many ciphers as there are 9's in the multiplier, and from the result subtract the multiplicand.*

Example. 6543 × 999 = ?

$$\begin{array}{r} 6543000 \\ \underline{6543} \\ 6536457 \end{array}$$

Solution. To 6543, the multiplicand, annex *three* ciphers and we have 6543000. Then, 6543000 − 6543 = 6536457, the product.

WRITTEN EXERCISE

1. Multiply 432 by 99.
2. Multiply 709 by 9.
3. Multiply 238 by 999.
4. Multiply 825 by 9999.
5. Multiply 235 by 19.
6. Multiply 455 by 49.
7. Multiply 354 by 199.
8. Multiply 9630 by 499.

CASE V

61. To multiply any number by 25.

RULE. *Divide the number by 4, if divisible by 4, and annex two ciphers; or add two ciphers and divide the result by 4.*

WRITTEN EXERCISE

1. Multiply 2345 by 25.
2. Multiply 6542 by 25.
3. Multiply 242 by 250.
4. Multiply 5460 by 2500.
5. Multiply 250 by 250.
6. Multiply 5600 by 5400.

CASE VI

62. To multiply when one part of the multiplier is a multiple of another part.

Example. $478 \times 217 = ?$

$$\begin{array}{rl} 478 & \\ \underline{217} & \\ 3346 & = 7 \times 478 \\ \underline{10038} & = 3 \times 3346 \\ 103726 & \end{array}$$

Solution. We observe that the tens and hundreds (21) of the multiplier make a number *three* times the units (7). Then multiplying the multiplicand by 7, and this product by 3, and placing the numbers as in ordinary multiplication and adding the two partial products, we have 103726, the complete product.

WRITTEN EXERCISE

1. Multiply 8643 by 123; by 155; by 182; by 273.
2. Multiply 564 by 255; by 82; by 279; by 546.

CASE VII

63. To multiply any number by 125.

RULE. *Annex three ciphers to the multiplicand and divide the result by 8.*

Example. $654 \times 125 = ?$

$$\begin{array}{r} 8\underline{)654000} \\ 81750 \end{array}$$

Solution. Annexing the three ciphers to 654 we have 654000, which divided by 8 gives the product 81750.

WRITTEN EXERCISE

1. Multiply 34562 by 125.
2. Find the cost of 125 yards of cloth at 56¢; 78¢; 98¢; $2.45; $3.56; $4.44; $3.89; $2.67.

CASE VIII

64. To multiply two numbers consisting of units and tens when both end in 5.

RULE. *To the product of the tens add one-half their sum and after the amount write 25, the product of the units.*

Example. 75×55 = ?

$$\begin{array}{r} 75 \\ \underline{55} \\ 4125 \end{array}$$

Solution. 5 units×5 = 25 units, which write for the two right-hand figures of the product. Then 7 tens multiplied by 5 plus one-half of 7+5 = 41. Writing the 41 to the left of the 25 we have the complete product, 4125.

WRITTEN EXERCISE

1. Multiply 45 by 65.
2. Multiply 85 by 45.
3. Multiply 35 by 75.
4. Multiply 55 by 65.
5. Multiply 95 by 75.
6. Multiply 45 by 105.

CASE IX

65. To multiply by any number that can be changed to 100, 1000, etc., by adding or subtracting a small number.

RULE. *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and then correct the result by subtracting or adding the product of the smaller number and multiplicand.*

Example 1. 346×98 = ?

$$\begin{array}{r} 34600 = 346\times100 \\ \underline{692} = 346\times2 \\ 33908 \end{array}$$

Solution. It will be seen that the difference between 98 and the unit of the next higher order (100) is 2. Therefore, 346×100 less 346×2 must be the product of 346×98, or 33908.

Example 2. 276×1002 = ?

$$\begin{array}{r} 276000 = 276\times1000 \\ \underline{552} = 276\times2 \\ 276552 \end{array}$$

Solution. It will be seen that the difference between the multiplier and the unit of the denomination (1000) is 2. Therefore, 276 × 1000 plus 276 × 2 must be the product of 276 × 1002, or 276552.

66. The **complement** of a number is the difference between the number and the unit of the next higher order.

WRITTEN EXERCISE

1. 765 × 99 = ?
2. 324 × 98 = ?
3. 472 × 998 = ?
4. 296 × 101 = ?
5. 364 × 1002 = ?
6. 275 × 10003 = ?

CASE X

67. When the multiplier can be changed to one figure followed by ciphers, by adding a small number.

RULE. *Multiply the multiplicand by the resulting round number, and then correct the result by subtracting the product cf the small number and multiplicand.*

Example. 382 × 499 = ?

$$\begin{array}{r} 382 \\ \underline{500} \\ 191000 \\ \underline{382} \\ 190618 \end{array}$$

Solution. It will be seen that the nearest resulting round number to the multiplier is 500. Hence, 382 × 500 = 191000. As this result is once the multiplicand too large, we correct the result by subtracting the product of 1 times 382, which gives the complete product, 190618.

WRITTEN EXERCISE

1. Multiply 674 by 28.
2. Multiply 789 by 59.
3. Multiply 456 by 399.
4. Multiply 458 by 599.
5. Multiply 750 by 898.
6. Multiply 245 by 399.

CASE XI

68. To multiply by the factors of a number.

RULE. *Separate the multiplier into two or more factors and find the continued product of the multiplicand and factors.*

NOTE. The factors of a number are the numbers which multiplied together will produce the number. (See 47.)

Example. 548 × 45 = ?

$$\begin{array}{r} 548 \\ \underline{5} \\ 2740 \\ \underline{9} \\ 24660 \end{array}$$

Solution. The factors of 45 are 5 and 9. Multiplying the multiplicand by 5 and the result thus obtained by 9 we have the required result, 24660.

Case XII

69. To multiply two numbers whose difference is divisible by 2.

Rule. *Square half the sum of the two factors, and from this subtract the square of half the difference of the factors.*

Example. $22 \times 28 = ?$

$$\begin{array}{r} 25^2 = 625 \\ \underline{3^2 = 9} \\ 616 \end{array}$$

Solution. Problems under this case involve the principles of the well-known theorem that, "The product of the sum and difference of two numbers is equal to the difference of their squares."

Half the sum of 22 and 28 is 25. Square 25 and we have 625. The difference between 28 and 22 is 6, and half of 6 squared is 9. Then, $625 - 9 = 616$, the product.

Note. Problems under this case are not practical except in cases where the work may be performed mentally.

ORAL EXERCISE

Find the product of:

1. 32×28.
2. 46×44.
3. 44×36.
4. 26×44.
5. 17×19.
6. 21×79.
7. 91×9.
8. 55×25.
9. 13×17.
10. 97×103.
11. 95×105.
12. 85×95.

Case XIII

70. To square any number.

Rule. *Add and subtract the same unit figure, so as to bring the number to be squared to a number that will have one cipher in one or both factors. To the product of these factors add the square of the number added or subtracted.*

Example. Square 57; 29.

Solution. $57^2 = 54 \times 60 + 9 = 3249.$
$29^2 = 28 \times 30 + 1 = 841.$

Note. This method holds good and practicable to 99; beyond that it may be extended, but not with any advantage.

ORAL EXERCISE

1. Square the following numbers: 23, 45, 76, 87, 98, 49, 34, 48, 42, 63, 77, 74, 62, 99.

Cross Multiplication

71. Lightning calculators acquire great rapidity by what is styled *cross multiplication*, but especial care should be exercised that mistakes do not occur by this method.

$$\begin{array}{r} 43 \\ \underline{34} \\ 1462 \end{array}$$

Solution. 4×3=12. Write the 2 and retain the 1 in mind. 4×4=16+1=17. Retain this in mind, multiplying 3×3, which, added to 17, makes 26; write the 6 and retain in mind the 2. 3×4=12, to which add the 2, making 14.

WRITTEN EXERCISE

1. Multiply 45 by 26; 37 by 28; 52 by 24; 36 by 27; 35 by 32.

ORAL REVIEW

1. Define a concrete number; an abstract number.
2. Can the multiplier ever be a concrete number? Explain.
3. Define *factors*. What are the factors of 36?
4. How does the presence of a zero in the multiplier affect the work?
5. Find the product of 89 by 11; 115 by 11.
6. Multiply 88 by 125; 98 by 50; 1600 by 25.
7. Multiply 87 by 83; 56 by 44; 25 by 35; 16 by 14.
8. Square 25; 35; 45; 65; 75; 85; 55; 105; 95.
9. Multiply 22 by 28; 32 by 28; 55 by 25; 44 by 36.
10. Multiply 200 by 49; 234 by 11; 444 by 25; 95 by 105.
11. Find the value of 131 barrels of sugar at $5 a barrel.
12. How much will a man earn in 21 weeks at $19 a week?
13. 60 is how many times 4; 75 times 15; 85 times 5; 105 times 21; 141 times 3; 369 times 9?
14. What is the cost of 12 barrels of apples at $3.60 per barrel and 8 bushels of potatoes at 75¢ a bushel?
15. Read aloud the following numbers mutliplied by 100: 26; 376; 4627; $27.50; 65¢; $225; $34.225; 9999.

WRITTEN EXERCISE

1. A man had $47 with which to buy 6 cords of wood at $7 a cord. How much money would he have left after buying the wood?

2. A man bought 2 bushels of apples at 75¢ a bushel, and sold them at 37¢ a peck. How much did he gain if one peck was bad?

3. A has $75, B has three times as much as A, and C has as much as both lacking $45. How much have they all?

4. Find a number to which if 369 be added the sum will be 1001 less than 9090.

5. If I receive $1600 salary, and pay $385 for board, $286.50 for clothing, $118.75 for books, and $196.65 for other expenses annually, what can I save in 3 years?

6. If a boy waste on the average two minutes every hour, how many minutes will he waste in four years?

7. A gardener sold from his garden, 120 bunches of onions at 12¢ a bunch, 18 bushels of potatoes at 62½¢ a bushel, 47 heads of cabbage at 7¢ a head, 6 dozen cucumbers at 18¢ a dozen; he expended $1.50 in spading, $1.27 for fertilizers, $1.87 for seeds, $2.30 in planting and hoeing. What were the profits of his garden?

8. Copy and find the amount of the following bill:

Harrisburg, Pa., July 16, 1914.

H. B. WEST & CO.,
342 Harrison Avenue, City.

Bought of EDWARD B. SMITH PROVISION COMPANY.

Terms—10 days.

156	bu.	Wheat	$1.04		
45	"	Oats	.34		
56	"	Barley	.65		
115	"	Potatoes	.86		
124	"	Apples	.48		
12	bbl.	Onions	1.35		

DIVISION

72. **Division** is the process of finding how many times one number is contained in another.

73. The **dividend** is the number to be divided. The **divisor** is the number by which we divide. The **quotient** is the result obtained by division, and shows how many times the divisor is contained in the dividend.

74. The **remainder** is the part of the dividend which remains when the division is not exact.

75. The **sign** of **division** is ÷ and is read *divided by.*

When this sign is placed between two numbers it shows that the one on the left is to be divided by the one on the right; as, $27 \div 3 = 9$, and is read 27 divided by $3 = 9$.

Division is also indicated by writing the dividend above the divisor; as, $\frac{27}{3} = 9$. It is read twenty-seven thirds = 9.

ORAL EXERCISE

1. How many times 9 cents are 72 cents? Is the result a concrete or an abstract number?

2. How do you find one of 9 equal parts of a number? Of 6 equal parts? Of 7 equal parts?

3. Distribute $35 equally among 5 men. How many dollars will each receive?

4. How much is 12 times $15? How many times is $15 contained in $90? What is $\frac{1}{15}$ of $60?

5. What is one of 5 equal parts of 30? Of 45? Of 60?

6. When the product is given and one factor is given, how do you find the other factor?

7. What is one of 9 equal parts of 72 cents? Is the result a concrete or abstract number?

76. Principles. *1. If the dividend and divisor are like numbers, the quotient must be an abstract number.*

2. If the dividend is a concrete number and the divisor abstract, the quotient is a concrete number.

3. A change in the dividend or divisor produces a like change in the quotient, but a like change in both dividend and divisor does not affect the quotient.

Division may be regarded as a short method of subtraction. Thus, $20-5=15$; $15-5=10$; $10-5=5$; $5-5=0$. Hence, we have performed *four* subtractions of 5, or there are *four* 5's in 20.

Division may also be regarded as the reverse of multiplication, since one number is contained in another as many times as it is a factor of the other.

ORAL EXERCISE

1. At 5¢ apiece, how many oranges can be bought for 30 cents? For 40 cents? For 35 cents?

2. Mary had 36 cents, and bought 5 oranges at 4¢ each. How many cents had she left?

3. If a field containing 12 acres produces 360 bushels of oats, what is the number of bushels per acre?

4. At $30 an acre, what is the cost of $\frac{1}{2}$ acre of land?

5. John bought 3 dozen oranges at 25 cents a dozen, and gave the man $1. How much change should he receive?

6. Divide 25 apples between two persons, so as to give one seven more than the other.

7. How many pineapples can be bought for $2.16 at 9¢ apiece?

8. If 5 yards of ribbon cost 55 cents, what will 9 yards cost?

9. How many sheep can be bought for $45, if 8 sheep cost $40?

10. I bought 12 yards of cloth for $2.88. What did it cost a yard?

11. The remainder is 6, the divisor 25, the quotient 4. What is the dividend?

12. What cost 75 horses at $75 each?

13. How many weeks in 35 days? In 56 days? In 98 days?

14. If 15 yards of cloth cost $60, for how much per yard must it be sold to gain $30? To gain $15?

15. How many days' labor at $4 a day will pay for 3 tons of coal at $6 a ton, and 5 cords of wood at $3 a cord? Ans. $8\frac{1}{4}$.

16. The product of two numbers is 132, and one of the numbers is 11. What is the other?

17. How long will it take 7 men to do the same work that 14 men can perform in 3 days?

18. The quotient of one number divided by another is 8. What would be the quotient if the divisor were multiplied by 2?

19. What is $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$ of 100?

20. How is one-half of any number or thing found? One-fourth? One-fifth? One-eighth?

21. The quotient is 15. If the dividend and divisor were each multiplied by 5, what would the quotient be?

77. It will be seen that the answer to the 15th problem in the above list is not a whole number, but that the remainder should be written as a part of the quotient by placing the divisor under it with a line between them.

One or more of the equal parts of a number is called a *fraction*. The names of the equal parts of a thing vary according to the *number* of these parts. Thus, $\frac{1}{2}$ is read *one-half;* $\frac{1}{4}$ is read *one-fourth;* $\frac{3}{4}$ is read *three-fourths*, etc.

The remainder cannot always be written as a part of the quotient. Thus, "How many calves at $6 a head can be bought for $25?" Evidently we cannot say, "$4\frac{1}{6}$ calves," but "4 calves and $1 remaining."

WRITTEN EXERCISE

78. Example 1. Divide 8742 by 6.

6)8742
  1457

Solution. Write the divisor at the left of the dividend with a curved line between them. Then 6 is contained in 8 one time with a remainder 2. Prefix 2 to 7 and you will have 27. Then 27 ÷ 6 = 4 and a remainder 3. Prefix 3 to 4 and you will have 34. Then 34 ÷ 6 = 5 and a remainder 4. Prefix 4 to 2 and you will have 42. Then 42 ÷ 6 = 7. Writing the result of the division in its place gives the complete quotient, 1457.

79. Proof. Find the product of the divisor and quotient and to this add the remainder if there is one. The result will be the dividend.

80. When the divisor is so small that the division may be performed mentally, as in the above example, the process is called **short division.**

Example 2. Divide 9730 by 35.

```
35)9730(278
   70
   ---
   273
   245
   ---
    280
    280
    ---
```

Solution. Write the divisor at the left of the dividend with a curved line between them. Then we find that the fewest figures which will contain the divisor are 97. $97 \div 35 = 2$ with 27 for a remainder. To this annex the next figure of the dividend, 3, and we have 273.

$273 \div 35 = 7$ with 28 for a remainder. To this remainder annex the next figure in the quotient, 0, and we have 280. Then we find that $280 \div 35 = 8$, with no remainder. These several quotients arranged as indicated in the illustration are the complete quotient, 278.

81. **Long division** is the process of dividing when the subtractions are *written*.

WRITTEN EXERCISE

1. Divide 87653 by 3.
2. Divide 54006 by 6.
3. Divide 15096 by 8.
4. Divide 605675 by 9.
5. Divide 760344 by 12.
6. Divide 1041835 by 11.
7. Divide 453880 by 25.
8. Divide 124864 by 32.
9. Divide $938.07 by 63.
10. Divide $3176.46 by 126.
11. Divide 5120401 by 587.
12. Divide 1041835 by 204.
13. $\frac{54320}{5} = ?$
14. $\frac{46840}{4} = ?$
15. $\frac{75399}{3} = ?$
16. $\frac{91280}{8} = ?$
17. $\frac{46277}{7} = ?$
18. $\frac{67894}{12} = ?$
19. $\frac{23985}{15} = ?$
20. $\frac{31364}{6} = ?$
21. $\frac{54657}{123} = ?$
22. Divide 2975019839 by 5423.
23. Divide 24715454386504, using 7 as a divisor 10 times in succession.
24. Of what number is 233 both divisor and quotient?
25. What will be the cost of 2547 feet of lumber at $7.50 for 283 feet?
26. A man buys 8 cows at $32 each and 5 cows at $27 each. At what price per head must he sell them to gain $51 on all?
27. At $66 an acre, how many acres of land can be bought for $2200?
28. How many tons of coal at $6¼ a ton can be bought for $37.50?

29. A farmer gave 49 bushels of oats for 196 pounds of cheese at 9¢ a pound; how much did each bushel of oats sell for?

30. A grocer bought a quarter of a ton of sugar for $40, and sold it at 9¢ a pound. How much did he gain?

31. Sixty loaves of bread were sold for $4.20 which was 60 cents less than cost. Find cost of 48 loaves.

32. The remainder is 43, the divisor 79, and the quotient one hundred three thousand two hundred forty-nine. Find the dividend.

33. A man made a fence across a garden that is 12 yards wide. If he puts in posts 2 yards apart to support the railing, how many posts did he use, provided there was a post at each end?

34. A farmer bought land from one man at $65 an acre and the same number of acres from another at $74 an acre. The whole amount paid was $66,025. How many acres did he purchase from each?

35. A woman has three children. She pays for each $15 a year for having their clothes made, $1.50 a month for mending, and $.35 a week for washing. How much could she save in a year if she knew how to wash, make clothes, and mend?

36. The *Northwest* sailed from Buffalo to Duluth via Mackinaw, a distance of 1100 miles, in 60 hours. If she stopped at Cleveland 4 hours and 20 minutes, at Detroit 4 hours, at Mackinac Island 1 hour and 40 minutes, what was her average rate per hour?

37. John runs 32 rods in a minute, and Henry pursues him at the rate of 44 rods in a minute; how long will it take Henry to overtake John, if John has 9 minutes the start?

38. A merchant bought 80 yards of linen at 55 cents per yard, and 200 yards of muslin at 20 cents per yard. He sold all the linen at 50 cents per yard; what must he charge per yard for the muslin in order to make up exactly his loss on the linen?

Short Methods of Division

CASE I

82. To divide any number by 10, 100, 1000, etc.

83. Removing any order of figures one place to the right, diminishes its value *ten times*, or divides by 10. For similar reasons, cutting off two figures divides by 100, and so on. The remaining figures are the quotient, and those cut off the remainder.

Example 1. Divide 26783 by 100.
Example 2. Divide 39456 by 1000.

1.

$$1|00\overline{)267|83}$$
267.83 rem., or $267\frac{83}{100}$

2.

$$1|000\overline{)39|456}$$
39.456 rem., or $39\frac{456}{1000}$

CASE II

84. When the divisor has ciphers on the right.

Example. Divide 29475 by 400.

$$4|00\overline{)294|75}$$
73.275 rem.

Solution. Resolve 400 into the factors 100 and 4. First divide by 100 by cutting off the two right-hand figures of the dividend. The quotient is 294 and a remainder of 75. Divide 294 by 4 and we have a quotient of 73 and a remainder of 2 hundreds, which prefixed to the first remainder 75, gives a *true* remainder of 275. Hence, the quotient $73\frac{275}{400}$.

WRITTEN EXERCISE

1. Divide 3425 by 100.
2. Divide 65705 by 1000.
3. Divide 457400 by 100.
4. Divide 74280 by 2000.
5. Divide 521076 by 500.
6. Divide 543 by 1000.
7. Divide 4536870 by 5600.
8. Divide 900700652 by 12300.

CASE III

85. To divide by the factors of a number.

Example 1. Divide 7840 by 32.

$$4\overline{)7840}$$
$$8\overline{)1960}$$
245

Solution. The divisor, 32, is equal to 4 times 8. Now 7840 ÷ 4 = 1960; but this quotient is 8 times too great, and must therefore be divided by 8. Hence, 1960 ÷ 8 = 245, the true quotient.

Example 2. Divide 37625 by 42, using its factors 2, 3, 7.

2)37625
3)18812..............1
7) 6270........2×2=4
895....5×3×2=30
True remainder....35

Solution. It will be seen that 42 is equal to 2×3×7. Then we may divide by 42, or by its factors 2, 3, and 7.

37625÷2=18812, and a remainder of 1, which must be a part of the true remainder. 18812÷3=6270, and a remainder of 2. Since a unit of the first quotient 18812, equals 2 units of the dividend, this second remainder 2 being a part of 18812, equals 2×2, or 4 units of the dividend.

6270÷7=895, and a remainder of 5. This third remainder results from dividing one-sixth of the dividend by 7, hence we must multiply the remainder, 5, by 2 times 3, the preceding divisors. These products and the first remainder added, make the true remainder; and the quotient **895$\frac{35}{42}$.**

WRITTEN EXERCISE

1. Divide 86357 by 42, or by 2, 3, 7.
2. Divide 5432604 by 135, or by 3, 5, 9.
3. Divide 98760034 by 210, or by 2, 3, 5, 7.
4. Divide 769104 by 420, or by 4, 3, 5, 7.

CASE IV

86. To divide when the divisor is one of the equal parts of 10, 100, 1000, etc.

1. **To divide by 5.** *Multiply the dividend by 2 and cut off one figure.*

2. **To divide by 20.** *Multiply the dividend by 5 and cut off two figures.*

3. **To divide by 25.** *Multiply the dividend by 4 and cut off two figures.*

4. **To divide by 50.** *Multiply the dividend by 2 and cut off two figures.*

5. **To divide by 75.** *To the dividend add one-third of it and cut off two figures from the result.*

6. **To divide by 125.** *Multiply the dividend by 8 and cut off three figures.*

7. **To divide by 150.** *From the dividend deduct one-third of it and cut off two figures from the result.*

8. **To divide by 250.** *Multiply the dividend by 4 and cut off three figures.*

It will be seen that this case applies to other divisors than those here given. Some of these, as may be found, are reserved for a subsequent section of the book.

87. As multiplication is the reverse of division, we may multiply by the numbers referred to in the above case by reversing the order of the work. Where we have employed multiplication, we must divide, and in cases where we have made use of division, multiplication must be substituted.

Example 1. Multiply 4326 by 75.
Example 2. Multiply 8203 by 125.

(1)
4)4326|00
108150
324450 Ans.

(2)
8)8203|000
1025375 Ans.

It will be seen in Example 1, that to multiply by 100 gives a result one-fourth too large as 75 is only three-fourths of 100. Hence, we deduct one-fourth of the result and we have the true product. In Example 2, when we multiply by 1000 our product is 8 times too large as 1000 is 8 times 125.

WRITTEN EXERCISE

1. Divide 43520 by 75.
2. Divide 658732 by 25.
3. Divide 98400 by 250.
4. Divide 546325 by 75.
5. Divide 1324860 by 125.
6. Multiply 65432 by 75.
7. Multiply 5432685 by 150.
8. Multiply 6587690 by 25.
9. Divide 546378400 by 150.
10. Divide 12343500 by 166⅔.

NOTE. Consistent practice on the foregoing contractions, in multiplication as well as in division, develops great skill and speed. Practical work is only possible, in the above rule, where the fractional parts of 10, 100, 1000, etc., are known at sight.

WRITTEN REVIEW

1. Write these numbers using Roman numerals: 74, 47, 333, 19, 1913.

2. At two shillings each, how many books can I buy for $20?

3. A clock strikes all the hours in January. How many blows does the hammer give the bell in that time?

4. The product of two numbers is 112112. What would the product have been if both multiplier and multiplicand had been nine times as large?

5. If 593 is subtracted 347 times from a certain number, the remainder is 287. Find the number.

6. Divide 816534936 by 168, using three factors, two of which are 7 and 6.

7. A man employed to chop wood at 80¢ a cord earned $48 in 5 weeks. How much did he cut daily?

8. The remainder is 68849 and the subtrahend 25763, find one-seventh of the minuend.

9. If 37 is added to a certain number, 85 can be subtracted from it 113 times. Find the number.

10. A farmer bought 75 tons of hay at $16 a ton, and gave in part payment 56 sheep at $3.75 each. The remainder he paid in butter at 33¢ a pound. How many pounds of butter were required.

THE VALUE OF SIGNS

88. A **sign** is a character used to indicate an operation, or to express the qualities or relations of numbers.

89. The matter of the *correct* use of the four signs, $+$, $-$, $\times$, and $\div$ is, perhaps, less a question of *right* or *wrong* than it is a question of "what is customary with mathematicians?"

1. Every *plus* or *minus* sign affects all that follows it, whether a parenthesis be used or not, till another plus or minus sign is reached.

2. The operations of multiplication and division must be performed first, and then the results be added or subtracted, according to the last preceding plus or minus sign, for the signs of multiplication and division connect factors while the signs of addition and subtraction connect terms. In other words, obey the signs in this order: $\times$, first; $\div$, second; $+$ or $-$, third. Hence the expression, "$5+11\times2-12\div4$," means not $5+11$, but 5 plus two times 11. This may be a little hard for some to get, but by reading it aloud, making a short pause after the word plus, and then read the "*two times eleven*" more rapidly than the "*5 plus*," you will quickly get the idea. Now we see that $5+11\times2=27$. Next we are to subtract, not 12, but 12 divided by 4, which is 3. Then, $27-3=24$, the correct result.

3. All persons who have studied algebra will understand this, $a+b\times c$, to mean that to the quantity a must be added the result obtained by multiplying b by c. Now if we give these letters values, say $a=5$, $b=11$, and $c=2$, we have $5+11\times2=27$.

90. The **signs of aggregation** are the parentheses ( ), braces { }, brackets [ ], and the vinculum ———. These signs are used to include several numbers which are to be treated as one number. Thus:

$$[86-6\times2+(3\times5)-4]\div5=?$$
$$[86-12+15-4]\div5=?$$
$$85\div5=17.$$

Hence to find the value of such expressions:

1st. Simplify the expressions within the parentheses by performing the operations indicated.

2d. When one parenthesis includes another, remove the inner one first.
3d. Combine the terms.

91. An **equation** is an expression of *equality* between two numbers or sets of numbers. It means that all on the right of the sign of equality is equal to all that is on the left; as, $2\times3+4=2\times5$.

WRITTEN EXERCISE

1. $84-(81\div9)+7=?$
2. $120\div6+30-6\times4=?$
3. $(8\times5\times6-180\div3)\div9=?$
4. $?+39\times2-16\div4=75+7.$
5. $(9\times12-7\times8)+105\div5=?$
6. $[75-7\times6+(3\times5)-6\times3]\div6=?$
7. $(325+45)\times(54\div9)-4+\overline{11-6\times1}+5\div(5\times1)=?$
8. $(9\times7)+(8\times12)-56\div8-(11\times7)=?$
9. $9\times7+12\div6+(7+2\times6)=100-28\times2+?$
10. To the difference of $97.28 and $56.37 add their sum.
11. From 5467 take 3462, multiply the remainder by 42, and divide the product by 5 times 42.
12. Divide 94,068 by 804, subtract the quotient from 89 times 89, and to the remainder add one-eighteenth of 1152.

Definitions and Analysis

92. The **fundamental rules of arithmetic** are addition, subtraction, multiplication, and division. A thorough knowledge of these is essential to the student as they are the basis of all arithmetical computations.

93. A **problem** is a question which requires a solution.

94. A **solution** is the process of explaining a problem.

95. A **rule** is a brief direction for the solution of a problem.

96. An **analysis** is a statement of the different steps in the solution of a problem.

In analyzing it is usual to reason from the *given number* to *one*, and then from *one* to the required number whether we are solving problems in simple numbers, fractions, or decimals. Thus:

If 5 hats cost $20, what will 7 hats cost?

Analysis. If 5 hats cost $20, 1 hat will cost one-fifth of $20, or $4; and since 1 hat costs $4, 7 hats will cost 7 times $4, or $28.

Again, If 6 men can do a piece of work in 8 days, in what time can 16 men do the same work?

Analysis. If 6 men can do a piece of work in 8 days, it would take one man 6 times 8 days, or 48 days, to do it. Since it would take one man 48 days to do the work, 16 men would do it in one-sixteenth of 48 days, which is 3 days.

ORAL EXERCISE

1. If a boy earn 35 cents in one day, how much will he earn in 4 days?
2. If 4 cows are worth $160, what are 12 cows worth?
3. How many men would be required to do the same work in 5 days that 15 men do in 8 days?
4. In 20 quires of paper there are 480 sheets. How many sheets are there in 5 quires?
5. A man earned 40 cents an hour, and a boy earned 15 cents. When the boy had earned 60 cents, what had the man earned? How much had both earned?
6. A boat sailed up stream for 12 hours at the rate of 10 miles an hour, but returned in 6 hours. How fast did it sail down stream? How much faster down stream than up?

UNITED STATES MONEY

97. **Money** is the measure of value used as a medium of exchange. It may consist of anything usually and lawfully employed in buying and selling.

98. **United States money** is the legal currency of the United States. It consists of gold coins, silver dollars, subsidiary coins (small change), gold certificates, silver certificates, United States notes (greenbacks), treasury notes (issued 1890), and national bank notes.

99. The legal coins of the United States are as follows:

1. *Gold.*—Double eagle, eagle, half eagle, and quarter eagle.
2. *Silver.*—Dollar, half dollar, quarter dollar, and dime.
3. The *nickel* five-cent piece and the *bronze* one-cent piece.

100. The paper money of the United States is as follows:

1. *United States Notes.*—They consist of promises of the government to pay to the bearer on demand a definite number of silver or gold dollars.
2. *Gold Certificates.*—Issued for gold deposited in the U. S. Treasury.
3. *Silver Certificates.*—Issued for silver deposited in the U. S. Treasury.
4. *Treasury Notes.*—Issued for silver bullion deposited in the U. S. Treasury. These notes are not now issued.
5. *National Bank Notes.*—Notes secured by U. S. bonds, and issued by national banks under the supervision of the government.

101. The term **legal tender** is applied to money which may be legally offered in the payment of debts.

All gold coins and silver dollars are *legal tender* for any amount. Silver coins, less than the dollar, are legal tender in amounts not exceeding ten dollars. Nickel and copper pieces are legal tender in amounts not exceeding twenty-five cents. Gold and silver certificates are not legal tender, but United States Treasury notes are legal tender in any amount. United States notes or "greenbacks" are legal tender in any amount, except for duties on imports and interest on the public debt. National bank notes are not legal tender.

In actual practice the national bank notes and the gold and the silver certificates are taken without question and pass as freely as any other kind of money, and their acceptance constitutes good payment.

102. The units or denominations of United States money are mill, cent, dime, dollar, and eagle. The unit of value is the gold dollar of 25.8 grains. The silver dollar contains 412.5 grains.

NOTE. The gold dollar is no longer coined.

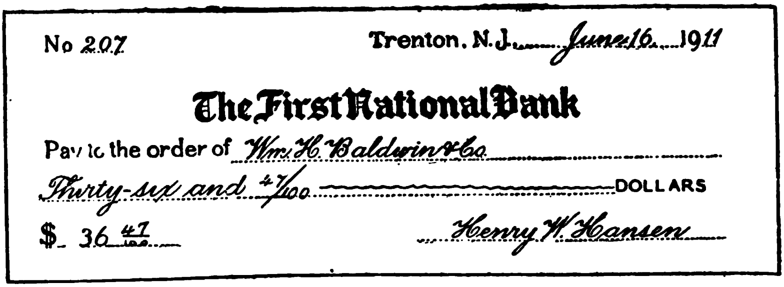
Table of United States Money

10 mills (m) = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

1. The units and the table as given above were established by an act of Congress, August 8, 1786.

2. Dollars and cents are principally used in business. The mill is not a coin, but 5 mills are frequently written ½ of a cent. Thus, $.375 = $.37½.

103. It is customary to express cents on negotiable paper, such as notes, checks, etc., as hundredths of a dollar, as in the following check:



No 207 Trenton, N.J. June 16 1911

The First National Bank

Pay to the order of Wm. H. Baldwin & Co.

Thirty-six and $\frac{47}{100}$ DOLLARS

$ 36 $\frac{47}{100}$ Henry W. Hansen

104. The gold and silver coins of the United States contain 9 parts, by weight, of pure metal and 1 part alloy. The alloy of *gold coins* is copper, or copper and silver; of *silver coins*, copper. The 1-cent piece is $\frac{19}{20}$ copper and $\frac{1}{20}$ tin and zinc, and the 5-cent piece $\frac{3}{4}$ copper and $\frac{1}{4}$ nickel.

105. Bullion is the pure uncoined gold and silver in the mass, and varies from coin value only by the charges for coinage made by the mint.

Notation of United States Money

106. United States currency is expressed according to the decimal system of notation and increases and decreases on a scale of ten.

107. The **dollar sign ($)** should always be written before the number which indicates dollars. If there are cents and mills they should form the fractional part of the number and be written to the right with a *decimal point* (.) between. Thus:

1. Two hundred thirty-six dollars, eighty-seven cents is written $236.87.
2. As cents are expressed as hundredths of a dollar, when the number of cents is less than 10, a cipher must be written in the first place to the right of the decimal point; as, $23.05.
3. In business calculations, if the number of mills in the final result is 5 or more, they are regarded as equal to 1 cent; but if less than 5, they are rejected.

Reduction of United States Money

108. Reduction is the process of changing the form of a number without altering its value. Hence, we may change dollars to cents or cents to dollars.

109. Rule. *Since there are 100 cents in a dollar, to reduce dollars to cents multiply the given dollars by 100, or if the number is expressed in dollars and cents remove the decimal point two places to the right. To reduce cents to dollars divide by 100 by removing the decimal point two places to the left.*

WRITTEN EXERCISE

Reduce:

1. 8 dollars to cents.
2. 654 dollars to cents.
3. $45.76 to cents.
4. 7600 cents to dollars.
5. 10010 cents to dollars.
6. $4532.05 to cents.
7. 610100 cents to dollars.
8. 8960\frac{1}{2}$ dollars to cents.
9. 4536\frac{3}{4}$ dollars to cents.
10. 2345 mills to dollars.

Express the following amounts by figures:

11. Twenty-seven dollars, sixteen cents, five mills.
12. Five thousand five dollars, five cents.
13. Three thousand three hundred dollars, eight cents.
14. One hundred fifty dollars, seven cents, five mills.
15. Nine hundred dollars, nine cents.
16. Nine hundred nine dollars, nine cents, nine mills.

Change the following:

17. 85 cents to mills.
18. 154 cents to mills.
19. 5674 dollars to mills.
20. $453 to mills.
21. $10.054 to mills.
22. 2345 mills to dollars.

To Add or Subtract in United States Money

110. Rule. *Write the numbers, placing dollars under dollars, cents under cents. Add or subtract as in simple numbers, placing the decimal point in the result directly under the points in the numbers added or subtracted.*

NOTE. In subtraction of U. S. money, if there are cents in the subtrahend and none in the minuend, suppose ciphers to be added to the minuend when subtracting.

WRITTEN EXERCISE

Carefully copy and add the following:

(1)	(2)	(3)	(4)
$456780.62	$342100.89	$948672.66	$345.005
3456.06	4532.67	143266.78	22.33
123894.89	23900.65	23452.63	1.56
32462.04	3496.99	6345.72	22.375
5693.57	346.87	284.63	10.45
845.45	306.26	306.62	5.875
856346.09	67405.87	853950.27	123.675
54320.56	2300.56	20004.99	22.25
1895.67	204.15	8885.15	10.87
65430.35	34.16	532.99	2.99

5. Add 9 dollars, 18 cents; 632 dollars, 30 cents; 87 cents; 909 dollars, 40 cents; 5006 dollars, 87 cents, 5 mills; 345 dollars, 57 cents, 5 mills; 75 cents; 90 dollars.

6. Add $98, $654, $235.87, $756.25, $6.54, $13.89.
7. Add $6543, $239.45, $.05, $6, $175, $85.34, $99.99.
8. Add $768.05, $234.67, $.25, $.09, $.72, $1.45, $40.07.
9. From $745698.05 subtract $97603.33.
10. From 4324 dollars, 45 cents subtract 3987 dollars.
11. Balance in bank March 1, $456.34. Deposited Mar. 10, $35.75; Mar. 31, $345; April 8, $78.65; May 1, $234.07; May 18, $134.50; May 28, $45.45; May 31, $239. Checks from March 1 to June 1, $65; $23.98; $345.65; $43.45; $32.54; $222.25. What is the balance in bank June 1?

To Multiply United States Money

111. **Rule.** *Multiply as in simple numbers without reference to the decimal point, and from the right of the product point off as many figures as there are figures to the right of the decimal point in the multiplicand.*

NOTE. In the multiplication of money the multiplier must be considered as an abstract quantity as money is a concrete expression. However, the terms may be interchanged for convenience.

Example. Find the cost of 23 horses at $153.25 each.

```
$153.25
     23
-------
  45975
 30650
-------
$3524.75
```

Solution. Multiply as in ordinary multiplication, without reference to the decimal point. Then, from the right of the product point off two places for the cents in the multiplicand.

WRITTEN EXERCISE

Multiply:

1. $156.23 by 5.
2. $76007.06 by 432.
3. 24 dollars, 5¢ by 16.
4. $.56 by 564; by 234; by 67.
5. $.07 by 456; by 239; by 238.
6. $48.08 by 54; by 99; by 999.

To Divide in United States Money

112. **Rule.** *Divide as in simple numbers without reference to the decimal point, and from the right of the quotient point off as many places as the places in the dividend exceed those in the divisor.*

NOTE. When the divisor alone contains cents, two ciphers should be annexed to the dividend before beginning to divide; or reduce both dividend and divisor to cents by annexing ciphers and omitting the decimal point.

Example. If 36 men earn $155.52 in one day, how much does one earn?

```
36)155.52($4.32
   144
   ---
    115
    108
    ---
      72
      72
```

Solution. Since 36 men earn $155.52, 1 man will earn as many as 36 is contained times in $155.52, or $4.32.

It will be seen that the dividend contains cents, therefore two places must be pointed off from the right of the quotient as shown in the illustration.

WRITTEN EXERCISE

1. Divide $45.54 by 9.
2. Divide $234.15 by 35.
3. Divide $2054.79 by 79.
4. Divide $1884 by 75.
5. Divide $149.04 by $2.07.
6. Divide $1728 by $.75.
7. Divide $3456 by $2.25.
8. Divide $63 by $2.625.
9. A man received $51.52 for 16 days' work; how much was that a day?
10. A man paid $30360 for 24 city lots. What was the average price per lot?
11. By selling land at $37 an acre that cost $31 an acre I gained $198; how many acres did I sell?
12. Two men paid $7000 for a farm. One took 80 acres at $40 an acre, the other the remainder at $25 an acre; how many acres in all?

WRITTEN REVIEW

1. Arrange in columns and add: $4.60, $7.25, $2130, $19.06, $8.52, and $1.87.
2. A grocer bought 87 bu. of potatoes at 50¢ a bushel, and retailed them at 18 cents a peck. How much did he gain?
3. How many yards of cotton at 12¢ a yard can you get for 8 lbs. of butter at 24¢ a pound?
4. If 17 pounds of sugar cost 51¢, how many pounds can you buy for $1?
5. If I get $180 for working 50 days of 4 hours each; how much is that per hour?

6. A owed me $534; he gave me 620 bu. of wheat at 80¢ a bushel. How much is yet due me?

7. B's income is $600. He spends 75¢ a day; how much will he save in 365 days?

8. Two men together receive $97.75, but one receives $18.25 more than the other. How much does each receive?

9. A grocer bought a barrel of molasses containing 36 gallons at 45¢ a gallon, and sold it at 55¢ a gallon; how much did he gain?

10. A man earns $75 a month, and pays $18 a month for house rent, and $45 a month for other expenses. How much can he save in 2 years?

11. A man is hired for six weeks but loses 5 days through sickness. He pays 50¢ a day for board, and earns $1.50 a day; how much does he save above his board in the 6 weeks?

12. 5 pounds of rice, and 7 pounds of dates together cost 95¢. If the rice cost 5¢ a pound, how much a pound did the dates cost?

13. $9\times7+8\times12-11\times12+27\div3=?$

14. A dairyman has fodder enough to keep 25 cows 6 months. If he sells 10 cows, how many months will the fodder last the rest?

15. Mr. B. invested $4000 in business. The first year he gained $765, and the second year he lost $375. The next two years his average yearly gain was $964. How much was he worth at the end of the four years?

16. I bought 16 yards of silk at $1.80 a yard; 31 yards of cotton at 12¢ a yard; a pair of gloves at $1.75; 4 quires of paper at 15¢ a quire; 5 books at 87¢ each; an album at $2.35; 4 magazines at an average cost of 12½¢ each. Find the amount of my bill.

PROPERTIES OF NUMBERS

ORAL EXERCISE

1. What two numbers, besides the number itself and 1, will give a product of 10? 16? 25? 28? 32?

2. What numbers will exactly divide 8? 16? 21? 36?

3. Of what sets of two numbers is 24 the product?

4. Of what sets of three numbers is 27 the product? 36? 45?

5. What are the factors of a number?

6. Of what number are 5 and 3 the factors? 2, 3, and 3. 3, 5, and 7? 3, 3, and 7? 2, 5, 7, and 9?

7. Name the numbers between 3 and 30, that have no other factors than the numbers themselves and 1.

8. Name all the numbers between 10 and 25 that have other factors than themselves and 1.

113. A number that has no exact divisor except itself and 1 is called a **prime number.**

Thus, 1, 2, 3, 5, 7, 11, etc., are *prime numbers.*
2 is the only even prime number

114. A number that has exact divisors besides itself and 1 is called a **composite number.**

Thus, 3 and 5 are factors of 15; 2, 3, and 7, of 42.

115. Integral numbers are: *prime* or *composite; even* or *odd.*

116. A **prime factor** is a prime number used as a factor.

117. A **composite factor** is a composite number used as a factor.

118. The **factors** of a number are those numbers which, multiplied together, produce that number.

The terms, ***number***, ***factor***, ***divisor***, ***multiple***, are used in the sense of ***integers***. See sec. 4.

119. An **even number** is a number that can be divided by 2 without a remainder; as, 6, 12, 26.

All *even* numbers end with 0, 2, 4, 6, or 8.

120. An **odd number** is a number that cannot be divided by 2 without a remainder; as, 7, 11, 23.

All *odd* numbers end with 1, 3, 5, 7, or 9.

121. An **exact divisor** is a number that divides any given number without a remainder.

The *exact divisors* of a number are also the factors of that number.

The terms *factor* and *divisor* differ only in use; *factor* suggesting the process of *multiplication*, and *divisor* the process of *division*.

Tests of Divisibility of Numbers

122. One number is said to be *divisible* by another when there is no *remainder*. Any number is divisible by:

1. *Two*, if it is an *even* number.
2. *Three*, if the sum of its digits is divisible by 3.
3. *Four*, if its two right-hand figures are ciphers, or express a number divisible by 4.
4. *Five*, if it ends with a cipher or 5.
5. *Six*, if it is an *even* number and divisible by 3.
6. *Eight*, if its three right-hand figures are ciphers, or express a number divisible by 8.
7. *Nine*, if the sum of its digits is divisible by 9.
8. *Ten*, if its right-hand figure is a cipher.
9. *Seven*, *eleven*, and *thirteen*, if it consists of but four places, the first and fourth being occupied by the same significant figures, and the second and third by ciphers; as, 1001, 4004, 5005, 9009, are divisible by 7, 11, and 13.

NOTE. A number which will not divide any given number without a remainder is not divisible by any multiple of that divisor.

Factoring

123. **Factoring** is the process of separating or resolving a composite number into its factors.

NOTE. It is not customary to consider the unit 1 and the number itself as factors; if they were, all numbers would be composite.

124. Principles. *1. Every composite number is equal to the product of its prime factors.*

2. A factor of a number is a factor also of any number of times that number.

3. A factor common to two or more numbers is a factor of their sum, and also of the difference of any two of them.

4. The prime factors of a number, or the product of any two or more of them, are the only exact divisors of that number.

125. To find all the prime factors of a composite number.

Example. What are the prime factors of 4620?

$$\begin{array}{r} 2\underline{)4620} \\ 2\underline{)2310} \\ 3\underline{)1155} \\ 5\underline{)385} \\ 7\underline{)77} \\ 11 \end{array}$$

Solution. Since the given number is even, divide it by 2, the least prime factor, and this result by 2, which gives an odd number for a quotient.

Next divide by the prime factors 3, 5, and 7, successively, obtaining for the last quotient 11, which is not divisible. Hence the divisors 2, 2, 3, 5, 7, and the last quotient 11, are the prime factors, or divisors of 4620.

126. Rule. *Divide the given number by any prime factor; then divide this quotient by another prime factor, and so on until the quotient obtained is a prime number. The several divisors, with the last quotient, are the prime factors required.*

ORAL EXERCISE

1. What is the only *even* prime number?
2. When are 2 numbers prime to each other?
3. Name three composite numbers which are prime to each other.
4. Name an *odd* number that is composite. An *even* number that is prime.
5. Prove that to divide by 625 is the same as to multiply by 16 and divide by 10000.
6. The quotient of a division is 834. What quotient would have been obtained if both dividend and divisor had been first multiplied by 13? Why?

WRITTEN EXERCISE

Find the prime factors of:

1. 135.
2. 252.
3. 951.
4. 1155.
5. 6300.
6. 2205.
7. 13981.
8. 21504.
9. 29925.
10. 12673.
11. 10010.
12. 31570.

Greatest Common Divisor

127. A **common divisor** of two or more numbers is a number that will exactly divide each of them.

128. The **greatest common divisor** of two or more numbers is the greatest number that will exactly divide each of them.

Thus, 8 is the *greatest common divisor* of 16, and 24; since it is the greatest number that divides each of them without a remainder.

129. Numbers are **prime to each other** when they have no common factors, or divisors.

Thus, 5 and 12 are prime to each other; so, also, are 7 and 15; 16 and 25, etc.

130. To find the greatest common divisor of two or more numbers.

Example. Find the greatest common divisor of 45, 75, and 90.

$$\begin{array}{r|l} 5 & 45-75-90 \\ \hline 3 & 9-15-18 \\ \hline & 3-\ 5-\ 6 \end{array}$$

Solution. Since the given numbers are exactly divisible by 5, and the resulting quotients by 3, they are also divisible by 3×5, or 15.

It may be readily seen that the last quotients 3, 5, and 6 have no common divisor as they are prime to each other. Hence:

131. Rule. *Write the numbers in a line, and divide by any prime factor common to all the numbers. Divide the successive quotients in like manner, and so continue the division till all the quotients are prime to each other. The product of all the divisors will be the greatest common divisor.*

132. Euclid, a famous Greek geometer, who lived about 300 B.C., gave the method of finding the g. c. d. by division. This method is useful if the prime factors of the numbers cannot be readily found.

133. To find the greatest common divisor of two or more numbers when they are not readily factored.

Example. Find the greatest common divisor of 679 and 1869.

```
679)1869(2
     1358
      511)679(1
          511
          168)511(3
              504
                7)168
                   24
```

Solution. The greatest common divisor of two or more numbers cannot be greater than the least of them. Dividing 1869, the greater number by 679, the less, we find that 679 is not a common divisor of the two numbers, as we have a remainder of 511; hence, we continue dividing the last divisor by the last remainder until there is no remainder. The last divisor is the *greatest common* divisor. Therefore the greatest common divisor of 679 and 1869 is 7.

NOTE. If more than two numbers are given, first find the greatest common divisor of two of them, and then of this divisor and one of the remaining numbers, and so on to the last. The last common divisor found is the g. c. d. of all the given numbers. If the remainder at any time is a prime number, and it is not contained in the last divisor, there is no common divisor greater than 1; it will therefore be useless to continue the division.

WRITTEN EXERCISE

Find the greatest common divisor of:

1. 33, 45, 66.
2. 40, 75, 100.
3. 72, 126, 216.
4. 825, 1372.
5. 2041, 8476.
6. 620, 1116, 1488.

7. A man has a lot the sides of which are 168, 182, 280, and 252 feet, respectively. He wishes to fence it with boards of equal length; what is the greatest length he can use?

8. A grain dealer put 312 bushels of corn, 376 bushels of oats, and 268 bushels of wheat into the largest possible sacks of equal size. What did each sack contain?

9. Three persons have respectively $630, $1134, and $1386, with which they agree to purchase horses at the highest price per head, that will allow each man to invest all his money. How many horses can each man buy?

10. Without actual division, determine what numbers less than 19 (except 7, 13, 14, 17) will divide 586080.

Multiples

ORAL EXERCISE

1. Name all the prime factors of 8; of 12; of 4 times 6.

2. Name a factor that is common to 35 and 49.

3. What numbers less than 50 are exactly divisible by 7?

4. Name a multiple of 5; of 7; of 9; of 11; of 15; of 75.

5. Name some numbers exactly divisible by 4 and 6; by 3 and 7; by 5 and 7; by 7 and 9.

6. By what prime numbers can 33 be divided? 42? 56?

7. Name some numbers of which 3 and 5 are factors.

8. Find the least number exactly divisible by 2, 5, and 7.

Least Common Multiple

134. A **multiple** of a number is a number that is exactly divisible by it.

Thus, 21 is a multiple of 7.

135. A **common multiple** of two or more numbers is a number exactly divisible by each of them.

Thus, 30 is a common multiple of 2, 3, 5, 10, and 15.

136. The **least common multiple** of two or more numbers is the least number exactly divisible by each of them.

Thus, 36 is the least common multiple of 4, 6, and 9.

137. Principles. *1. A multiple of a number contains all the prime factors of that number.*

2. A common multiple of two or more numbers may be found by multiplying the given numbers together.

3. The least common multiple of two or more numbers is the least number that contains all the prime factors of each of those numbers.

4. Two or more numbers may have any number of common multiples but only one least common multiple.

138. To find the Least Common Multiple of two or more numbers.

Example. Find the least common multiple of 30, 42, and 60.

$30 = 2 \times 3 \times 5$
$42 = 2 \times 3 \times 7$
$60 = 2 \times 2 \times 3 \times 5$
$2 \times 2 \times 3 \times 5 \times 7 = 420$

Explanation. The least common multiple cannot be less than the largest number 60, since it must contain 60; hence it must contain all the prime factors of 60, which are 2, 2, 3, and 5, as well as the prime factors of the other numbers, not found in 60. It will be seen that 7 is the only prime factor not found. Therefore the product of 2, 2, 3, 5, and 7 is the least common multiple of the numbers, or 420.

$$\begin{array}{r|l} 2 & 30—42—60 \\ \hline 3 & 15—21—30 \\ \hline 5 & \ \ 5— \ \ 7—10 \\ \hline & \ \ 1— \ \ 7— \ \ 2 \end{array}$$

$2 \times 3 \times 5 \times 7 \times 2 = 420$

Solution. Divide the given numbers by the prime factor 2 as indicated in the margin, since 2 is a prime factor of two or more of the given numbers. Write the quotients underneath and so continue the division until there is no prime factor which will divide two or more of them.

It will be seen in the third division by 5 that 7 is not a multiple of 5, therefore we bring it to the line below. Hence the product of the several divisors 2, 3, 5, and the remaining quotients 7 and 2 will be the least common multiple.

139. In finding the least common multiple of numbers, all numbers that are factors of other given numbers, may be disregarded.

Thus, the common multiples of 5, 10, 15, 30, 60, 80, and 120 are the same as the common multiples of 60, 80, and 120.

140. Rule. *Write the numbers in a horizontal line with a line between them and divide by any prime factor that will divide two or more of them without a remainder.*

In like manner divide the quotients obtained, and continue the process until the quotients are prime to each other. The product of the divisors and the final undivided quotients is the least common multiple.

A Criticism. Up to this stage of the work the pupil should know addition, subtraction, multiplication, and division and their phraseology, but they have never called a product or a dividend a multiple, nor is there any reason why they should have done so, for the term multiple is confusing. It means dividend, and dividend implies that there is somewhere a divisor. *Common dividend* implies that numbers have more than one dividend, and that the least of these should be called the *Least Common Dividend,* but for the sake of harmonizing with other books on arithmetic we continue to call it the *Least Common Multiple.*

WRITTEN EXERCISE

Find the least common multiple of:

1. 30, 42, 66.
2. 18, 24, 54.
3. 16, 48, 80, 32, 66.
4. 16, 60, 140, 210.
5. Find the least common multiple of all the *odd* numbers between 17 and 25, inclusive.
6. Find the least common multiple of all the *composite* numbers between 15 and 25, inclusive.
7. Find the least number of which 20, 21, 24, 60, and 84 are factors.
8. Show the prime factors that make the l. c. m. and g. c. d. of the following numbers: 36, 42, 54.
9. Four men start at the same place to walk around a garden. A can go around in 9 minutes, B in 10 minutes, C in 12 minutes, and D in 15 minutes; in what time will they all meet at the starting point?
10. What is the product of the l. c. m. of 12, 16, 24, and 32, multiplied by their g. c. d.?

CANCELLATION

ORAL EXERCISE

1. What factor remains if 3, a factor of 12, is omitted?
2. Divide 36 by 12. Divide one-fourth of 36 by 3.
3. What is the difference between the quotient of 48 by 12 and the quotient of one-fourth of 48 by one-fourth of 12?
4. What is the quotient of $5 \times 4 \times 7$ divided by 4×5? $7 \times 4 \times 3$ divided by $7 \times 2 \times 3$?
5. $(3 \times 16) \div (4 \times 3) = 16 \div 4$. Why?

141. Cancellation is the process of shortening operations in division by striking out common factors from both dividend and divisor, and using only the remaining factors.

142. **Principle.** *Rejecting equal factors from both dividend and divisor does not affect the value of the quotient.*

Example. Divide the product of 8, 18, 15, and 5 by the product of 4, 3, 25, and 10.

$$\frac{\overset{\cancel{2}}{\cancel{8}}\times\overset{6}{\cancel{18}}\times\overset{3}{\cancel{15}}\times\cancel{5}}{\cancel{4}\times\cancel{3}\times\underset{\cancel{5}}{\cancel{25}}\times\underset{5}{\cancel{10}}}=\frac{6\times3}{5}=\frac{18}{5}=3\frac{3}{5}$$

Solution. As per the illustration indicate the operations to be performed by writing the dividend above and the divisor below a horizontal line. Begin the reduction by dividing 4 into 8 and write 2 in the dividend, then 3 into 18 and write 6 in the dividend. Next 5 into 25 and write 5 in the divisor, then 5 into 15 and write 3 in the dividend, and finally 2 into 10 and write 5 in the divisor as all the common factors are cancelled. The product of the remaining factors of the dividend, divided by the remaining factors of the divisor, is the required quotient, or $3\frac{3}{5}$.

143. **Rule.** *Write the numbers forming the dividend above a horizontal line, and the numbers forming the divisor below the line, then cancel all the factors common to both dividend and divisor.*

Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.

Note. When equal factors cancel each other, the unit 1 remains, since a number divided by itself gives a quotient of 1. If the 1 belongs in the divisor it need not be retained, since it cannot affect the quotient; but if it belongs to the dividend, it must be retained, since it may be the only factor of the quotient.

WRITTEN EXERCISE

1. Divide 13×7×5×3 by 3×5×7.
2. Divide 17×14×13×12 by 7×3×26×34.
3. Divide 42×18×6×4 by 36×21×6.
4. What is the quotient of 140×39×13×7, divided by 7×26×21?
5. How many times is 81×35 contained in 90×21×3?
6. Find the quotient of (7×153×19)÷(7×228×14).
7. How many times is the continued product of 14, 9, 3, 20, 5, and 6 contained in the continued product of 183, 18, 70, 12, and 5?

8. 26×39×16×2×3 divided by 13×3×8×2×4=?

9. 3×75×125×33×28 divided by 3×14×25×150×22=?

10. 84×96×108×27 divided by 14×12×9×16×3 = ?

11. How many bushels of corn at 55¢ a bushel will pay for 9 tons of coal at $6.80 a ton?

12. At $.85 a day, how long will it take a boy to earn as much as a man earns in 35 days at $2.55 a day?

13. A farmer exchanged 240 bushels of corn, worth 75¢ a bushel, for an equal number of bushels of wheat, worth $1 a bushel, and oats, worth 50¢ a bushel. How many bushels of each did he receive?

14. How many chests of tea, each containing 60 pounds, at 32¢ a pound, must be given for 30 firkins of butter, each containing 56 pounds, at 24¢ a pound?

WRITTEN REVIEW

1. From the sum of 26348 and 14275 take their difference.

2. A grocer bought two kinds of syrup, one for 54¢ a gallon, and the other for 62¢. What was the average cost per gallon?

3. Divide $21 between A and B, so that A will have $3 more than B.

4. How many times can I spend three-fourths of a dollar, if I have $69.75?

5. The product of four numbers is 7776, and three of the numbers are respectively 24, 9, 3. Find the other number.

6. Divide the product of 16×34×7×21×5 by 8×17×6×7.

7. The sum of two numbers is 289, and one number is greater than the other by 25; find the numbers.

8. Seven men engage to pave a street in 42 days, but owing to sickness one man is unable to work. How long will it take the remainder of the men to do the work?

9. James had 25¢, gave John 11, when John had 8 more than James had left. How many had John at first?

COMMON FRACTIONS

ORAL EXERCISE

1. When anything is divided into *two* equal parts, what is each part called? Into *three* equal parts? Into *four* equal parts? Into *six* equal parts? Into *nine* equal parts? Into *twelve* equal parts? Into *sixteen* equal parts?

2. How many *fourths* are in a unit, or in anything? How many *fifths?* How many *sevenths?* How many *eighths?* How many *twelfths?*

3. What is meant by one-half of a unit? By one-sixth? By three-fourths? By five-sixths? By nine-tenths?

4. What are 4 of the 7 equal parts of a week called?

5. Which is greater, one-half or one-third of a unit? One-fourth, or one-fifth? One-sixth, or one-eighth?

6. Which are the smaller parts of a unit, thirds or fourths? Sixths or sevenths? Eighths or tenths?

7. Read the following fractions in the order of their size, the smallest first: $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{12}$.

144. A **common fraction** is the expression of one or more of the equal parts of a unit.

NOTE. A fraction is only another form for an expression of division, the number above the line being the dividend, and the number below the line the divisor.

145. The number written above the line is called the **numerator** of the fraction, and the number written below the line is called the **denominator.**

146. The *denominator* tells into how many parts the number is divided, the *numerator* tells how many of these parts are taken.

Thus, in the fraction, $\frac{2}{3}$, 3 tells into how many parts the unit is divided while the 2 shows how many of these parts have been taken.

147. Principles. *1. The greater the number of equal parts into which a unit is divided, the less is the value of each part.*

2. The less the number of equal parts into which a unit is divided, the greater is the value of each part.

148. The **terms** of a fraction are its numerator and denominator taken together.

Thus, 3 and 4 are the *terms* of the fraction $\frac{3}{4}$.

149. A **fractional unit** is one of the equal parts into which any unit is divided.

Thus, $\frac{1}{3}$ is the *fractional unit* of $\frac{2}{3}$; $\frac{1}{8}$, of $\frac{5}{8}$. Numbers formed from these units are called *fractional numbers;* as, $\frac{2}{3}$, $\frac{5}{8}$.

150. To read fractions, name first the number of fractional units, and then the kind of them.

Thus, $\frac{1}{3}$, $\frac{4}{7}$, $\frac{5}{9}$, are read, *one-third, four-sevenths, five-ninths.*

Classes of Fractions

151. According to *value,* fractions are divided into *proper, improper,* and *mixed* numbers. As to *form* into *simple, complex,* and *compound.*

152. A **proper fraction** is one whose numerator is less than its denominator; as, $\frac{3}{4}$, $\frac{6}{7}$, $\frac{5}{11}$.

153. An **improper fraction** is one whose numerator equals or exceeds its denominator. Its value is equal to, or greater than a unit; as, $\frac{5}{5}$, $\frac{7}{6}$, $\frac{12}{3}$.

Any fraction which is not in its simplest form may be said to be improper.

154. A **mixed number** is an integer and a fraction united; as, $3\frac{1}{2}$, $4\frac{7}{9}$, $16\frac{3}{5}$.

155. A **simple fraction** is a single fraction, both of whose terms are integers. It may be *proper* or *improper;* as $\frac{9}{13}$, $\frac{17}{20}$, $\frac{29}{16}$.

156. A **complex fraction** is a fraction having one or both of its terms fractional; as, $\frac{\frac{5}{8}}{6}$, $\frac{12\frac{2}{3}}{15}$, $\frac{\frac{6}{7}}{\frac{3}{4}}$, $\frac{\frac{2}{3}\times\frac{5}{7}}{\frac{1}{2}+\frac{2}{3}}$.

157. A **compound fraction** is a fraction of a fraction. It consists of two or more fractions connected by "of" or the sign of multiplication, ×; as, $\frac{1}{8}\times\frac{6}{9}\times\frac{3}{16}\times3\frac{7}{9}$; $\frac{5}{9}$ of $\frac{6}{15}$.

158. The **reciprocal** of a number is 1 divided by that number; as, the reciprocal of 8 is $1\div8$; of 12, is $1\div12$.

159. The **reciprocal** *of a* **fraction** is 1 divided by that fraction, or it is the fraction inverted; as, the reciprocal of $\frac{5}{8}$ is $1\div\frac{5}{8}=\frac{8}{5}$.

160. Since every fractional number indicates a division of its numerator by its denominator, all changes in the *terms* of a fraction will affect the *value* of the fraction according to the laws of division; hence, the following:

General Principles

1. Multiplying the numerator, or dividing the denominator, multiplies the fraction.

2. Dividing the numerator, or multiplying the denominator, divides the fraction.

3. Multiplying or dividing both numerator and denominator by the same number does not alter the value of the fraction.

WRITTEN EXERCISE

1. Express in figures: 5 sixths; 8 ninths; 6 nineteenths: 17 fifty-fifths; 55 ninety-eighths; 45, and 5 twelfths; 11 tenths; 16, and 23 twenty-fourths; 10 tenths; 125, and 32 forty-fifths.

2. A son receives $\frac{1}{2}$ of his father's estate, and the daughter $\frac{1}{4}$. If the son's portion is $24500, what is the daughter's?

3. Rearrange the following fractions in the order of their size, the largest first: $\frac{3}{8}$, $\frac{9}{16}$, $\frac{30}{32}$, $\frac{1}{2}$, $\frac{31}{32}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$.

Reduction of Fractions

161. **Reduction of fractions** is the process of changing their form without changing their value.

CASE I

162. **To reduce a fraction to its lowest terms.**

163. A fraction is reduced to its *lowest terms* when its numerator and denominator are prime to each other; as, $\frac{4}{14}=\frac{2}{7}$.

Example. Reduce $\frac{48}{64}$ to its lowest terms.

$8|\frac{48}{64}=\frac{6}{8}$

$2|\frac{6}{8}=\frac{3}{4}$

Solution. Dividing both terms of the fraction, $\frac{48}{64}$, by the common divisor, 8, the result is $\frac{6}{8}$; and the terms of this fraction by the common divisor, 2, the result is $\frac{3}{4}$. Since these terms, 3 and 4, have no common divisor, the fraction is reduced to its lowest terms (163).

Since the numerator and denominator have been divided by the same numbers, the value of the fraction has not been changed (160, Prin. 3).

164. Rule. *Divide the terms of the fraction by any common divisor, and continue the division until no number will divide them without a remainder.*

WRITTEN EXERCISE

Reduce to their lowest terms:

1. $\frac{20}{45}$
2. $\frac{44}{76}$
3. $\frac{55}{66}$
4. $\frac{55}{80}$
5. $\frac{120}{168}$
6. $\frac{105}{225}$
7. $\frac{84}{96}$
8. $\frac{75}{135}$
9. $\frac{156}{208}$
10. $\frac{168}{192}$
11. $\frac{175}{210}$
12. $\frac{108}{144}$
13. $\frac{264}{480}$
14. $\frac{420}{660}$
15. $\frac{132}{144}$
16. $\frac{210}{300}$
17. $\frac{126}{504}$
18. $\frac{96}{144}$
19. $\frac{18}{144}$
20. $\frac{90}{135}$
21. $\frac{125}{1000}$
22. $\frac{275}{625}$
23. $\frac{1296}{1728}$
24. $\frac{78}{192}$
25. $\frac{96}{192}$
26. $\frac{86}{946}$
27. $\frac{6125}{10000}$
28. $\frac{125}{625}$
29. $\frac{2688}{3072}$
30. $\frac{1440}{1728}$
31. $\frac{528}{1760}$
32. $\frac{1001}{1144}$
33. $\frac{4004}{8008}$
34. $\frac{567}{783}$
35. $\frac{9691}{14977}$
36. $\frac{7610}{8371}$

CASE II

165. To reduce a fraction to higher terms.

166. A fraction is reduced to *higher terms* when the numerator and denominator are expressed in larger numbers; as, $\frac{3}{7}=\frac{9}{21}$.

Example. Change $\frac{4}{5}$ to a fraction whose denominator is 35.

$35\div5=7$

$$\frac{4\times7}{5\times7}=\frac{28}{35}$$

Solution. First, divide 35, the required denominator, by 5, the denominator of the given fraction. Since both terms must be multiplied by the same number to preserve the value of the fraction, we multiply the terms by 7, the factor required to produce the required denominator, and we have the required fraction, $\frac{28}{35}$ (160, Prin. 3).

167. Rule. *Divide the required denominator by the denominator of the fraction, multiply the numerator by the quotient thus obtained and write the product over the required denominator.*

WRITTEN EXERCISE

1. Reduce $\frac{5}{8}$ to a fraction whose denominator is 32.
2. Reduce $\frac{11}{13}$ to a fraction whose denominator is 91.
3. Reduce to twelfths: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{2}{3}$.
4. Reduce to thirty-seconds: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{7}{8}$.
5. Reduce to seventy-fifths: $\frac{1}{25}$, $\frac{3}{5}$, $\frac{7}{25}$, $\frac{4}{15}$, $\frac{2}{3}$, $\frac{16}{25}$.

CASE III

168. To reduce an improper fraction to an integer or mixed number.

Example. Reduce $\frac{124}{5}$ to a mixed number.

$\frac{124}{5} = 124 \div 5 = 24\frac{4}{5}$

Solution. Since 5 fifths equals 1, 124 fifths must equal as many times 1 as there are times 5 in 124, which are $24\frac{4}{5}$.

169. Rule. *Divide the numerator by the denominator.*

WRITTEN EXERCISE

Reduce to integers or mixed numbers:

1. $\frac{278}{56}$
2. $\frac{375}{25}$
3. $\frac{496}{15}$
4. $\frac{237}{7}$
5. $\frac{472}{24}$
6. $\frac{9006}{18}$
7. $\frac{375}{15}$
8. $\frac{2968}{16}$
9. $\frac{3789}{50}$
10. $\frac{103407}{1046}$
11. $\frac{4009}{11}$
12. $\$\frac{2960}{16}$

CASE IV

170. To reduce integers and mixed numbers to improper fractions.

Example 1. Change 16 to the form of a fraction having 64 for its denominator.

$16 \times 64 = 1024$
$16 = \frac{1024}{64}$

Solution. Since 1 is equal to 64 *sixty-fourths,* 16 is equal to 16 times 64 sixty-fourths, or 1024 sixty-fourths. Therefore, $16 = \frac{1024}{64}$.

Example 2. Change $34\frac{5}{8}$ to an improper fraction.

$34\frac{5}{8}$
$\underline{\quad 8}$
272 eighths
$\underline{\quad 5}$ eighths
277 eighths.

Solution. Since 1 is equal to 8 *eighths*, 34 is equal to 34 times 8 eighths, or 272 eighths; to which add 5 *eighths*, and the result is 277 eighths. Therefore, $34\frac{5}{8} = \frac{277}{8}$.

To reduce an *integer* to a fractional *form*, write 1 under it for a denominator; thus, $16 = \frac{16}{1}$.

171. Rule. ***Multiply the integer by the given denominator, and to the product add the numerator of the fraction, and write the result over the given denominator.***

ORAL EXERCISE

Reduce the following to improper fractions: $2\frac{1}{2}$; $6\frac{1}{2}$; $3\frac{1}{3}$; $4\frac{2}{3}$; $5\frac{2}{3}$; $5\frac{3}{4}$; $6\frac{5}{6}$; $2\frac{6}{7}$; $5\frac{3}{5}$; $3\frac{3}{4}$; $2\frac{5}{9}$; $8\frac{2}{3}$; $4\frac{3}{8}$; $6\frac{5}{7}$; $3\frac{7}{8}$; $8\frac{3}{4}$; $5\frac{7}{8}$; 9; 16; 24; 15.

WRITTEN EXERCISE

Reduce the following to improper fractions:

1. $18\frac{2}{3}$
2. $24\frac{5}{7}$
3. $171\frac{5}{6}$
4. $324\frac{7}{8}$
5. $165\frac{12}{13}$
6. $16\frac{9}{15}$ oz.
7. $37\frac{3}{17}$ rds.
8. $27\frac{3}{4}$ bu.
9. $36\frac{9}{17}$
10. $473\frac{3}{19}$
11. $265\frac{7}{9}$
12. $800\frac{62}{87}$
13. $321\frac{9}{34}$
14. $75\frac{6}{21}$ T.
15. $38\frac{5}{16}$ A.
16. $\$375\frac{3}{4}$
17. $479\frac{3}{16}$ lb.
18. $450\frac{21}{24}$ gal.
19. $678\frac{3}{7}$ mi.
20. $792\frac{8}{9}$ pk.

21. Change 9 to 4ths; 16 to 8ths; 42 to 7ths; 39 to 40ths; 19 to 32nds; 21 to 18ths.

CASE V

172. To reduce dissimilar fractions to equivalent fractions having a common denominator.

173. **Dissimilar fractions** are fractions that have not the same denominator. *Similar fractions* have like denominators.

174. A **common denominator** is a denominator common to two or more fractions. The *Least Common Denominator* is the least denominator to which they can all be reduced.

The least common denominator of several fractions is the least common multiple (136) of their denominators.

Example. Change $\frac{3}{7}$, $\frac{2}{3}$, and $\frac{5}{14}$ to equivalent fractions having the least common denominator.

$$\begin{array}{ll} 7\underline{)7-3-14} & \frac{3}{7}=\frac{18}{42} \\ \quad 1-3-\ 2 & \frac{2}{3}=\frac{28}{42} \\ 7\times3\times2=42 & \frac{5}{14}=\frac{15}{42} \end{array}$$

Solution. First find the least common multiple of the denominators of the fractions, which is 42. By Art. 167, $\frac{3}{7}$ is reduced to $\frac{18}{42}$, $\frac{2}{3}$ to $\frac{28}{42}$, and $\frac{5}{14}$ to $\frac{15}{42}$. The value of each fraction remains unchanged, since both terms have been multiplied by the same number.

In finding the least common denominator of mixed numbers, they must first be reduced to improper fractions.

175. **Rule.** *Find the least common multiple of the denominators of the given fractions for their least common denominator, and proceed as in 167.*

ORAL EXERCISE

Change to similar fractions, having a common denominator:

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$
2. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$
3. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$
4. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{1}{15}$
5. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{9}$
6. $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{16}$
7. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$
8. $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{12}$

WRITTEN EXERCISE

Reduce the following to their least common denominator:

1. $\frac{1}{3}$, $\frac{5}{7}$, $\frac{8}{9}$
2. $\frac{3}{11}$, $\frac{2}{14}$, $\frac{5}{7}$
3. $\frac{5}{12}$, $\frac{8}{15}$, $\frac{9}{20}$
4. $\frac{3}{5}$, $\frac{11}{12}$, $\frac{9}{21}$
5. $\frac{16}{21}$, $\frac{13}{16}$, $\frac{11}{14}$
6. $\frac{3}{10}$, $\frac{8}{9}$. $\frac{7}{45}$, $\frac{4}{15}$
7. $\frac{4}{9}$, $\frac{7}{12}$, $\frac{11}{36}$, $\frac{19}{24}$
8. $\frac{11}{30}$, $\frac{8}{15}$, $\frac{19}{60}$, $\frac{7}{40}$
9. $\frac{5}{9}$, $\frac{13}{21}$, $\frac{13}{27}$, $\frac{5}{12}$
10. $\frac{13}{17}$, $\frac{25}{51}$, $\frac{19}{102}$
11. $\frac{4}{15}$, $\frac{3}{20}$, $\frac{4}{25}$, $\frac{3}{35}$
12. $\frac{7}{9}$, $\frac{5}{11}$, $\frac{13}{18}$, $\frac{6}{22}$
13. $\frac{5}{12}$, $\frac{8}{21}$, $\frac{9}{28}$, $\frac{13}{42}$
14. $4\frac{2}{3}$, $6\frac{5}{8}$, $\frac{11}{24}$, $\frac{5}{12}$
15. $3\frac{7}{8}$, $6\frac{4}{5}$, $3\frac{7}{9}$, $\frac{13}{15}$

ADDITION OF FRACTIONS

176. **Addition of fractions** is the process of finding the sum of two or more fractions.

177. **Principle.** *Fractions can be added only when they have a common denominator, and when they express parts of like units.*

Example 1. What is the sum of $\frac{3}{8}$, $\frac{1}{8}$, and $\frac{3}{8}$?

$\frac{3}{8}+\frac{1}{8}+\frac{3}{8}=\frac{7}{8}$

Solution. As these fractions have a common denominator, we add their numerators, and write the sum over the common denominator, 8, and we have $\frac{7}{8}$.

Example 2. What is the sum of $\frac{2}{3}$, $\frac{2}{9}$, and $\frac{3}{4}$?

$$\frac{2}{3}+\frac{2}{9}+\frac{3}{4}=\frac{24+8+27}{36}=\frac{59}{36}=1\frac{23}{36}$$

Solution. Since the given fractions are not similar, reduce them to equivalent fractions having the least common denominator, which is 36 (174). Add their numerators, and write the sum, 59, over the common denominator 36, and $\frac{59}{36}=1\frac{23}{36}$, the required result.

Example 3. What is the sum of $16\frac{2}{3}$, $14\frac{5}{6}$, and $7\frac{7}{12}$?

$16 — \frac{2}{3} = \frac{8}{12}$
$14 — \frac{5}{6} = \frac{10}{12}$
$7 — \quad \frac{7}{12}$
$2\frac{1}{12} \quad 2\frac{1}{12}$
$39\frac{1}{12}$

Solution. First find the sum of the fractions as explained in Example 2, which is $2\frac{1}{12}$. Then the sum of the fractional parts placed under the whole number of parts and the several numbers added, gives the required result, $39\frac{1}{12}$.

178. Rule. *Reduce the given fractions to equivalent fractions having the least common denominator, add their numerators, and place the sum over the common denominator.*

When there are mixed numbers or integers, add the fractions and integers separately, then add the results.

ORAL EXERCISE

By inspection, find the sum of:

1. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{5}{4}, \frac{4}{4}, \frac{3}{4}, \frac{7}{4}$.
2. $\frac{3}{7}, \frac{1}{7}, \frac{6}{7}, \frac{5}{7}, \frac{4}{7}, \frac{2}{7}, \frac{7}{7}$.
3. $\frac{1}{9}, \frac{6}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}, \frac{3}{9}$.
4. $\frac{1}{13}, \frac{5}{13}, \frac{6}{13}, \frac{8}{13}, \frac{4}{13}, \frac{7}{13}, \frac{2}{13}, \frac{9}{13}$.
5. $\frac{4}{16}, \frac{5}{16}, \frac{7}{16}, \frac{11}{16}, \frac{15}{16}, \frac{4}{16}, \frac{2}{16}, \frac{12}{16}$.
6. $\frac{16}{20}, \frac{4}{20}, \frac{5}{20}, \frac{10}{20}, \frac{15}{20}, \frac{18}{20}, \frac{2}{20}, \frac{10}{20}$.

Add the following without using pencil:

1. $\frac{1}{3}+\frac{1}{2}$
2. $\frac{1}{3}+\frac{1}{8}$
3. $\frac{2}{3}+\frac{1}{4}$
4. $\frac{2}{3}+\frac{1}{8}$
5. $\frac{1}{3}+\frac{1}{4}$
6. $\frac{1}{3}+\frac{3}{8}$
7. $\frac{2}{3}+\frac{1}{2}$
8. $\frac{2}{3}+\frac{3}{8}$
9. $\frac{1}{3}+\frac{3}{4}$
10. $\frac{1}{3}+\frac{5}{8}$
11. $\frac{2}{3}+\frac{3}{4}$
12. $\frac{2}{3}+\frac{5}{8}$
13. $\frac{1}{3}+\frac{1}{6}$
14. $\frac{1}{3}+\frac{7}{8}$
15. $\frac{2}{3}+\frac{1}{6}$
16. $\frac{2}{3}+\frac{7}{8}$

WRITTEN EXERCISE

Find the sum of the following:

1. $\frac{2}{3}, \frac{4}{5}, \frac{7}{10}$
2. $\frac{5}{7}, \frac{3}{14}, \frac{4}{21}, \frac{5}{14}$
3. $\frac{5}{12}, \frac{8}{15}, \frac{15}{36}, \frac{8}{24}, \frac{1}{3}$
4. $\frac{3}{10}, \frac{8}{9}, \frac{3}{15}, \frac{7}{18}, \frac{6}{45}$
5. $\frac{8}{9}, \frac{6}{7}, \frac{2}{3}, \frac{9}{27}, \frac{4}{21}$
6. $\frac{1}{2}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{3}{4}, \frac{1}{3}$
7. $7\frac{8}{9}, 4\frac{3}{4}, 5\frac{7}{12}, 16\frac{2}{3}$
8. $1\frac{1}{10}, 2\frac{5}{7}, \frac{6}{35}, 17\frac{3}{5}$
9. $8\frac{5}{7}, \frac{1}{2}, 9\frac{3}{10}, 4\frac{13}{15}$
10. $\frac{3}{7}, \frac{6}{11}, 12\frac{5}{11}, 14\frac{6}{7}$
11. $140\frac{3}{4}, 265\frac{7}{8}, 375\frac{2}{16}$
12. $1\frac{3}{8}, 5\frac{6}{15}, 2\frac{5}{30}, 75\frac{7}{40}$

13. A man bought $7\frac{1}{3}$ cords of wood at one time and $12\frac{2}{3}$ cords at another time. How much did he buy in all?

14. A man bought a house for \$7850, furniture for \$$825\frac{7}{9}$, a horse for \$$255\frac{3}{8}$, and a carriage for \$$127\frac{7}{15}$. What was the cost of all?

15. Kate has \$$75\frac{9}{11}$ and John has \$$38\frac{7}{10}$ more than she has. How many dollars has John?

16. A merchant bought cloth for \$$42\frac{1}{9}$; silk, for \$$13\frac{5}{8}$; muslin, for \$$12\frac{3}{4}$; cotton, for \$$15\frac{1}{2}$; and velvet for \$$52\frac{2}{5}$. What was the amount of his bill?

17. A merchant bought 10 pieces of cloth, containing respectively 48^1, 50^3, 52^2, 49, 45^1, 51^3, 54^2, 47, 48^2, and 53 yards. How many yards in all did he buy?

Note. In billing, it is common in the dry-goods business to express fourths (quarters) without denominators by placing the numerators in small figures a little above and to the right of the integer. Thus, $48^1 = 48\frac{1}{4}$; $52^3 = 52\frac{3}{4}$.

18. By horizontal addition find the sum of 6 pieces of M. shirting, containing 40^2, 41^1, 46^3, 51^1, 45^2, and 50^1 yards.

Short Methods in Addition

ORAL EXERCISE

179. *Mental work* in adding fractions is of the utmost importance as well as of great intellectual value. It is not so difficult as at first appears. The business man's fractions are usually of the simplest kind, and in most cases may be added with equally as much ease as integers.

Example 1. Add two fractions whose numerators are 1.

$\frac{1}{7}+\frac{1}{6}=\frac{7+6}{7\times 6}=\frac{13}{42}$ *Solution.* Add the denominators, and place the sum over the product of the denominators.

Example 2. Add two fractions whose numerators are alike and greater than 1.

$\frac{6}{7}+\frac{6}{11}=\frac{6\,(7+11)}{7\times 11}=\frac{108}{77}=1\frac{31}{77}$ *Solution.* Add the denominators and multiply the sum by the numerator of either of the fractions, and write the product over the product of the denominators.

By inspection, find the sum of:

3. $\frac{1}{6}+\frac{1}{4}$
4. $\frac{1}{6}+\frac{1}{8}$
5. $\frac{1}{8}+\frac{3}{4}$
6. $\frac{1}{6}+\frac{1}{2}$
7. $\frac{1}{6}+\frac{3}{8}$
8. $\frac{1}{8}+\frac{1}{2}$
9. $\frac{1}{6}+\frac{3}{4}$
10. $\frac{1}{6}+\frac{5}{8}$
11. $\frac{1}{8}+\frac{1}{4}$
12. $\frac{1}{6}+\frac{1}{3}$
13. $\frac{1}{6}+\frac{7}{8}$
14. $\frac{1}{8}+\frac{2}{3}$

In the following problems add the first two fractions and to their sum add the third; as, $\frac{1}{2}+\frac{1}{5}+\frac{1}{6}=\frac{1}{2}+\frac{1}{5}=\frac{7}{10}$, and $\frac{7}{10}+\frac{1}{6}=\frac{52}{60}$ $=\frac{13}{15}$.

15. $\frac{1}{3}+\frac{1}{4}+\frac{1}{7}$
16. $\frac{1}{5}+\frac{1}{8}+\frac{1}{9}$
17. $\frac{1}{4}+\frac{1}{5}+\frac{1}{10}$
18. $\frac{1}{6}+\frac{1}{8}+\frac{1}{24}$
19. $\frac{1}{9}+\frac{1}{3}+\frac{1}{5}$
20. $\frac{1}{10}+\frac{1}{5}+\frac{1}{3}$

ORAL EXERCISE

180. In adding small fractions, the least common denominator can be determined quite easily by inspection and the fractions added without much hesitancy. Thus:

Example 1. Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$.

$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{8}=\frac{29}{24}=1\frac{5}{24}$

Solution. By inspection, find the l. c. d. to be 24. Then reduce, mentally, each fraction to 24ths, and add as in whole numbers. Thus, 12, 20, 26, $\frac{29}{24}$, or $1\frac{5}{24}$.

Example 2. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

$\frac{2}{3}+\frac{3}{4}+\frac{5}{6}=\frac{27}{12}=2\frac{1}{4}$

Solution. By inspection, find the l. c. d. to be 12. Then add as in whole numbers. Thus, 8, 17, $\frac{27}{12}$, or $2\frac{1}{4}$.

Here are some reasonably simple examples:

3. To $\frac{1}{2}$ add $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.
4. To $\frac{1}{3}$ add $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.
5. To $\frac{1}{4}$ add $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$.
6. To $\frac{1}{5}$ add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{9}$.
7. To $\frac{1}{6}$ add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{5}{12}$.
8. To $\frac{2}{3}$ add $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{10}$.

ORAL EXERCISE

By inspection, add the following:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$2\frac{1}{2}$	$3\frac{1}{3}$	$4\frac{1}{4}$	$6\frac{1}{2}$	$12\frac{1}{6}$	$13\frac{1}{7}$	$5\frac{1}{3}$	$4\frac{1}{6}$	$7\frac{1}{7}$
$1\frac{3}{4}$	$5\frac{2}{3}$	$2\frac{3}{5}$	$3\frac{3}{8}$	$5\frac{3}{4}$	$7\frac{4}{7}$	$3\frac{1}{5}$	$14\frac{2}{7}$	$13\frac{1}{3}$

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
$8\frac{1}{2}$	$7\frac{2}{3}$	$6\frac{1}{3}$	$2\frac{2}{5}$	$5\frac{1}{4}$	$4\frac{1}{6}$	$7\frac{5}{7}$	$9\frac{1}{8}$	$10\frac{1}{2}$
$10\frac{3}{4}$	$15\frac{3}{5}$	$8\frac{1}{6}$	$22\frac{1}{10}$	$9\frac{3}{8}$	$12\frac{1}{4}$	$53\frac{6}{7}$	$16\frac{5}{8}$	$21\frac{1}{6}$
$3\frac{1}{8}$	$3\frac{1}{15}$	$2\frac{1}{9}$	$6\frac{1}{2}$	$16\frac{1}{12}$	$6\frac{1}{3}$	$1\frac{2}{7}$	$5\frac{7}{8}$	$2\frac{1}{12}$

(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
$4\frac{1}{2}$	$2\frac{1}{3}$	$6\frac{1}{4}$	$3\frac{1}{5}$	$1\frac{1}{6}$	$5\frac{1}{7}$	$3\frac{1}{8}$	$7\frac{1}{9}$	$4\frac{1}{10}$
$3\frac{1}{11}$	$3\frac{1}{12}$	$1\frac{1}{16}$	$3\frac{1}{15}$	$4\frac{1}{18}$	$2\frac{1}{14}$	$7\frac{1}{16}$	$2\frac{1}{18}$	$2\frac{1}{15}$
$4\frac{1}{3}$	$5\frac{1}{4}$	$6\frac{1}{8}$	$2\frac{3}{10}$	$3\frac{5}{9}$	$4\frac{5}{28}$	$2\frac{1}{4}$	$1\frac{1}{2}$	$3\frac{3}{5}$
$3\frac{5}{33}$	$2\frac{3}{4}$	$4\frac{7}{32}$	$5\frac{1}{2}$	$6\frac{2}{3}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$3\frac{5}{9}$	$4\frac{7}{30}$
$5\frac{5}{66}$	$4\frac{2}{3}$	$5\frac{1}{2}$	$4\frac{2}{3}$	$2\frac{1}{6}$	$3\frac{3}{14}$	$4\frac{1}{2}$	$6\frac{7}{18}$	$5\frac{2}{3}$

The student should be given a thorough drill in adding exercises like the above. If facility in simple addition and the grouping of figures has not been thoroughly acquired, it should be carefully reviewed at this time.

WRITTEN EXERCISE

Copy and add the following problems:

(1)	(2)	(3)	(4)	(5)	(6)
$3452\frac{1}{8}$	$4530\frac{4}{7}$	$7206\frac{1}{9}$	$2399\frac{1}{4}$	$3425\frac{5}{13}$	$4703\frac{5}{14}$
$7805\frac{2}{3}$	2563	$8004\frac{1}{3}$	$3297\frac{7}{9}$	$2376\frac{4}{5}$	4523
5045	$4003\frac{3}{8}$	$2353\frac{1}{6}$	$1234\frac{2}{3}$	4321	$5436\frac{1}{2}$
$8213\frac{4}{5}$	8134	2314	4516	$432\frac{3}{13}$	$654\frac{3}{4}$
$2578\frac{1}{2}$	$1709\frac{5}{12}$	$3498\frac{7}{18}$	$2340\frac{5}{6}$	$234\frac{1}{2}$	$456\frac{3}{7}$
1023	$9484\frac{3}{4}$	$5673\frac{3}{4}$	1502	4287	9301
$2394\frac{3}{4}$	$9876\frac{1}{16}$	5432	$1987\frac{4}{15}$	$6543\frac{7}{65}$	$1346\frac{9}{28}$
2310	3487	$3487\frac{6}{24}$	$6510\frac{1}{3}$	2319	$4576\frac{13}{56}$
$5643\frac{5}{24}$	$1234\frac{9}{12}$	$9867\frac{2}{3}$	$1234\frac{1}{2}$	$3456\frac{7}{13}$	$8769\frac{7}{8}$

SUBTRACTION OF FRACTIONS

181. Subtraction of fractions is the process of finding the difference between two fractions.

182. Principle. *Only similar fractions can be subtracted.*

Example 1. Find the difference between $\frac{2}{3}$ and $\frac{3}{8}$.

$\frac{2}{3}-\frac{3}{8}=\frac{16}{24}-\frac{9}{24}=\frac{7}{24}$

Solution. Since the fractions are not similar they must be reduced to equivalent fractions having a common denominator. The l. c. d. is 24. $\frac{2}{3}=\frac{16}{24}$ and $\frac{3}{8}=\frac{9}{24}$. $\frac{16}{24}-\frac{9}{24}=\frac{7}{24}$.

Example 2. Find the difference between $24\frac{3}{8}$ and $16\frac{2}{3}$.

$$\begin{array}{r} 24\frac{3}{8} = 24\frac{9}{24} \\ 16\frac{2}{3} = 16\frac{16}{24} \\ \hline 7\frac{17}{24} \end{array}$$

Solution. Change the given fractions to similar fractions as in Example 1. It is evident that $\frac{16}{24}$ cannot be taken from $\frac{9}{24}$, hence 1 taken from 24 and united with the $\frac{9}{24}$, makes $\frac{33}{24}$. Then $\frac{33}{24}-\frac{16}{24}=\frac{17}{24}$, and 16 from 23 the remainder of the minuend, and we have $7\frac{17}{24}$.

183. Rule. *Reduce the fractions to a common denominator, take the difference of the numerators and write it over the common denominator.*

When there are mixed numbers or integers, subtract the fractions and integers separately.

Mixed numbers may be reduced to improper fractions and subtracted according to the first part of the rule.

ORAL EXERCISE

184. In subtracting small fractions the work should be performed mentally after the manner of adding such fractions as shown in 179; as, $\frac{5}{6}-\frac{1}{8}=\frac{40-6}{6\times 8}=\frac{34}{48}$, or $\frac{17}{24}$.

Find the value of the following:

1. $\frac{2}{3}-\frac{1}{6}$.
2. $\frac{3}{4}-\frac{3}{8}$.
3. $\frac{8}{9}-\frac{2}{3}$.
4. $\frac{9}{10}-\frac{1}{5}$.
5. $\frac{7}{10}-\frac{1}{4}$.
6. $\frac{17}{20}-\frac{4}{5}$.
7. $\frac{8}{15}-\frac{1}{3}$.
8. $\frac{9}{14}-\frac{4}{7}$.
9. $2-\frac{5}{6}$.
10. $3-1\frac{3}{4}$.
11. $2\frac{3}{4}-1\frac{1}{2}$.
12. $5\frac{1}{2}-3\frac{3}{4}$.

13. John is $19\frac{7}{12}$ years of age and William is $4\frac{1}{4}$ years younger. How old is William?

14. From a box containing 7 dozen eggs, $6\frac{2}{3}$ dozen were sold. How many eggs were left in the box?

15. From $20\frac{2}{7}$ take $8\frac{3}{14}$. From $24\frac{2}{5}$ take $19\frac{3}{10}$.

WRITTEN EXERCISE

Find the difference between:

1. $\frac{7}{10}$ and $\frac{7}{15}$.
2. $\frac{18}{35}$ and $\frac{7}{25}$.
3. $\frac{19}{20}$ and $\frac{5}{16}$.
4. $\frac{65}{77}$ and $\frac{47}{88}$.
5. $\frac{42}{49}$ and $\frac{23}{98}$.
6. $\frac{3}{4}$ and $\frac{7}{13}$.
7. $\frac{39}{40}$ and $\frac{7}{15}$.
8. $\frac{69}{85}$ and $\frac{23}{70}$.
9. $\frac{21}{34}$ and $\frac{1}{3}$.
10. $\frac{5}{105}$ and $\frac{1}{70}$.
11. $5\frac{1}{4}$ and $2\frac{1}{7}$.
12. $6\frac{7}{8}$ and $5\frac{2}{3}$.
13. $24\frac{9}{10}$ and $3\frac{7}{45}$.
14. $32\frac{2}{3}$ and $5\frac{4}{51}$.
15. 234 and $34\frac{3}{4}$.

Find the value of:

16. $\frac{1}{5}+\frac{2}{3}-\frac{1}{6}$.
17. $\frac{3}{4}+\frac{5}{6}-\frac{1}{2}$.
18. $\frac{5}{7}+\frac{2}{3}+\frac{5}{8}-\frac{1}{7}-\frac{3}{4}$.
19. $\frac{8}{15}+\frac{6}{7}-\frac{1}{2}-\frac{1}{6}+\frac{2}{3}$.
20. $\frac{7}{10}+\frac{2}{3}+\frac{5}{6}-\frac{1}{2}-\frac{3}{4}-\frac{1}{3}$
21. $6\frac{2}{3}+8\frac{1}{7}+16-5-12\frac{5}{6}$.
22. $7\frac{1}{2}-\frac{3}{4}-3\frac{1}{2}-\frac{5}{6}+2\frac{3}{8}-1$.
23. $234\frac{7}{8}-156\frac{5}{6}-50\frac{2}{3}$.
24. $4\frac{3}{4}+16\frac{5}{9}+54\frac{2}{3}-16-2\frac{7}{8}$.
25. $65-23\frac{3}{8}-\frac{7}{9}-17\frac{3}{4}-22$.

26. From a field containing 297 acres, $110\frac{1}{9}$ acres were sold to one man and $87\frac{5}{7}$ acres to another. How many acres were left?

27. Frank's father gave him \$$\frac{5}{8}$ on Monday, \$$1\frac{2}{3}$ on Tuesday, and \$$2\frac{1}{2}$ on Wednesday. On Thursday, Frank spent \$$3\frac{1}{4}$. How much money had he left?

28. Robert has \$$\frac{9}{11}$ and James has \$$\frac{2}{5}$ less than he has. How much have both together?

29. A certain sum of money is to be divided among five persons; A is to have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder. How much is E's share?

30. $16-3\times4+15\div3-3+2\frac{3}{7}-5\frac{2}{3}=?$

31. A boy was asked how much money he had. He replied, "If I had \$$8\frac{2}{7}$ more than I now have, I should have then \$$27\frac{5}{8}$." How much money did he have?

32. If 5 is added to each term of $\frac{3}{4}$, is the value of the fraction increased or diminished, and how much?

33. From \$$234\frac{5}{8}$+\$$34\frac{2}{3}$ take \$$105\frac{5}{6}$−\$$89\frac{3}{8}$.

34. A farm which was bought for \$$12{,}248\frac{3}{4}$ was sold at auction for \$$10{,}811\frac{7}{12}$. What was the amount of reduction in price?

MULTIPLICATION OF FRACTIONS

185. **Multiplication of fractions** is the process of finding a product when one or both factors are fractions.

1. 6 times \$2 are how many dollars? 6 times $\frac{1}{9}$ are how many ninths?
2. How much is $\frac{1}{2}$ of $\frac{1}{2}$ of a dollar? $\frac{1}{3}$ of $\frac{3}{4}$ of a dollar?
3. How much is $\frac{1}{2}$ of $\frac{1}{5}$ of a dollar? $\frac{1}{5}$ of $\frac{1}{2}$ of an apple?
4. If 4 men can do a piece of work in 6 days, how long will

it take 1 man to do it? What part of the work do they perform in 1 day? What part does 1 man do in 1 day?

5. What part of 24 is 4? What part of 24 is 6?

CASE I

186. To multiply a fraction by an integer.

Example. Multiply $\frac{5}{8}$ by 4.

$$\frac{5}{8}\times 4=\frac{20}{8}=2\frac{1}{2}, \text{ or}$$
$$\frac{5}{8}\times 4=\frac{5}{8\div 4}=\frac{5}{2}=2\frac{1}{2}.$$

Solution. Multiplying the numerator of a fraction multiplies that fraction (160, 1). Hence, 4 times $\frac{5}{8}=\frac{20}{8}$, or $2\frac{1}{2}$. Or,

Dividing the denominator of a fraction multiplies that fraction (160, 1). Hence, divide the denominator by 4 and write the result under the numerator as shown in the illustration and it equals $\frac{5}{2}$, or $2\frac{1}{2}$.

187. Rule. *To multiply a fraction by an integer, multiply the numerator or divide the denominator by the integer, then reduce the result to its simplest form.*

ORAL EXERCISE

1. Multiply $\frac{1}{8}$ by 4. $\frac{2}{9}$ by 3. $\frac{1}{12}$ by 5.
2. If cloth is worth \$$\frac{3}{8}$ a yard, what is 12 yards worth?
3. How much is 5 times $\frac{2}{5}$ of a rod? $\frac{5}{6}$ of a rod?

Find the product of:

4. $\frac{1}{2}$ of 16.
5. $\frac{1}{3}$ of 24.
6. $\frac{2}{3}\times 15$.
7. $\frac{3}{4}\times 8$.
8. $\frac{2}{9}\times 8$.
9. $\frac{6}{7}\times 7$.
10. $\frac{3}{5}\times 6$.
11. $\frac{3}{7}\times 5$.
12. $\frac{2}{3}$ of 9.
13. $\frac{5}{9}\times 6$.
14. $\frac{8}{11}$ of 22.
15. $\frac{6}{13}\times 26$.

CASE II

188. To multiply an integer by a fraction, or a fraction by a fraction.

Example 1. If a ton of coal cost \$7, what will $\frac{3}{4}$ of a ton cost?

$$\$7\times\frac{3}{4}=\frac{21}{4}=\$5\frac{1}{4}, \text{ or}$$
$$\frac{7}{1}\times\frac{3}{4}=\frac{21}{4}=5\frac{1}{4}.$$

Solution. If 1 ton costs \$7, $\frac{3}{4}$ of a ton will cost 3 times what $\frac{1}{4}$ of a ton costs, or 3 times $\frac{7}{4}=\frac{21}{4}$, or \$$5\frac{1}{4}$. Or,

Write the whole number, 7, in the form of a fraction and multiply as in the illustration,

Example 2. Multiply $\frac{15}{16}$ by $\frac{4}{5}$.

$$\frac{\overset{3}{\cancel{15}}}{\underset{4}{\cancel{16}}}\times\frac{\cancel{4}}{\cancel{5}}=\frac{3}{4}$$

Solution. $\frac{15}{16}$ multiplied by 1 is $\frac{15}{16}$, hence $\frac{15}{16}$ multiplied by $\frac{4}{5}=\frac{4}{5}$ of $\frac{15}{16}$. Applying cancellation, and reducing as shown in the illustration, we have $\frac{3}{4}$, the product.

189. Rule. *Multiply the numerators together for the numerator of the product, and the denominators for the denominator of the product.*

1. When possible use cancellation.
2. Reduce mixed numbers to improper fractions before multiplying.
3. An integer may be expressed as a fraction by writing 1 under it as a denominator.

ORAL EXERCISE

Find the value of:

1. $3\times\frac{3}{4}$. 3. $6\times\frac{3}{8}$. 5. $\frac{5}{6}\times\frac{6}{7}$. 7. $2\frac{1}{2}\times2\frac{1}{2}$.
2. $5\times\frac{3}{5}$. 4. $\frac{2}{3}\times\frac{3}{4}$. 6. $\frac{7}{8}\times\frac{8}{9}$. 8. $3\frac{1}{2}\times\frac{2}{7}$.

9. If tea is worth $\$\frac{3}{4}$ a pound, what will $\frac{2}{3}$ of a pound cost?

10. Multiplying by $\frac{1}{3}$ is the same as taking what part of a number?

11. What will $1\frac{1}{2}$ lbs. of sugar cost at $6\frac{2}{3}$¢ a pound?

WRITTEN EXERCISE

Find the product of:

1. $\frac{2}{3}\times\frac{7}{16}$. 3. $3\frac{7}{9}\times4\frac{6}{17}$. 5. $\frac{3}{10}\times\frac{25}{27}$. 7. $2\frac{1}{2}\times3\frac{1}{4}$.
2. $\frac{7}{15}\times\frac{3}{21}$. 4. $2\frac{3}{8}\times6\frac{4}{19}$. 6. $\frac{3}{14}\times6\frac{2}{3}$. 8. $5\times6\frac{3}{4}$.

9. How much will $10\frac{1}{2}$ yards of cloth cost at $\$2\frac{4}{7}$ a yard?

10. A man owned $\frac{15}{27}$ of a farm and sold $\frac{3}{5}$ of his share. What part of the whole farm did he sell?

CASE III

190. To multiply mixed numbers together.

Example. Multiply $256\frac{3}{4}$ by $26\frac{1}{3}$.

$$\begin{array}{r}
256\frac{3}{4}\\
26\frac{1}{3}\\
\hline
\frac{1}{4}\\
85\frac{1}{3}\\
19\frac{1}{2}\\
1536\phantom{\frac{1}{2}}\\
512\phantom{0\frac{1}{2}}\\
\hline
6761\frac{1}{12}
\end{array}$$

Solution. Write the multiplier under the multiplicand as in simple numbers. Commencing at the right, first multiply the fraction of the multiplicand by the fraction of the multiplier and we have $\frac{1}{4}$. Next multiply the integer in the multiplicand by the fraction in the multiplier and obtain $85\frac{1}{3}$. Multiply the fraction in the multiplicand by the integer in the multiplier and obtain $19\frac{1}{2}$. Multiply the integers together and add the partial products. The result is $6761\frac{1}{12}$, the required product.

NOTE. The above method is recommended when the fractional numbers are large.

191. Rule. *Multiply the fractions together, each integer by the fraction of the other number, and the integers together. The sum of the partial products will be the product required.*

WRITTEN EXERCISE

Find the product of:

1. $45\frac{1}{2}\times22\frac{3}{5}$.
2. $86\frac{2}{3}\times45\frac{1}{3}$.
3. $73\frac{1}{4}\times24\frac{1}{6}$.
4. $234\frac{2}{3}\times65\frac{5}{6}$.
5. $432\frac{1}{7}\times21\frac{1}{8}$.
6. $840\frac{2}{7}\times56\frac{1}{6}$.
7. $754\frac{1}{6}\times89\frac{3}{5}$.
8. $602\frac{1}{4}\times88\frac{3}{7}$.
9. $865\frac{4}{9}\times72\frac{1}{5}$.

10. A man rented a house at $\$37\frac{3}{4}$ a month, and remained in it $7\frac{1}{2}$ years; what did his rent amount to during the time?

11. Paid $\$275\frac{3}{4}$ for a horse, and sold him for $\frac{4}{5}$ of what he cost. What was the loss?

12. How much more than 4 ten-dollar bills is needed to pay for $4\frac{1}{2}$ times 4 barrels of apples at $\frac{2}{3}$ of $\$3\frac{3}{5}$ a barrel?

13. Find the value of $(129-76\frac{5}{9})\times(12\frac{1}{4}-2\frac{2}{3})+33\frac{1}{3}$.

14. Mr. A bought $62\frac{1}{2}$ bushels of wheat at $87\frac{1}{2}$¢ a bushel, and sold $\frac{1}{3}$ of it at 90¢, and the remainder at $92\frac{2}{3}$¢ a bushel; what did he clear by the transaction?

15. A merchant bought a piece of cloth for $\$65\frac{3}{4}$, but was obliged to sell it for $\frac{6}{7}$ of what it cost him. How much did he lose?

16. What is the product of $\frac{1}{6}$ of $\frac{2}{3}\times3\frac{1}{3}\times\frac{3}{5}$?

Solution. $\frac{1}{\underset{3}{\cancel{6}}}\times\frac{\cancel{2}}{3}\times\frac{\overset{2}{\cancel{10}}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{5}}=\frac{2}{9}$.

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1. The word "of" between fractions, is equivalent to the sign of *multiplication.*

2. In multiplying *compound fractions* (157) the same rule applies as in Case II.

17. What is the product of $\frac{3}{4}$ of $\frac{4}{5}$ of $21\frac{1}{3}\times 2\frac{1}{2}$?
18. What is the product of $\frac{2}{7}$ of $96\times\frac{7}{10}$ of $26\frac{2}{3}$?
19. What is the product of $\frac{6}{63}$ of $21\times\frac{6}{19}$ of $\frac{11}{8}$ of $\frac{3}{5}$?
20. Simplify $\frac{2}{3}$ of $\frac{5}{8}+1\frac{1}{14}$ of $\frac{63}{75}+\frac{7}{10}-\frac{1}{60}$.

Short Methods in Multiplication

192. *To multiply together two numbers when the multiplier is a mixed number and a convenient fractional part of 10, 100, 1000, etc.*

193. *To multiply any number by:*

$1\frac{2}{3}$. *Annex one cipher and divide by 6;* as, $234\times 1\frac{2}{3}=2340\div 6=390$.

$2\frac{1}{2}$. *Annex one cipher and divide by 4;* as, $165\times 2\frac{1}{2}=1650\div 4=412\frac{1}{2}$.

$3\frac{1}{3}$. *Annex one cipher and divide by 3;* as, $432\times 3\frac{1}{3}=4320\div 3=1440$.

$6\frac{2}{3}$. *Annex one cipher and deduct $\frac{1}{3}$ of the result;* as, $144\times 6\frac{2}{3}=1440-(\frac{1}{3}$ of $1440)=960$.

$7\frac{1}{2}$. *Annex one cipher and deduct $\frac{1}{4}$ of the result;* as, $84\times 7\frac{1}{2}=840-(\frac{1}{4}$ of $840)=630$.

$8\frac{1}{3}$. *Annex one cipher and deduct $\frac{1}{6}$ of the result;* as, $256\times 8\frac{1}{3}=2560-(\frac{1}{6}$ of $2560)=2133\frac{1}{3}$.

$12\frac{1}{2}$. *Annex two ciphers and divide by 8;* as, $84\times 12\frac{1}{2}=8400\div 8=1050$.

$13\frac{1}{3}$. *Annex one cipher and add $\frac{1}{3}$ of the result;* as, $255\times 13\frac{1}{3}=2550+(\frac{1}{3}$ of $2550)=3400$.

This method of multiplying may be continued advantageously, using other aliquot parts of 100, 1000, etc.; as, $14\frac{2}{7}$, $16\frac{2}{3}$, $33\frac{1}{3}$, $66\frac{2}{3}$, etc.

In short methods of multiplying we should remember that $12\frac{1}{2}$¢ = \$$\frac{1}{8}$; 25¢ = \$$\frac{1}{4}$; $37\frac{1}{2}$¢ = \$$\frac{3}{8}$; 50¢ = \$$\frac{1}{2}$; $62\frac{1}{2}$¢ = \$$\frac{5}{8}$; 75¢ = \$$\frac{3}{4}$; $87\frac{1}{2}$¢ = \$$\frac{7}{8}$.

NOTE. A great deal of drill should be given upon these principles. Consistent practice develops great skill and speed. The table of aliquot parts will be found later in the study.

WRITTEN EXERCISE

Find the value of the following:

1. $365 \times 2\frac{1}{2}$.
2. $206 \times 6\frac{2}{3}$.
3. $562 \times 1\frac{2}{3}$.
4. $532 \times 7\frac{1}{2}$.
5. $7543 \times 8\frac{1}{3}$.
6. $2349 \times 14\frac{2}{7}$.
7. $9423 \times 28\frac{4}{7}$.
8. $2365 \times 33\frac{1}{3}$.
9. $345 \times 3\frac{1}{3}$.
10. $437 \times 16\frac{2}{3}$.
11. $534 \times 12\frac{1}{2}$.
12. $503 \times 66\frac{2}{3}$.
13. $2994 \times 13\frac{1}{3}$.
14. $5643 \times 133\frac{1}{3}$.
15. $7632 \times 333\frac{1}{3}$.
16. $346 \times 666\frac{2}{3}$.

194. *To multiply together two mixed numbers when the integers are alike and the sum of the fractional parts is equal to 1.*

Example. What will $9\frac{2}{3}$ yards of cloth cost at $9\frac{1}{3}$¢ per yard?

$$\begin{array}{r} 9\frac{1}{3}¢ \\ 9\frac{2}{3} \\ \hline 90\frac{2}{9}¢ \end{array}$$

Solution. Proceed in the same manner as shown in (58) multiplying together two numbers whose tens are alike, and the sum of whose units is ten. Hence, we say $\frac{2}{3}$ times $\frac{1}{3}$ and write the product, $\frac{2}{9}$. Then 9 units multiplied by 1 more than itself for the remaining figures $= 90$.

ORAL EXERCISE

By inspection, find the value of:

1. $3\frac{1}{3}$ gallons at $3\frac{2}{3}$¢.
2. $2\frac{1}{2}$ yards at $\$2\frac{1}{2}$.
3. $4\frac{3}{4}$ tons at $\$4\frac{1}{4}$.
4. $5\frac{1}{6}$ bushels at $\$5\frac{5}{6}$.
5. $7\frac{1}{2}$ pounds at $7\frac{1}{2}$¢.
6. $10\frac{1}{7}$ acres at $\$10\frac{6}{7}$.
7. $12\frac{1}{2}$ acres at $\$12\frac{1}{2}$.
8. $11\frac{1}{3}$ tons at $\$11\frac{2}{3}$.
9. $6\frac{2}{3}$ yards at $6\frac{1}{3}$¢.
10. $9\frac{1}{4}$ yards at $9\frac{3}{4}$¢.
11. Square $3\frac{1}{2}$; $6\frac{1}{2}$; $15\frac{1}{2}$; 25; 35; 45; 55; 75.

195. *To multiply together any two mixed numbers whose fractional parts are alike.*

Example. Multiply $72\frac{3}{4}$ by $9\frac{3}{4}$.

$$\begin{array}{r} 72\frac{3}{4} \\ 9\frac{3}{4} \\ \hline \frac{9}{16} \\ 60\frac{3}{4} \\ 648 \\ \hline 709\frac{5}{16} \end{array}$$

Solution. Multiply $\frac{3}{4}$ by $\frac{3}{4}$ and write the result, $\frac{9}{16}$. Since $\frac{3}{4}$ of $72 + \frac{3}{4}$ of 9 equals $\frac{3}{4}$ of $(72+9)$, write $\frac{3}{4}$ of the sum of the integers, $60\frac{3}{4}$. Then write the product of the integers, 9 times 72, or 648. Adding the several products and we have $709\frac{5}{16}$, the required result.

WRITTEN EXERCISE

What is the product of:

1. $34\frac{1}{2} \times 27\frac{1}{2}$.
2. $65\frac{1}{3} \times 24\frac{1}{3}$.
3. $78\frac{3}{4} \times 42\frac{3}{4}$.
4. $25\frac{2}{3} \times 65\frac{2}{3}$.
5. $43\frac{3}{5} \times 84\frac{3}{5}$.
6. $45\frac{1}{5} \times 88\frac{1}{5}$.
7. $234\frac{1}{6} \times 32\frac{1}{6}$.
8. $432\frac{4}{5} \times 86\frac{4}{5}$.
9. $540\frac{1}{4} \times 44\frac{1}{4}$.

In business practice, a fractional part of a cent is counted as 1 cent if it is one-half or more, and nothing if less than one-half. Hence, in the multiplication of mixed numbers when the numbers are small and of inferior financial value the fraction may be disregarded, except when it equals or exceeds $\frac{1}{2}$, in which case regard it as 1.

10. What cost $11\frac{1}{4}$ yards of cloth at $26\frac{2}{3}$¢ per yard?

$$\begin{array}{r} 26\frac{2}{3}¢ \\ 11\frac{1}{4} \\ \hline 14 \\ 286 \\ \hline \$3.00 \end{array}$$

Solution. The multiplication of $\frac{2}{3}$ by $\frac{1}{4}$ is disregarded. We take $\frac{1}{4}$ of $26\frac{2}{3}$, to the nearest unit, which is 7; then $\frac{2}{3}$ of 11, which is 7, and $7+7=14$, which write as shown in the illustration. To this add the product of 11 times 26 and we have 300, or $3.00, the total product.

DIVISION OF FRACTIONS

196. Division of fractions is the process of finding the quotient of two numbers when one or both of them are fractional.

1. If 3 oranges cost 15¢, what will 1 orange cost?

2. If you divide $\frac{3}{4}$ of an orange equally among 3 boys, what part of $\frac{3}{4}$ does each boy get? What part of the orange?

3. Divide $\frac{3}{4}$ by 3; $\frac{6}{9}$ by 6; $\frac{4}{7}$ by 2; $\frac{8}{9}$ by 4; $\frac{10}{11}$ by 5.

4. Dividing by 3 is the same as multiplying by what fraction? Dividing by 4? By 5? By 6?

5. If $\frac{2}{5}$ of a pound of coffee cost 10¢, what will 1 pound cost?

Solution. Since $\frac{2}{5}$ of a pound of coffee cost 10¢, $\frac{1}{5}$ of a pound will cost $\frac{1}{2}$ of 10¢, or 5¢; and $\frac{5}{5}$, or 1 pound, will cost 5 times 5¢, or 25¢.

6. If $\frac{3}{8}$ of a pound of coffee cost 15¢, what will 2 pounds cost?

197. Principle. *Dividing the numerator or multiplying the denominator, divides the fraction* (160, 2).

Inverting the Divisor

ORAL EXERCISE

1. How many halves in 1? How many thirds? How many fifths?

2. $1 \div \frac{1}{3} = ?$ $1 \div \frac{1}{4} = ?$ $1 \div \frac{1}{5} = ?$ Since $1 \div \frac{1}{3} = 3$, $1 \div \frac{2}{3} =$ what part of 3? In simplest form $= 1\frac{1}{2}$.

3. Since $1 \div \frac{1}{4} = 4$; $1 \div \frac{3}{4} = \frac{4}{3}$, or $1\frac{1}{3}$. $1 \div \frac{4}{5} = ?$ $1 \div \frac{3}{7} = ?$ $1 \div \frac{5}{9} = ?$ $1 \div \frac{3}{8} = ?$ Hence,

198. *When the dividend is 1 and the divisor a fraction, the quotient is equal to the terms of the divisor inverted.*

Since $1 \div \frac{3}{5} = \frac{5}{3}$, $6 \div \frac{3}{5} = 6$ times $\frac{5}{3}$, or $\frac{30}{3} = 10$. Or, that since $1 \div \frac{3}{5} = \frac{5}{3}$, $\frac{2}{7} \div \frac{3}{5} = \frac{2}{7}$ of $\frac{5}{3} = \frac{10}{21}$. Hence,

199. *Any number, integral or fractional, divided by a fraction is equal to that number of times the divisor inverted.*

200. The **reciprocal** of an **integer** is 1 divided by that integer; as, the reciprocal of 6 is $\frac{1}{6}$.

201. The **reciprocal** of a **fraction** is 1 divided by that fraction; as, 1 divided by $\frac{1}{6}$ is $\frac{6}{1}$, the divisor inverted.

CASE I

202. To divide a fraction by a fraction.

Example. Divide $\frac{7}{8}$ by $\frac{2}{3}$, or find how many times $\frac{2}{3}$ is contained in $\frac{7}{8}$.

$\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16}$, or $1\frac{5}{16}$

Solution. Since $\frac{2}{3}$ is contained in one $\frac{3}{2}$ times, in $\frac{7}{8}$ it will be contained $\frac{7}{8}$ of $\frac{3}{2}$ times, or $\frac{21}{16}$ times, or $1\frac{5}{16}$ times.

203. Rule. *Invert the terms of the divisor and proceed as in multiplication.*

1. When possible, use cancellation.
2. Reduce integers and mixed numbers to improper fractions.

WRITTEN EXERCISE

Find the quotient of:

1. $\frac{3}{8} \div \frac{3}{4}$.
3. $\frac{5}{9} \div \frac{7}{8}$.
5. $\frac{20}{27} \div \frac{5}{7}$.
7. $5\frac{5}{7} \div 3\frac{2}{7}$.
2. $\frac{7}{9} \div \frac{3}{10}$.
4. $\frac{20}{49} \div \frac{4}{7}$.
6. $2\frac{3}{4} \div \frac{3}{4}$.
8. $16\frac{1}{2} \div 2\frac{3}{4}$.
9. Divide $\frac{25}{34} \times \frac{17}{5}$ by $\frac{2}{7}$ of $\frac{3}{4}$.
10. Divide $\frac{3}{5} \times \frac{15}{18} \times \frac{6}{7} \times \frac{3}{4}$ by $\frac{9}{12} \times \frac{3}{8} \times \frac{6}{7}$.
11. Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{5}{9}$ by $\frac{5}{6}$ of $\frac{8}{9}$ of $\frac{3}{10}$ of 4.
12. Divide $\frac{3}{5}$ of $3\frac{1}{3}$ of $1\frac{7}{8}$ of 9 by $1\frac{2}{3} \times \frac{3}{5} - \frac{3}{4}$ of $\frac{7}{9}$.

CASE II

204. To divide a fraction by an integer, or an integer by a fraction.

Example 1. Divide $18\frac{3}{4}$ by 7.

$$\begin{array}{r}7\overline{)18\frac{3}{4}}\\ \hline 2\frac{19}{28}\end{array}$$

Solution. The division may be performed as in simple numbers, by saying that 7 is contained in $18\frac{3}{4}$ 2 times, with a remainder of $4\frac{3}{4}$; and $4\frac{3}{4}$ or $\frac{19}{4} \div 7 = \frac{19}{28}$. Therefore the quotient is $2\frac{19}{28}$.

Example 2. Divide 25 by $3\frac{3}{4}$.

$$\begin{array}{rl}3\frac{3}{4}) & 25\\ 4 & 4\\ \hline 15) & 100(6\frac{2}{3}\\ & 90\\ \hline & 10\end{array}$$

Solution. It is frequently convenient to multiply both dividend and divisor by the least common multiple of the denominators of the fractions, or fraction, and then divide the numerator of the dividend by the numerator of the divisor. Thus, $3\frac{3}{4} = 15$ fourths, and $25 = 100$ fourths. Then perform the division as in any other case of division, and we have the quotient $6\frac{2}{3}$.

NOTE. In examples like the above the integers may be changed to a fractional form and the work performed as in Case I. In fact, in many problems this may be the better method, especially when the mixed numbers are small.

WRITTEN EXERCISE

Find the quotient of:

1. $24\frac{1}{2} \div 6$.
2. $44\frac{2}{3} \div 7$.
3. $65\frac{3}{4} \div 8$.
4. $235 \div 6\frac{1}{4}$.
5. $752 \div 9\frac{3}{5}$.
6. $605 \div 7\frac{1}{2}$.
7. $5432\frac{2}{3} \div 24\frac{1}{3}$.
8. $8345\frac{3}{8} \div 16\frac{5}{8}$.
9. $2004\frac{5}{6} \div 12\frac{3}{4}$.

10. If a barrel of flour costs \$$6\frac{3}{4}$, how many barrels can be bought for \$$45\frac{1}{2}$?

11. What must be divided by $11\frac{1}{20}$ to produce $7\frac{4}{5}$?

12. How many packages of berries, each containing $2\frac{2}{3}$ quarts, at $4\frac{1}{2}$¢ a quart, must be given for three 8-lb. jars of butter at 22¢ a pound?

13. Divide $658349\frac{3}{4}$ by $12\frac{2}{3}$.

$$\begin{array}{rr}12\frac{2}{3}) & 658349\frac{3}{4}\\ & 12\\ 152 & \overline{)7900197}\\ \hline & 51974\frac{149}{152}\end{array}$$

Explanation. As has been previously shown, multiply both dividend and divisor by the least common multiple of the denominator of the fractions and then divide the numerator of the dividend by the numerator of the divisor as in simple numbers.

Divide:

14. $54326\frac{1}{2}$ by $25\frac{2}{3}$.
15. $32007\frac{3}{4}$ by $16\frac{3}{8}$.
16. $657893\frac{3}{5}$ by $9\frac{3}{10}$.
17. $237456\frac{3}{7}$ by $24\frac{2}{3}$.

18. A farmer exchanged $19\frac{1}{3}$ tons of straw at \$$15\frac{1}{2}$ a ton for cloth at $\frac{3}{4}$ of a dollar a yard. How many yards did he receive?

19. Find the cost of $13\frac{2}{3}$ bbls. of flour, when $\frac{2}{7}$ of a bbl. cost $\$1\frac{3}{7}$.

20. If $8\frac{3}{4}$ tons of coal cost $\$30\frac{5}{8}$, how many tons can be bought for $\$127\frac{3}{4}$?

Case III

205. To reduce complex fractions to their simplest form.

Example. Simplify $\dfrac{26-(\frac{2}{3}\times\frac{3}{4})+\frac{1}{3}}{\frac{3}{7}\div\frac{3}{4}}+42$.

Solution. First perform the multiplication indicated in the numerator, and obtain $\frac{1}{2}$. Then $26-\frac{1}{2}+\frac{1}{3}=25\frac{5}{6}$, the result of the numerator. Next perform the division indicated in the denominator, $\frac{3}{7}\div\frac{3}{4}=\frac{4}{7}$. Then the result of the numerator $25\frac{5}{6}$, divided by the result of the denominator $\frac{4}{7}$, gives $45\frac{5}{24}$, to which add 42, and we have the final result $87\frac{5}{24}$.

WRITTEN EXERCISE

1. $84+6\div9-(81-3\div13)+42-12\times3\frac{1}{3}=?$

2. From 1 subtract $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}$, divide the remainder by $5\frac{1}{6}-4\frac{7}{8}$, and find the difference between 1 and the quotient obtained.

3. Subtract $\frac{51}{59}$ from $\dfrac{40\frac{5}{7}}{10\frac{15}{28}}$.

4. Simplify $\dfrac{5\frac{5}{24}}{4\frac{4}{9}}-(\frac{9}{80}\text{ of }3\frac{3}{4})$.

5. From $17\frac{1}{2}$ take $\frac{3}{5}$ of $6\frac{1}{4}$, and multiply the remainder by $\frac{2}{3}$.

6. Simplify $\dfrac{\frac{6}{7}\times\frac{3}{4}-2\frac{1}{3}\times\frac{1}{14}}{8+6-5\times2-3}$.

The Signs "×" and "of"

7. Simplify $\dfrac{(\frac{1}{4}\div\frac{1}{5}\text{ of }\frac{1}{7})-(\frac{1}{4}\div\frac{1}{5}\times\frac{1}{7})}{\frac{8}{7}\text{ of }\frac{3}{10000}\div\frac{1}{4}}$.

Explanation. The above problem is given to illustrate the difference in the use of the signs "×" and "of" in fractional questions. The first step in the solution of the problem is not to divide $\frac{1}{4}$ by $\frac{1}{5}$ as we might be inclined to work it. $\frac{1}{5}$ of $\frac{1}{7}$ forms a compound fraction and must be simplified and the result used as the divisor. In the second part of the fraction it is different. The $\frac{1}{4}$ is divided by $\frac{1}{5}$ at once and the result multiplied by $\frac{1}{7}$. Of course short methods of doing this should be adopted; as, for instance, $\frac{1}{4}\times\frac{5}{1}\times\frac{1}{7}$. The point to remember is that $\frac{1}{5}$ of $\frac{1}{7}$ forms a compound fraction, while $\frac{1}{5}\times\frac{1}{7}$ does not. The answer to the problem is 6250.

8. Simplify $\frac{2}{3}\times\frac{4}{5}\div\frac{6}{7}$.
10. Simplify $\frac{2}{3}\div\frac{4}{5}\times\frac{6}{7}$.
9. Simplify $\frac{2}{3}$ of $\frac{5}{9}+\frac{3}{7}\div\frac{4}{5}$.
11. Simplify $\frac{3}{7}+\frac{5}{9}$ of $\frac{10}{3}-\frac{2}{21}$.
12. Simplify $\dfrac{5\frac{1}{2}\text{ of }\frac{2}{9}\text{ of }2\frac{4}{7}-1\div(\frac{1}{5}+\frac{1}{2})}{15+3\times15-4\div\frac{2}{3}}$.

WRITTEN EXERCISE

1. If $3\frac{1}{2}$ bushels of apples cost $2.40, what will $\frac{2}{3}$ of a bushel cost?

2. The product of four factors is $\frac{34}{35}$, and three of them are respectively $\frac{2}{7}$, $3\frac{5}{21}$, and 49. Find the fourth factor.

3. A woman sold two tubs of butter, weighing together 75 lbs., one being $5\frac{1}{2}$ lbs. heavier than the other. How much did each tub weigh?

4. What is the average price of 25 cows, if 13 of them cost $50 a head and the others $75 a head?

5. A workman received $23.40 for making brooms at $7.20 per C. How many did he make?

6. Simplify $\dfrac{(3\frac{1}{5}-2\frac{1}{2})\div\frac{5}{6}\text{ of }\frac{3}{8}}{2\frac{2}{3}\div(\frac{1}{2}+\frac{1}{4})}$.

7. A grocer bought 10 doz. oranges at 15¢ a doz., kept 2 out of every score for his own use, and sold the remainder, half at 2 for 5¢ and half at 3 for 10¢. Find his gain on the whole cost.

8. Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}\times15$ by $6\frac{2}{3}\times\frac{5}{8}\times\frac{3}{5}$.

9. Four men in partnership gain $$7654\frac{2}{3}$. If they share equally, what will be each one's share of the gain?

FRACTIONAL RELATIONS

206. The **relation of numbers** is their relative value as compared with one another.

Thus, the relation of 6 to 3 is the number of times 6 contains 3, which is 2 times. The relation of 3 to 6 is the part 3 is of 6, which is $\frac{3}{6}$, or $\frac{1}{2}$.

207. Principle. *Only like numbers are so related as to be compared with each other.*

ORAL EXERCISE

1. What part of 6 is 5?

Solution. Since 1 is $\frac{1}{6}$ of 6, 5 is 5 times $\frac{1}{6}$, or $\frac{5}{6}$; or, it is 5 divided by 6, which is $\frac{5}{6}$.

2. What part of \$8 is \$1? Of \$9 is \$2.

3. What part of 7 yards is 1 yard? What part of $\frac{5}{9}$ is $\frac{1}{9}$?

4. What part of \$45 is \$15? \$60 is \$40? \$72 is \$60?

5. What part of $4\frac{1}{2}$ is 3?

Solution. $4\frac{1}{2}=\frac{9}{2}$, and $3=\frac{6}{2}$. $\frac{6}{2}$ is $\frac{6}{9}$ of $\frac{9}{2}$. Therefore 3 is $\frac{6}{9}$ of $4\frac{1}{2}$.

What part of:

6. $3\frac{1}{3}$ is 2?
7. $5\frac{1}{4}$ is 3?
8. $6\frac{2}{5}$ is 4?
9. $7\frac{1}{2}$ is 6?
10. $8\frac{3}{4}$ is 7?
11. $6\frac{2}{3}$ is 5?
12. $9\frac{3}{8}$ is 5?
13. $10\frac{1}{2}$ is 3?
14. $12\frac{1}{2}$ is 5?

CASE I

208. To find what part one number is of another.

Example. What part of $\frac{10}{21}$ is $\frac{3}{7}$?

$$\frac{\frac{3}{7}}{\frac{10}{21}}=\frac{3}{7}\div\frac{10}{21}=\frac{9}{10}$$

Solution. $\frac{3}{7}$ is such a part of $\frac{10}{21}$ as $\frac{3}{7}$ divided by $\frac{10}{21}$, which is $\frac{9}{10}$. Or, we may say $\frac{3}{7}=\frac{9}{21}$, and therefore $\frac{9}{21}$ is $\frac{9}{10}$ of $\frac{10}{21}$.

209. Rule. *Divide the number denoting the part by the number denoting the whole, and the quotient will be the relation required.*

WRITTEN EXERCISE

What part of:

1. 72 is 60?
2. 75 is $\frac{2}{5}$?
3. 75 is $18\frac{3}{4}$?
4. $37\frac{1}{2}$ is $6\frac{1}{4}$?
5. \$100 is \$$16\frac{2}{3}$?
6. $15\frac{1}{2}$ in. is 12 in.?
7. 1000 is 125?
8. \$10 is \$$1.66\frac{2}{3}$?
9. $16\frac{4}{5}$ is $\frac{7}{8}$?
10. $\frac{9}{10}$ is $\frac{3}{7}$?
11. $\frac{15}{16}$ is $\frac{4}{5}$?
12. $\frac{9}{7}$ is $\frac{8}{7}$?

13. If from $18\frac{3}{4}$ yds. of cloth $2\frac{7}{8}$ yds. are cut, what part of the whole is taken?

14. If 7 yds. of cotton cost \$$2\frac{3}{5}$, what part of a yard can be bought for \$$\frac{2}{7}$?

15. If 15 tons of coal cost \$$112\frac{1}{2}$, what part of \$$112\frac{1}{2}$ will $\frac{3}{4}$ of a ton cost?

CASE II

210. To find a number when a fractional part of it is given.

Example. $16\frac{2}{3}$ is $\frac{5}{7}$ of what number?

$16\frac{2}{3} \div \frac{5}{7} = 23\frac{1}{3}$

Solution. Since $16\frac{2}{3}$ is $\frac{5}{7}$ of a number, $\frac{1}{7}$ of the number is $\frac{1}{5}$ of $16\frac{2}{3}$, or $3\frac{1}{3}$; and $\frac{7}{7}$ or the whole is 7 times $3\frac{1}{3}$, which is $23\frac{1}{3}$. Or, $16\frac{2}{3} \div \frac{5}{7} = 23\frac{1}{3}$.

211. Rule. *Divide the number denoting the part by the given fraction, and the quotient will be the number required.*

ORAL EXERCISE

1. 3 is $\frac{1}{2}$ of what number?
2. 9 is $\frac{1}{5}$ of what number? $\frac{1}{7}$ of what? $\frac{1}{9}$ of what?
3. 10 is $\frac{2}{5}$ of what number? $\frac{5}{7}$ of what? $\frac{1}{10}$ of what?
4. $\frac{1}{2}$ of 20 is $\frac{2}{5}$ of what? $\frac{5}{7}$ of what? $\frac{10}{3}$ of what?
5. $\frac{4}{7}$ of 28 is $\frac{8}{9}$ of 3 times what number?

WRITTEN EXERCISE

Find the number of which:

1. 75 is $\frac{3}{4}$. 3. 180 is $\frac{4}{5}$. 5. $24\frac{3}{4}$ is $\frac{2}{5}$. 7. 234 is $\frac{2}{3}$.
2. 84 is $\frac{7}{8}$. 4. 324 is $\frac{3}{7}$. 6. $43\frac{2}{7}$ is $\frac{2}{3}$. 8. 643 is $\frac{8}{11}$.

9. A man sold $\frac{5}{9}$ of his wheat and had 396 bushels left. How many bushels did he sell?

10. If 6 pounds of butter are worth $\$3\frac{3}{4}$, how many pounds can I buy for $\$6\frac{3}{4}$?

11. If $\frac{19}{24}$ of the value of a piece of property is $389.50, what is the value of the remainder?

12. D started on a journey of $89\frac{7}{16}$ miles, and the first day traveled $66\frac{1}{4}$ miles. What part of the distance remained for the second day?

13. If $\frac{2}{7}$ of a line is 60 feet, how long is $\frac{3}{7}$ of the line?

14. A boy spent $\frac{1}{5}$ of his money for a 40-cent knife. How much did he have left?

15. If a man receives $\$2\frac{1}{2}$ for $\frac{2}{3}$ of a day's work, how much will he receive if he works 4 days?

CASE III

212. To find a number that is intermediate between two or more other numbers.

213. The process of finding a medial sum or quantity, or a mean proportion between two or more numbers is called **average.**

Example. If a grocer mixes tea worth respectively 30¢, 40¢, and 60¢ a pound, what is 1 lb. of the mixture worth?

$$\begin{array}{r} 30¢ \\ 40 \\ \underline{60} \\ 3\overline{)130¢} \\ 43\frac{1}{3}¢ \end{array}$$

Solution. Since the whole mixture is worth the sum of 30¢, 40¢, and 60¢, or 130¢, 1 lb. is worth $\frac{1}{3}$ of 130¢, or $43\frac{1}{3}$¢.

NOTE. To find the average of consecutive numbers, add the highest number to the lowest, and divide by 2.

WRITTEN EXERCISE

1. In an orchard, 94 apple trees yield 17 bushels of apples each, and 45 others yield 27 bushels each. What is the average yield per tree?

2. If different grades of coffee, worth respectively 18, 22, and 28 cents per pound, be mixed in the following proportions: 5 lb. of the first, 10 lb. of the second, and 15 lb. of the third, what will the mixture be worth per pound?

3. If I buy oranges at 10¢ a dozen and sell them at 4 for 5¢, how many must I sell to gain the cost price of 30 oranges?

4. What is the cost of 12 doz. pencils, at the rate of 3 for 5 cents?

5. A merchant's profits and losses for 5 consecutive years were as follows: first year $2765.50 profit; second year $234 loss; third year $4320 profit; fourth year $3568.75 profit; fifth year $6235.65 profit. What were his average profits per year?

6. If a boy buys peaches at the rate of 5 for 2¢, and sells them at the rate of 4 for 3¢, how many must he buy and sell to make a profit of $4.20?

CASE IV

214. To divide a number into parts, proportional to other given numbers.

215. The process of dividing a number into parts proportional to several given numbers is called **partitive proportion.**

Example 1. Divide 24 into parts to each other as 2, 3, and 7.

Solution. Since the parts are proportional to 2, 3, and 7, the number must be divided into 12 parts (the sum of 2, 3, and 7). Then one number must be $\frac{2}{12}$ of 24, or 4; another $\frac{3}{12}$ of 24, or 6; and the third $\frac{7}{12}$ of 24, or 14. Therefore the parts are 4, 6, and 14.

Example 2. Divide $798 into parts proportional to $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{2}{5}$.

Solution. Fractions have the ratios of their numerators when their denominators are the same; hence change the fractions to their least common denominator, and we have $\frac{40}{120}$, $\frac{45}{120}$, $\frac{48}{120}$. Then divide $798 into parts proportional to 40, 45, and 48 as shown in Example 1 and the parts will be found to be $240, $270, and $288.

WRITTEN EXERCISE

1. Divide $600 among A, B, and C, in the ratio of 3, 4, and 5.

2. A bankrupt owes four creditors as follows: A $2500; B $3300; C $4200; and D $4000. His property is worth $10500, what does each creditor receive?

3. Divide 440 into three parts that shall be in the ratio of 3, $3\frac{1}{3}$, and 7.

4. Divide $82.60 among 27 men and 37 boys, so that each man may have three times as much as a boy.

5. Divide $640 among A, B, and C, so that B may have 6 times as much as A, and C $1\frac{1}{2}$ times as much as B.

REVIEW OF FRACTIONS

ORAL EXERCISE

Find the value of:

1. $5 \div \frac{2}{3}$. 3. $16 \div \frac{2}{5}$. 5. $\frac{2}{5} \div 5$. 7. $\frac{2}{7} \div 7$.
2. $9 \div \frac{3}{4}$. 4. $18 \div \frac{2}{7}$. 6. $\frac{3}{8} \times 6$. 8. $\frac{3}{4} \div 6$.

9. What part of 27 is $\frac{1}{2}$ of 13?

10. A man earns $18 in 11 days; how much can he earn in $5\frac{1}{2}$ days?

11. What is the cost of $9\frac{1}{2}$ yds. of cloth at $9\frac{1}{2}$¢ a yard?

12. Divide $35 between two men so that one gets $4 as often as the other gets $3.

13. What is $\frac{3}{4}$ of 60? Of what is 60 three-fourths?

14. What part of a dollar is 5¢, $12\frac{1}{2}$¢, $33\frac{1}{3}$¢, $16\frac{2}{3}$¢, $66\frac{2}{3}$¢, $87\frac{1}{2}$¢, $8\frac{1}{3}$¢, $83\frac{1}{3}$¢?

15. (a) $\frac{1}{4}$ of 80 is how many times $\frac{1}{6}$ of 30?
    (b) 35 is $\frac{5}{6}$ of how many?
    (c) $\frac{3}{4}$ of 44 is $\frac{11}{12}$ of what number?
    (d) $\frac{3}{5}$ of 25 is $\frac{1}{3}$ of what number?

16. If a man cut 8 cords of wood in $\frac{4}{5}$ of a week, how much can he cut in $2\frac{1}{2}$ weeks?

17. If 4 bbls. of apples cost $\$14\frac{2}{5}$, what will $\frac{5}{12}$ of a bbl. cost?

18. If $2\frac{1}{2}$ bu. of wheat cost $5, what will $\frac{4}{5}$ bu. cost?

19. Divide 21 into three parts, that the second shall be double the first, and the third double the second.

20. A house and lot cost together $2400; the lot cost $\frac{1}{7}$ of what the house cost. Find the cost of each.

Solution. Let $\frac{7}{7}$ equal the cost of the house, and the lot $\frac{1}{7}$; hence both will cost $\frac{8}{7}$, which is equal to $2400 the cost of both. Then, since $\frac{8}{7} = \$2400$, $\frac{1}{7}$ will equal $\frac{1}{8}$ of $2400, or $300 the cost of the lot, and $\frac{7}{7}$ will cost 7 times $300, or $2100 the cost of the house. Therefore the lot cost $300, and the house $2100.

21. A man invested $\frac{3}{4}$ of his money in a farm and had $1500 left. How much did he pay for the farm?

22. A man after spending $\frac{3}{4}$ of his money, found that $\frac{2}{9}$ of what remained equaled $20, what was his money?

23. A pole is $\frac{2}{5}$ in the water, $\frac{1}{4}$ in the mud, and 14 feet out of the water. How long is the pole?

24. $\frac{1}{4}$ of 6 is what part of 9? What part of 15?

25. Divide the number 85 into two such parts that $\frac{3}{4}$ of the first shall equal $\frac{2}{3}$ of the second.

26. Bought 30 oranges and for $\frac{1}{3}$ of them I paid 2¢ for 3, for the remainder 3¢ for 2. For what must I sell them to double my money?

27. $\frac{2}{3}$ of the sum of two numbers is 1000. Their difference is 500, what are the numbers?

28. A can do a piece of work in 8 days, and B can do the same work in 6 days. In how many days can they both do it, working together?

Solution. Since A can do the work in 8 days, he can do $\frac{1}{8}$ of it in 1 day, and B can do $\frac{1}{6}$ of it in 1 day. Together they can do the sum of $\frac{1}{8}$ and $\frac{1}{6}$ or $\frac{7}{24}$ of it in 1 day. If together they do $\frac{7}{24}$ of the work in a day, to do $\frac{24}{24}$ or the whole work requires as many days as $\frac{7}{24}$ is contained in $\frac{24}{24}$, or $3\frac{3}{7}$ days.

29. A and B working together can do a certain piece of work in 6 days, A alone can do it in 10 days. How long will it take B to do it working alone?

30. A bicyclist rides 12 miles an hour and walks 4. His wheel breaks and he is obliged to walk to his home, which he reaches 8 hours after starting. How far did he go?

31. A boy found 20¢, and then lost $\frac{3}{5}$ of what he found, and then had $\frac{3}{2}$ as much as he had at first; how much had he at first?

32. After losing $\frac{2}{5}$ of my money I found 70¢. I then had 10¢ more than I had at first. How much had I at first?

33. Two men traveling in the same direction are 60 miles apart; the one in advance travels 5 miles an hour, and the other 7 miles an hour. In how many hours will the latter overtake the former?

34. What part of $\$5\frac{1}{4}$ is $\$1\frac{1}{2}$?

35. Reduce $5\frac{1}{2}$ to 4ths. $\frac{4}{5}$ to 16ths.

36. A can cut $\frac{1}{2}$ cord of wood in a day; B can cut a cord in $\frac{1}{2}$ a day. How long will it take both working together to cut 3 cords?

37. If $\frac{1}{7}$ of a piece of land is worth $80, how much is half the remainder worth?

38. A boy agreed to carry 25 glasses to a certain place for 6¢ apiece, on condition that for each one he broke he should forfeit 10¢. He received 70¢, how many did he break?

39. If 4 horses eat 21 bushels of grain in 6 days, how many bushels will 8 horses eat in 12 days?

40. A can earn $10 in $2\frac{1}{2}$ days, and B in $3\frac{1}{3}$ days. In how many days can they together earn $48?

41. A coat and vest cost $27. The vest cost $2 more than $\frac{1}{4}$ the cost of the coat. What was the cost of each?

42. Find the cost of 1250 ft. of lumber at $12 per M.

WRITTEN REVIEW

1. (a) $27+3\times5-48\div6=?$ (b) $\frac{2}{3}\div\frac{5}{6}$ of $\frac{9}{35}=?$

2. $\dfrac{27+9\times7}{18\div3+8\frac{2}{5}}-\dfrac{2\frac{3}{5}}{1\frac{6}{7}}=?$

3. If $\frac{3}{4}$ of $\frac{4}{5}$ of a cord of wood costs $\frac{3}{7}$ of \$2.10, how many cords can be bought for \$33?

4. What number increased by $\frac{3}{7}$ of itself equals 970?

5. Three men invested \$1200; the first put in \$200, the second \$700, and the third \$300. They gained \$368; what was each man's share?

6. The factors of the dividend are $18\frac{1}{2}$, 15, $24\frac{3}{4}$, and $7\frac{1}{3}$, while those of the divisor are $17\frac{3}{5}$, $8\frac{2}{9}$, and $19\frac{4}{5}$. Find the quotient by cancellation.

7. Divide 456 into two such parts that the second shall be $\frac{11}{17}$ of the first.

8. The sum of three numbers is 940. The first number equals $\frac{5}{9}$ of the second, and the second equals $\frac{7}{10}$ of the third. Find the numbers.

9. Reduce $\frac{18}{32}$ (a) to a fraction whose denominator is 48; (b) to a fraction whose denominator is 63.

10. A real-estate agent bought land for \$9750, and sold it so as to gain $\frac{1}{13}$ of the cost, the gain being \$5 an acre; how many acres did he buy?

11. What is the greatest length of boards I can use in fencing my garden, the sides of which are 168 ft., 182 ft., 280 ft., and 252 ft.?

12. A merchant bought a quantity of flour for \$$2173\frac{1}{2}$, and sold it for $\frac{22}{23}$ of the cost, thereby losing 25¢ on a barrel. How many barrels of flour did he purchase?

13. A and B traded upon equal capitals; A gained a sum equal to $\frac{2}{5}$ of his capital, and B a sum equal to $\frac{11}{40}$ of his; B's gain was \$500 less than A's; what was the capital of each?

14. A retailer bought 6 bbl. of flour at \$6.25 per barrel, and 465 lb. of sugar at $4\frac{3}{4}$¢ a pound; he gave in payment a fifty and a ten-dollar bill. How much change should he receive?

15. Divide 2010 into three parts having the ratio of $\frac{4}{5}$, 1, and $2\frac{2}{3}$.

16. A and B worked for 17 days and received \$72.25; $\frac{2}{3}$ of A's wages equal $\frac{3}{4}$ of B's daily wages. How much should each receive?

17. Find the number which is 207 more than $\frac{1}{5}$ and $\frac{1}{7}$ of itself.

18. At a certain election there were 327,140 votes cast for three candidates; the successful candidate received a majority of 28 over the other two, whose votes were in the ratio of 1 to 3. How many votes were cast for each candidate?

19. A, B, and C engage in partnership, A investing $3000, B $4000, and C $5000. What part of the total capital did each invest?

20. Divide $990 among A, B, and C, giving B twice as much as C, and A $30 more than $\frac{2}{3}$ of the remainder.

21. A fruit peddler had 10 pears which he offered at 2 for 5¢, and 15 apples at 3 for 5¢. A boy bought the 25 at 5 for 10¢. Did the peddler gain or lose, and how much?

22. From a piece of cloth measuring $36\frac{1}{2}$ yds., a merchant sold $8\frac{1}{2}$ yds. at $1.25 a yard; $12\frac{3}{4}$ yds. at $\$1.12\frac{1}{2}$; and the remainder at 95¢ a yard, throwing in $\frac{1}{4}$ of a yard that was damaged. How much did he make on the whole, if it cost him $87\frac{1}{2}$¢ a yard?

23. If $40 exceeded by $\$10\frac{1}{2}$ is $\frac{3}{4}$ of what I gained by the sale of a house and lot, how much did I gain?

24. Two trains are $97\frac{3}{4}$ miles apart, and running toward each other on double tracks, one at the rate of $48\frac{1}{2}$ miles an hour, the other at the rate of $36\frac{3}{4}$ miles an hour; how far apart will they be in two hours?

25. $\frac{5}{8}$ of a number is $24\frac{3}{4}$ less than the number. What is the number?

26. A can do a piece of work in 14 days, and A and B together can do the same work in 8 days. How many days ought it to take B alone to do the work?

27. If peaches are worth $\$\frac{4}{5}$ a basket, and a man sells them at $\$\frac{5}{4}$ a basket, how many baskets will it take to yield a profit of $81?

28. A train passes 15 telegraph poles every minute; if the distance between poles is 60 yards, at what rate per hour is the train going?

29. The profits of a garden for two years were $1456; the

profits of the second year being $\frac{27}{25}$ of those of the first. Find the profits of each year.

30. A pipe can fill a cistern in 3 minutes, and another can fill it in 5 minutes, while a third can empty it in $7\frac{1}{2}$ minutes. If it be empty, in what time will the cistern be filled, when all pipes are open?

31. From a cask of wine $\frac{1}{4}$ is drawn off, and the cask is filled up with water; $\frac{1}{4}$ of the mixture is then drawn off, and the cask again filled up with water. After this has been done four times altogether, what fraction of the original quantity of wine will be left in the cask?

32. A merchant purchased 15 pieces of prints, containing 45^3 44, 51^2, 47^1, 43, 52^2, 49^3, 54, 45^2, 46, 52^1, 43, 48^1, 50^2, 53^3 yards respectively; what was the cost at 4^2 cents per yard?

33. A workman in a factory receives $4 per day and 60¢ per hour for overtime. His time for one week is as follows: Monday, $9\frac{1}{2}$ hr.; Tuesday, 8 hr.; Wednesday, $8\frac{1}{2}$ hr.; Thursday, $10\frac{1}{2}$ hr.; Friday, 9 hr.; Saturday, $12\frac{1}{2}$ hr. How much should he be paid for the week's work, assuming that a day's work is 8 hours?

34. What number is that which, being increased by its half, its fourth, and 23 more will be doubled?

35. $\frac{4}{5}$ of a farm is given to a man's oldest son; $\frac{2}{3}$ of the remainder to the second son, and the remainder to the third son; the second son gets 30 acres more than the third. How many acres in the farm?

36. A can do a piece of work in $\frac{1}{4}$ da., B in $\frac{1}{5}$ da., and C in $\frac{1}{7}$ da. In what time can all do it working together?

37. How many acres does a farm contain, if $\frac{3}{8}$ of it is in grass, $\frac{5}{16}$ in corn, $\frac{1}{4}$ in wheat, and the remaining 16 acres in oats?

38. Simplify $\dfrac{5\frac{1}{2}-3\frac{3}{4}}{\frac{2}{5}+\dfrac{15}{3\frac{1}{2}+2\frac{3}{4}}}$ showing the work.

39. A man bought a span of horses, a harness and a carriage, for $320. If he paid $1\frac{1}{2}$ times as much for the carriage as for a horse and twice as much for a horse as for the harness, how much did he pay for the carriage?

40. In an 80-bushel mixture of peas and oats, the peas were $\frac{5}{8}$ of the whole. How many bushels of oats must be added to make the mixture half and half?

41. A man breathes on an average 17 times a minute, and takes in about $\frac{5}{7}$ of a quart of air at each breath; how many gallons of air will he breathe in 12 hours?

42. If a mixed metal be composed of 11 parts of tin to 100 parts of copper, how much copper and how much tin will be required to make 24 cwt. of this metal?

43. A and B together can do a piece of work in 12 days. If A can do only $\frac{3}{4}$ as much as B, how long will it take each of them to do the work?

44. C can dig a well in 25 days, and C and D in 15 days. How long will it take D to dig what remains after C has dug $\frac{1}{4}$ of it?

45. Six lots of land measuring $6\frac{3}{4}$, $8\frac{2}{3}$, $12\frac{7}{8}$, $9\frac{1}{2}$, $15\frac{5}{12}$, and $8\frac{1}{8}$ acres respectively, were bought at \$187.12½ an acre and sold at \$210 an acre. Find the gain resulting from the transaction.

46. Two men were born on the first day of January, one in the year 1876, the other in 1882; both died 27 years after they were born, upon the first of January; required the difference of their ages.

47. The shadow of a tree measures 42 feet; a staff 40 inches in length casts a shadow 18 inches at the same time. What is the height of the tree?

48. If 5 men, 10 women, or 15 boys can do a work in 33 days, in what time will 30 men, 30 women, and 30 boys do the same work, if they all work at it together?

49. Reduce $\frac{2\frac{1}{2}-\frac{5}{6}}{2\frac{1}{2}+\frac{5}{6}}+\frac{7}{12}$ of $\frac{9\times 10}{14\times 3}-\frac{22\frac{1}{2}}{30}$ to its simplest form.

50. A man can dig 25 bushels of potatoes in a day, and pick up 75 bushels in the same time. How many bushels can he dig and pick up in 8 days?

51. It costs 20¢ a line to insert an advertisement in a newspaper the first time, 5¢ a line for each of the ten subsequent insertions, and then 2½¢ a line. If a man is charged \$4.80 for a space of 6 lines, how often should his advertisement appear?

52. By what must the sum of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{3}{10}$, $3\frac{1}{3}$, and $6\frac{1}{3}$ be divided to equal $3\frac{3}{4}$?

53. A stove manufacturer purchases old iron at 75¢ per hundred pounds, and makes out of it stoves weighing 150 pounds each, which he sells at $17.50 apiece. How much does he gain on each 500 pounds of old iron?

54. If a garrison of 1000 men have provisions for 12 months, how long will the provisions last if at the end of 2 months they be reinforced by 500 men?

55. A person mixed 25 lb. of tea at 25¢ a pound; 34 lb. at 40¢ a pound, and 41 lb. at 55¢; what must the mixture be sold per pound to gain $\frac{1}{4}$ of the cost?

56. A train 150 yards long, running 25 miles per hour, crosses a bridge in $22\frac{1}{2}$ seconds. Find the length of the bridge.

57. How many yards of cloth at $5 a yard, must a merchant buy, that by selling it at a profit of $\frac{1}{3}$ the cost he may gain $100?

58. Find the quantity of coal consumed by a steamer for a voyage of 4043 miles, supposing her rate per hour to be $14\frac{1}{25}$ knots, and the coal used 87 tons per day, considering a knot as $\frac{311}{270}$ of a mile?

59. Mr. B sold $\frac{3}{8}$ of his farm for $2400, at $30 an acre, losing $480 on the part sold. For how much per acre should he sell the remainder of the farm to gain $500 on the whole farm?

DRILL EXERCISE

1. Define arithmetic; integer; fraction; mathematical sign.

2. State the difference between an abstract and a concrete number, and illustrate; between like and unlike numbers.

3. Name the fundamental operations in arithmetic. What is the difference between the simple and the local value of a figure?

4. Define prime number; composite number; least common multiple; greatest common divisor; an equation; a problem; a rule.

5. Write in Roman notation 2349; in Arabic notation MMMXLIV; in Arabic notation, fifty million three thousand nine.

6. Give an exact divisor of 21; a common multiple of $\frac{2}{3}$ and 5; two factors of $3\frac{2}{11}$ one of which is an integer.

7. Give a full classification of each of the following numbers: 264; $\frac{3}{7}$ lb.; 11 horses. Write in words $301\frac{26}{41}$.

8. Express the number which is just as much less than 120 as it is larger than 30; the average of 12, 16, 19, and 21.

9. Write a number which will be odd, composite, integral, concrete.

10. Discriminate between the terms: difference and remainder; unit of a fraction and fractional unit.

11. Define improper fraction; mixed number; horizontal vinculum; compound fraction; cancellation.

12. What is the effect upon the value of a fraction of dividing the denominator by an integer; of multiplying both terms by the same numbers; of adding the same number to both terms?

13. A careless student in attempting to divide a fraction by $\frac{11}{17}$, inverted the dividend instead of the divisor, and thereby got the quotient, $1\frac{2}{61}$. What was the fraction?

14. Count by $3\frac{1}{3}$ to 40; by $3\frac{1}{8}$ to 50; $8\frac{1}{3}$ to 100.

15. What is the reciprocal of a fraction? Show how the general principles of division apply to fractions.

16. Define factor; multiple; exact divisor; power. Square: $2\frac{1}{2}$, 75, $7\frac{1}{2}$, 25, $5\frac{1}{2}$, $6\frac{1}{2}$, $10\frac{1}{2}$, 65, 35. Multiply orally: 23 times 27; 56 times 54; 44 times 46.

DECIMAL FRACTIONS

216. A **Decimal Fraction** or a *decimal* is one or more of the decimal divisions of a unit, and has for its denominator 10 or some power of 10; as, $\frac{2}{10}$, $\frac{6}{100}$, $\frac{7}{1000}$.

Decimal fractions are quite similar to other fractions, except that common fractions arise from the division of a unit into any number of equal parts; while *decimal fractions* arise from the division of a unit into ten, one hundred, one thousand, etc., equal parts, and can have only 10, 100, 1000, etc., as the denominator.

217. The trouble of writing the denominators of decimals is obviated by writing the numerator only, the denominator being indicated by the position of a point (.); as,

We may write $55\frac{5}{10}$, or 55.5; the point (.) is the *unit point*, or the decimal point, and shows us that the figure to the left is units. The figure to the right is *tenths*. Note that the figure to the *left* of the units is *tens;* the figure to the *right* of units is *tenths*.

The decimal point is not to be magnified. It has no more to do with "decimals" than with whole numbers. All that it ever does is to locate *units;* they locate everything else.

218. One of the equal decimal parts into which a unit is divided is called a **decimal unit;** as, .1 is a *decimal unit;* so, also, is .01, .001, etc.

219. The places at the right of the decimal point are called *decimal places*. The first place at the right is *tenths*, the second place is *hundredths*, etc.

220. It must be kept in mind that the position of the decimal unit indicates the *denominator* and determines the value of the decimal expression; as,

$\frac{5}{10}$ is written .5 $\quad\quad$ $\frac{125}{1000}$ is written .125
$\frac{5}{100}$ " " .05 $\quad\quad$ $\frac{125}{10000}$ " " .0125
$\frac{5}{1000}$ " " .005 $\quad\quad$ $\frac{16}{100000}$ " " .00016

221. The method of expressing decimal fractions and their relation to integers is but a continuation of the method of notation for integers. It is clearly shown by the following:

NUMERATION TABLE

Millions	Hundred-tho'ds.	Ten-thousands	Thousands	Hundreds	Tens	Units	Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-tho'dths	Millionths
5	4	6	2	7	4	8	.	1	2	3	4	8	7
Integers								Decimals					

The above number is read, 5 million 462 thousand 748, and 123 thousand 487 millionths.

222. In decimals, as in integers, ***units*** is the starting-point of notation and of numeration. The scale is extended to the *left* of the units' place in writing ***integers***, and to the ***right*** of the units' place in writing *decimals*.

223. Rule. *1.* ***Numerate toward the decimal point to determine the numerator, and from the decimal point to the right to determine the denominator of the decimal.***

2. ***Read the decimal as though it were an integer and give it the name of the right-hand order.***

224. In reading integers and decimals as mixed decimals the word *and* should be omitted, except between the integer and the decimal.

Thus in reading 3.5, say *three, and five tenths;* 40000.0325, say *forty thousand, and three hundred twenty-five ten-thousandths.*

In writing decimals, *vacant orders* must be filled with ciphers.

The *numerator* of a decimal fraction when expressed alone, must have as many decimal places as there are ciphers in the denominator. Thus, $\frac{8}{100}=.08$; $\frac{15}{1000}=.015$, etc.

ORAL EXERCISE

Express in words:

1. 23.02
2. 65.023
3. 40.005
4. 11.111
5. 0.705
6. 0.0070
7. 0.00255
8. 6.600606
9. 9009.900009
10. 5000.05005505
11. 1234.12345678
12. 4321.000000007

13. Read the following: .02; .002; .0002; .00002; 230.023; 16.66$\frac{2}{3}$; 200.133$\frac{1}{3}$; 87.87$\frac{1}{2}$.

14. Read the following: $8.875; .0045 T.; there are $4.8665 in a pound sterling; there are 2150.42 cu. in. in a bushel; there are 1.152$\frac{2}{3}$ statute miles in a *knot*.

225. A **pure decimal** is one which has only decimal figures; as, .245; .0545.

226. A **mixed decimal** number is one which contains an integer and a decimal ; as, 25.055.

227. A **complex decimal** is one which has a common fraction at the right of the decimal; as, .235$\frac{2}{3}$.

228. General Principles. *1. Annexing a cipher to a decimal does not alter its value, since it does not change the place of any figure of the decimal.*

2. Each removal of the decimal point one place to the left divides the value of the decimal by 10.

3. Each removal of the decimal point one place to the right multiplies the value of the decimal by 10.

NOTE. There should be considerable practice in the reading and writing of decimals, using the numbers to be found in the regular work. They should all be read by the pupils, and should then be written by them from the teacher's reading before any work is done.

Notation of Decimals

229. Write 75 thousandths in the form of a decimal.

Solution. Since *thousandths* occupy the third place, three figures are required to express the decimal. Hence it is necessary to prefix a cipher to bring the right-hand figure in the proper place. Therefore write *point, naught, seven, five,* or .075 the decimal required.

230. Rule. *Write the numerator of the decimal as if it were an integer, and so place the decimal point that the right-hand figure shall stand in the place denoted by the denominator of the decimal.*

NOTE. The author has found this an excellent method in writing decimals:

write the ***numerator*** of the decimal as if it were an integer, then at the right of the number written numerate, beginning with ***tenths***, and so continue the numeration until the required denominator is reached, filling ***vacant orders*** with ciphers as the orders are named. Then fix the point, and the result is the required decimal.

WRITTEN EXERCISE

Write decimally:

1. 45 hundredths.
2. 33 thousandths.
3. 245 thousandths.
4. 16 ten-thousandths.
5. 256 millionths.
6. 34 hundred-thousandths.
7. Six hundred six and sixteen hundredths.
8. Four thousand and four thousandths; four hundred and four hundredths.
9. Two hundred fifty-six and two hundred fifty-six millionths.
10. Seven hundred and seven hundredths; seven hundred seven hundredths; ten thousand ten-thousandths.
11. Five thousand five and five hundred-thousandths; eight million eighty thousand and eighty-five millionths.
12. 103 thousand 10 and 406 ten-millionths; 8 billion 801 million 725 thousand 540 and 125 thousand 305 hundred-billionths.
13. Seven hundred and two thousandths; seven hundred two thousandths; three hundred and five ten-thousandths.
14. Write the following as decimals, so that the decimal points stand in the same vertical line: 9 tenths; 25 hundredths; 10 and 16 ten-thousandths: 2345 millionths; 23 ten-thousandths; 245 thousandths; 306 ten-millionths; 1728 ten-thousandths.
15. Express decimally $\frac{15}{100}$, $\frac{2}{1000}$, $19\frac{250}{1000}$, $234\frac{65}{10000}$, $23\frac{42}{1000}$.
16. Express the following in words: .0245; 65.0005; 2002.003003.
17. Read aloud the following: 0.600; 25.0010; 0.050; 0.0000400. Are these decimals in their simplest form?

Reduction of Decimals

231. To reduce a decimal to a fraction.

Example 1. Reduce .375 to a common fraction.

$$.375 = \frac{375}{1000} = \frac{3}{8}$$

Solution. .375 expressed as a common fraction is $\frac{375}{1000}$, or $\frac{3}{8}$, when it is reduced to its lowest terms.

Example 2. Reduce $.133\frac{1}{3}$ to a common fraction.

$$.133\frac{1}{3} = \frac{133\frac{1}{3}}{1000} = \frac{133\frac{1}{3} \times 3}{1000 \times 3} = \frac{400}{3000} = \frac{2}{15}$$

Solution. Reduce the complex fraction to a simple fraction by multiplying both terms by the denominator 3, and reducing to its lowest terms gives us the required fraction, $\frac{2}{15}$.

232. Rule. *Omit the decimal point, supply the proper denominator and reduce the fraction to its simplest form.*

ORAL EXERCISE

Reduce to common fractions in their lowest terms:

1. .25
2. .15
3. .40
4. .30
5. .75
6. $.16\frac{2}{3}$
7. $.33\frac{1}{3}$
8. $.12\frac{1}{2}$
9. .125
10. .025

WRITTEN EXERCISE

Reduce to equivalent fractions in their simplest form:

1. .35
2. .56
3. .85
4. $.41\frac{2}{3}$
5. $.37\frac{1}{2}$
6. .105
7. $.066\frac{2}{3}$
8. .512
9. .372
10. $.097\frac{2}{9}$
11. $.104\frac{1}{6}$
12. .216
13. .425
14. $.008\frac{1}{3}$
15. $.313\frac{1}{3}$
16. .0144
17. .1878
18. $.053\frac{4}{7}$
19. 2.125
20. $.197\frac{2}{9}$
21. $27.12\frac{1}{2}$
22. $16.042\frac{6}{7}$
23. $13.0133\frac{1}{3}$
24. 48.016

233. To reduce a fraction to a decimal.

Example. Reduce $\frac{5}{8}$ to an equivalent decimal.

$$8\overline{)5.000}$$
$$.625$$

Solution. Since $\frac{5}{8} = \frac{1}{8}$ of 5 units, and 5 units equals 5000 *thousandths*, $\frac{1}{8}$ of 5000 *thousandths* is 625 *thousandths*, or .625.

234. Rule. *Annex ciphers to the numerator and divide by the denominator, pointing off as many places in the result as there are ciphers annexed.*

The sign + is sometimes placed after the result to indicate that there is still a remainder. Thus, $\frac{1}{3} = .333+$, or $.333\frac{1}{3}$.

235. When the denominator of a common fraction in its lowest terms contains other factors than 2 and 5, the division cannot be exact; and when the same figure or set of figures is repeated in the quotient, the decimal is called a **repeating decimal,** or **circulating decimal,** and the figure or the set of figures repeated is the **repetend.**

Thus, $\frac{7}{27} = .259259+$, a *repeating decimal.*

WRITTEN EXERCISE

Reduce to equivalent decimals:

1. $\frac{7}{8}$
2. $\frac{1}{16}$
3. $\frac{5}{8}$
4. $\frac{5}{32}$
5. $\frac{55}{64}$
6. $\frac{17}{125}$
7. $\frac{7}{12}$
8. $\frac{3}{50}$
9. $\frac{5}{7}$
10. $\frac{31}{32}$
11. $\frac{13}{24}$
12. $\frac{7}{175}$
13. $12\frac{5}{8}$
14. $34\frac{1}{40}$
15. $75\frac{2}{15}$

ADDITION AND SUBTRACTION OF DECIMALS

236. The only requirement in addition and subtraction of decimal fractions is that the decimal points be in a column, since decimals increase and decrease uniformly by a scale of ten in the same manner as integers.

Example 1. Add 12.07, .0045, 1.909, and 345.00324.

$$\begin{array}{r@{}l} 12&.07 \\ &.0045 \\ 1&.909 \\ 345&.00324 \\ \hline 358&.98674 \end{array}$$

Solution. Write the numbers so that units of the same order stand in the same column and with decimal points directly under each other, then add as in simple numbers and place the point in the sum directly below the points in the numbers added.

Complex decimals should be reduced to similar decimals before beginning the work of adding or subtracting.

Example 2. From 5.345 subtract 1.0056.

$$\begin{array}{r@{}l} 5&.345 \\ 1&.0056 \\ \hline 4&.3394 \end{array}$$

Solution. Write the given numbers as in addition, the subtrahend under the minuend and subtract as in simple numbers, placing the point directly under the point in the numbers subtracted.

NOTE. If the minuend has not as many decimal places as the subtrahend, suppose decimal ciphers to be annexed until the right-hand figures are of the same order.

WRITTEN EXERCISE

1. Add ninety-five and seven hundred forty-eight thousandths; seven and nine thousand six hundred eighty-five ten-thousandths; six thousand five hundred seven and four tenths; seven hundred nine and eighty-seven hundredths; two hundred twenty-two and twenty-two hundredths; and seventy-five ten-thousandths.

2. Add eighty-four and sixty-three thousandths; three hundred and five ten-thousandths; three hundred five ten-thousandths; seven hundred and two thousandths; and seven hundred two thousandths.

3. Add .301; .0013; .0103; 1.03; .001003; 31.013; 1.003.

4. Add 384; .384; 384.384; 3.84; 38.4; .00384; .30840; and .03804.

5. Add \$1.35; \$34.50; \$12.37$\frac{1}{2}$; \$56.56$\frac{1}{4}$; \$300; \$2.78$\frac{1}{4}$.

6. Paid for building a house \$3256.75, for painting the same \$318$\frac{5}{8}$, for furniture \$1204.37$\frac{1}{2}$, for carpets \$708$\frac{4}{5}$. What was the cost of the whole?

7. Add .13245$\frac{1}{3}$; 13.375; 215.25; 65.14$\frac{2}{7}$; .012; .0008$\frac{2}{3}$.

$$\begin{array}{r} .13245\frac{1}{3} \\ 13.37500 \\ 215.25000 \\ 65.14285\frac{5}{7} \\ .01200 \\ .00086\frac{2}{3} \\ \hline 293.91317\frac{5}{7} \end{array}$$

Explanation. In examples like the one here shown the complex decimals must be reduced to the lowest order of decimals, before adding.

In practice the fractions may be rejected if the decimals are carried one place, at least, farther than accuracy is required.

8. Add 56.45$\frac{4}{7}$; 1.00033$\frac{1}{3}$; .1234; 200.12$\frac{2}{9}$; 33.015234; 15.37$\frac{1}{2}$; .0156234; and 42.18$\frac{5}{12}$.

9. Find the difference between .0002 and .00005.

10. Find the difference between 15.00342$\frac{1}{4}$ and 12.06786$\frac{1}{3}$, and to the result add $\frac{2}{3}$.

11. From \$234.87$\frac{1}{2}$ subtract \$34.76$\frac{1}{4}$, \$32.87$\frac{1}{2}$, \$5.37$\frac{1}{2}$, \$0.75, and \$54.88.

12. The difference between two numbers is 3.074, and the smaller number is .826. Find the greater number.

13. A man bought an overcoat for \$35$\frac{3}{4}$, a suit for \$21$\frac{2}{3}$,

and a hat for $4.25, and gave in payment one fifty, and two ten-dollar bills. What change should he receive?

14. I owned at the beginning of the year, property valued at $15,620.50. Bought during the year property amounting to $6540.75. Sold during the year property amounting to $12,400.75. Value of property on hand at the close of the year, $9020.75. Find my gain or loss for the year.

15. A has $5065, B has $3478.75, C has $565.50 more than A and B together, and D has as much money as all the rest. How much money has D?

16. A merchant had on hand January 1, $240. He had on deposit in the Merchants' Bank, $524.80. He received during January, $485.50, and paid during January, $562.80. He deposited cash, $55. Find the amount of cash on hand and the amount on deposit January 31.

17. From 2 and .002 take .02 and .000002.

18. A grocer bought 14 bushels of potatoes at 75¢ a bushel, and retailed them at 12¢ a half-peck. How much did he gain?

MULTIPLICATION OF DECIMALS

237. The method of multiplying decimals is the same as in simple numbers. No reference to the decimal point need be made until after the factors are multiplied.

Example. Find the product of 2.06×.0045.

$$\begin{array}{r} 2.06 \\ .0045 \\ \hline 1030 \\ 824 \\ \hline .009270 \end{array}$$

Solution. Multiply the numbers as though they were integers. Since *hundredths* multiplied by *ten-thousandths* produces *millionths*, the product must contain six decimal places. Prefix ciphers to supply the deficiency, fix the point and we have the required product.

238. Rule. *Multiply as in whole numbers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

1. If there are not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply by 10, 100, 1000, etc., remove the decimal point in the multiplicand as many places to the *right* as there are ciphers in the multiplier.

ORAL EXERCISE

1. What is 6 times .3? 5 times .5? 7 times .04?
2. What is .03 times .07? .15 times .06? 8 times .5?
3. What is 8 times .05? 8 times .005? 8 times .0005?
4. Multiply 123.45 by 10; by 100; by 1000; by 10000.
5. Multiply .000032 by 100; by 10000; by 1; by .1.
6. Multiply .006 by .2; by .02; by .002; by 2; by 20.
7. Multiply $45.25 by 10; by 100; by .01; by .001; by 2.
8. What part of .5 is .005? of .75 is .0075? of 235 is 23.5?

WRITTEN EXERCISE

1. What is $14.5\times.65$?
2. What is $1456.025\times.045$?
3. What is $.007\times.075$?
4. $43.099\times.00044=?$
5. $65.045\frac{2}{3}\times.048=?$
6. $.00444\frac{2}{7}\times14.56\frac{1}{4}=?$

7. On Monday I spend .5 of my money; on Tuesday .25 of what is left; on Wednesday .125 of what is still left. What decimal part of my money is left on Wednesday night?

8. From 101 hundredths take 683 thousandths, and multiply the difference by 22 thousandths.

9. What is the product of $7.45\times.25\times.8$ of $\frac{7}{8}\times.56\frac{1}{4}$?

10. A merchant imported 1550 yards of tapestry carpet valued at 80¢ a yard. What was the duty at $42\frac{1}{2}$% *ad valorem?*

The symbol % stands for hundredths and is read *per cent.*

11. A farmer sold 150 bushels of oats at $37\frac{1}{2}$¢ a bushel, and 540 bushels of corn at 56.25¢ a bushel. He received in payment 76 pounds of sugar at $05\frac{1}{4}$¢ a pound, 16 pounds of tea at 42¢ a pound, 4 barrels of flour at $\$6.56\frac{1}{2}$ a barrel, and the balance in cash. How much cash did he receive?

12. Simplify $.02\frac{1}{7}-.001\frac{2}{3}+\frac{5}{21}-(.075\times.0075)-.0014375$.

13. What is the product of 456.75 cords $\times15\frac{3}{8}$?

14. What must I pay at $\$1\frac{1}{3}$ a foot, to fence my lot, which is 4 rods by 8 rods, each rod being 16.5 feet?

15. The cost of beef is $10\frac{3}{8}$ cents per pound, and of flour $3\frac{7}{8}$ cents per pound. A ration consists of 1 lb. 4 oz. of beef, and 1 lb., 6 oz. of flour. What will be the cost of 10,840 rations at the above figures?

16. Three men divided $1624 among them so that the first got .35 of it, the second $\frac{3}{8}$ of it, and the third the remainder. How much did each man receive?

Short Methods

239. The same short methods for multiplying which have been illustrated in sections 57 to 71 of simple numbers, and the methods for multiplying by mixed numbers, as shown in Section 192, will apply in the multiplication of decimals.

1. To multiply a number by .1, .01, .001, etc., move the decimal point as many places to the left as there are places in the multiplier; as, 567.04 × .001 = .56704.

2. To multiply together numbers ending in .5, or the sum of whose right-hand figures is 10, proceed as explained in Section 58; as, .65 × .065 = .04225; .044 × .046 = .002024.

DIVISION OF DECIMALS

240. Divide in decimals as if the numbers were integers, but the dividend must contain at least as many decimal places as the divisor before beginning the division, since the dividend is the product of the divisor and quotient.

Example. Divide 27.3 by .013.

```
.013)27.300(2100.
     26
     --
      13
      13
      --
       00
```

Solution. First annex ciphers to the dividend to make the number of decimal places equal to the number in the divisor for the reason as stated in 239. Divide as in integers, without reference to the decimal point, and we obtain 2100 as shown in the illustration.

241. Rule. *1. When the dividend does not contain as many decimal places as the divisor annex sufficient ciphers to make them equal.*

2. Divide as in integers, and from the right of the quotient point off as many decimal places as the number of places in the dividend exceeds those in the divisor.

1. If there be a remainder after dividing the dividend, the division may be continued by annexing ciphers (228, 1). Such ciphers must be considered as parts of the dividend.

2. In pointing off, if the figures of the quotient are fewer in number than the decimal places required, supply the deficiency by prefixing ciphers.

3. If the divisor and dividend have the same number of decimal places, the quotient obtained will be a whole number.

4. Remember, do not commence the division until the number of decimal places in the dividend is equal to, or exceeds the number in the divisor. In most business transactions, the division is considered sufficiently exact when the quotient is carried to 4 decimal places, unless greater accuracy is required.

5. To divide by 10, 100, 1000, etc., remove the decimal point in the dividend as many places to the *left* as there are ciphers in the divisor.

ORAL EXERCISE

1. Divide the following by 6: $24; .24; 2.4; .024; .0024.
2. Divide the following by 12: $36.36; 3.636; 363.6; .3636.
3. Divide 3 by 3; .3 by 3; .03 by 3; .03 by .03; .003 by 3.
4. Divide 77 by 7; .777 by 7; .777 by .7770; .077 by .07.
5. Divide .008 by 8; 800 by .08; 800 by 8000; 8 by .8.
6. Divide 5.60 by 7; 56 by .7; .56 by .007; 560 by .07.

WRITTEN EXERCISE

Divide:

1. 88.476 by 1.2.
2. .05475 by 15.
3. 2200 by .44.
4. 5.27 by 1.24.
5. .0264 by .033.
6. 88.476 by 1.04.
7. $135 by $$.37\frac{1}{2}$.
8. 100 by .00001.
9. $60.40 by $2416.
10. .36 by .00009.
11. 35.88 by .345.
12. 75 by .0125.
13. 45.367 by $.016\frac{2}{3}$.
14. .1898 by $.33\frac{1}{3}$.
15. 4011 by $2.7\frac{2}{7}$.

Find the sum of the quotients:

(16)	(17)	(18)
99.99 ÷ .03.	34.4 ÷ 2.	36 ÷ 120.
99.99 ÷ .003.	34.4 ÷ 200.	3600 ÷ .12.
99.99 ÷ .0003.	34.4 ÷ 4000.	360 ÷ 12000.
99.99 ÷ .00003.	34.4 ÷ 80000.	.036 ÷ 12.
.36 ÷ .02.	.0036 ÷ .9.	.0036 ÷ 1200.
.36 ÷ .003.	.0036 ÷ .18.	.00036 ÷ .12.
.36 ÷ .0004.	.0036 ÷ .006.	.000036 ÷ 120.
.36 ÷ .00009.	.0036 ÷ .00003.	.036 ÷ .012.

19. Multiply eight hundred and fifteen thousandths by seven hundred sixty-four millionths, and divide the product by thirty-four ten-thousandths. (Correct to two decimal places.)

20. Divide $\frac{1}{10}$ by 1; 1 by $\frac{1}{10}$; 3 by .003; .003 by 3, and add the results.

Note. To divide a number by .1, .01, .001, etc., move the decimal point as many places to the *right* as there are places in the divisor.

21. If the bricks in a building cost \$716.75 at \$7.625 per M., how many bricks were purchased?

22. The product of three factors is .03519, and two of them are .2875 and .1632. Required the third.

23. Two men start at the same time from two towns 315 miles apart and travel toward each other. They meet at the end of 30 hours. One has traveled 4.6 miles per hour. At what rate has the other traveled?

WRITTEN REVIEW

1. A farmer gave 20 sheep worth \$3.27½ each, 3 cows worth \$37.25 each, and 2 tons of hay worth \$14.37½ each for a horse, and afterward sold the horse for \$200; how much did he make or lose by his trades?

2. Add .026, .0137, and .4; from the sum subtract .3998, and divide the difference by 21.

3. From one-tenth take one-thousandth; multiply the remainder by 10,000, and divide the product by one million.

4. Find the difference between the two smallest decimals that can be expressed by 9, 8, and 2.

5. A sum of money was divided among A, B, and C. A received .93 of $\frac{15}{31}$ of it; B, $\frac{3}{11}$ of it, and C \$3007.30. Find the amount divided.

6. If the factors of a divisor are 16, .385, $4\frac{1}{8}$, and 5, and those of the dividend are $3\frac{2}{3}$, 14, and .011, what is the quotient? (Solve by cancellation.)

7. What number subtracted 99 times from 70.1 will leave .008 as a remainder?

8. Simplify $\frac{\frac{2}{3}\times\frac{3}{4}}{.001}+\frac{10.101\times.001}{.000001}-601.$

9. A shoe manufacturer had on hand January 1, boots and shoes valued at $5921.47, and stock valued at $2320. He bought during the year, boots and shoes at a cost of $3562. Manufactured the stock into boots and shoes at an expense of $1462. He sold during the year, $9634, and gave free to friends goods valued at $145. If he had on hand at the close of the year boots and shoes valued at $4694, what was his gain or loss for the year?

10. A merchant during the first year of business gained a sum equal to .32 of his capital, which he added thereto; during the second year his losses amounted to .12½; he then retired with a capital of $11,550. How much was his net gain during the two years?

11. A merchant bought 2340 yards of prints at $.05¼ per yard, and sold .35 of his purchase at $.06⅓ per yard. At what price per yard must he sell the remainder to gain $35.50 on the whole?

12. To what must $\frac{.8}{.15}\times\frac{3}{4}$ be added to give $\frac{1.5\times.15}{.06\times.06}-12.5$?

13. A could do a piece of work in a day of 10 hours, B could do it in .8 of A's time, and C in .9 of A's time. A worked by himself 2 hours, was then joined by B, and they worked together for 3 hours, when C joined them. How long did it take the three to finish the work?

ALIQUOT PARTS

242. The **aliquot parts** of a number are the fractional parts of it.

Thus, 5, $8\frac{1}{3}$, 10, $12\frac{1}{2}$, $16\frac{2}{3}$, 25, are aliquot parts of 50.

The aliquot parts of 100 and 360 are the most commonly used. All *composite numbers* contain aliquot parts.

Any fraction having 1 as its numerator is an aliquot part of a unit.

243. Many of the ordinary business computations may be materially shortened by the use of aliquot parts. The parts frequently employed to express price as parts of a dollar are shown on this and the following page.

ORAL EXERCISE

1. How many cents in $\$\frac{1}{3}$? in $\$\frac{1}{6}$? in $\$\frac{1}{8}$? in $\$\frac{1}{12}$? in $\$\frac{1}{15}$?

2. What aliquot part of $1 is 25¢? $33\frac{1}{3}$¢? 50¢? $12\frac{1}{2}$¢? $16\frac{2}{3}$¢?

3. What aliquot part of 25¢ is $12\frac{1}{2}$¢? $6\frac{1}{4}$¢? 5¢? $2\frac{1}{2}$¢? $8\frac{1}{3}$¢?

4. What aliquot part of $1 is $11\frac{1}{9}$¢? $14\frac{2}{7}$¢? $6\frac{1}{4}$¢? $6\frac{2}{3}$¢? $3\frac{1}{3}$¢?

5. Formulate a short method for finding the cost when the quantity is given and the price is 25¢; 50¢; $33\frac{1}{3}$¢; $14\frac{2}{7}$¢.

6. Name four aliquot parts of 50; of 75; of 30; of 25.

7. At $6\frac{1}{4}$¢ a pound what will 16 pounds of sugar cost?

8. Read answers to the following: $24\times16\frac{2}{3}$¢; $33\times33\frac{1}{3}$¢; $16\times6\frac{1}{4}$¢; $54\times11\frac{1}{9}$¢; $96\times8\frac{1}{3}$¢.

244. **Quantity**, in business transactions, is the entire amount of any commodity bought or sold.

Aliquot Parts of 100 ($1.00)

$ ¢
$\frac{1}{2} = 50$
$\frac{1}{3} = 33\frac{1}{3}$
$\frac{1}{4} = 25$
$\frac{1}{5} = 20$
$\frac{1}{6} = 16\frac{2}{3}$
$\frac{1}{7} = 14\frac{2}{7}$
$\frac{1}{8} = 12\frac{1}{2}$
$\frac{1}{9} = 11\frac{1}{9}$
$\frac{1}{10} = 10$
$\frac{1}{11} = 9\frac{1}{11}$
$\frac{1}{12} = 8\frac{1}{3}$
$\frac{1}{14} = 7\frac{1}{7}$
$\frac{1}{15} = 6\frac{2}{3}$
$\frac{1}{16} = 6\frac{1}{4}$
$\frac{1}{18} = 5\frac{5}{9}$
$\frac{1}{20} = 5$
$\frac{1}{25} = 4$
$\frac{1}{30} = 3\frac{1}{3}$
$\frac{1}{32} = 3\frac{1}{8}$
$\frac{1}{40} = 2\frac{1}{2}$
$\frac{1}{50} = 2$

The pupil should memorize the above.

245. **Price** is the value of one article, or of the *unit* of the commodity.

246. Cost is the value of the entire quantity.

Case I

247. To find the cost when the price or quantity is an aliquot part of 100.

Example 1. Find the cost of 96 lb. of coffee at $33\frac{1}{3}$¢ per pound.

Solution. If the coffee had cost $1 per pound the entire cost would be $42 for the 42 pounds; but, since it is $\$\frac{1}{3}$ per pound it will cost only $\frac{1}{3}$ of $42, or $14.

Example 2. Find the cost of 25 yd. of cloth at 32¢ per yard.

Solution. If 1 yd. cost 32¢, 25 yards would cost $\frac{1}{4}$ of what 100 yards would cost, since 25 is $\frac{1}{4}$ of 100. 100 would cost $32, therefore 25 yd. costs $\frac{1}{4}$ of $32, or $8.

Multiples of 100 ($1.00)

$	¢
$\frac{2}{15}$ =	$13\frac{1}{3}$
$\frac{3}{20}$ =	15
$\frac{3}{16}$ =	$18\frac{3}{4}$
$\frac{5}{16}$ =	$31\frac{1}{4}$
$\frac{3}{8}$ =	$37\frac{1}{2}$
$\frac{5}{12}$ =	$41\frac{2}{3}$
$\frac{9}{16}$ =	$56\frac{1}{4}$
$\frac{5}{8}$ =	$62\frac{1}{2}$
$\frac{2}{3}$ =	$66\frac{2}{3}$
$\frac{3}{4}$ =	75
$\frac{4}{5}$ =	80
$\frac{13}{16}$ =	$81\frac{1}{4}$
$\frac{5}{6}$ =	$83\frac{1}{3}$
$\frac{6}{7}$ =	$85\frac{5}{7}$
$\frac{7}{8}$ =	$87\frac{1}{2}$
$\frac{8}{9}$ =	$88\frac{8}{9}$
$\frac{9}{10}$ =	90
$\frac{15}{16}$ =	$93\frac{3}{4}$
$\frac{19}{20}$ =	95
$\frac{39}{40}$ =	$97\frac{1}{2}$

The pupil should memorize the above.

WRITTEN EXERCISE

Find the cost of the following, making all extensions mentally:

(1)

420 lb. at 20¢.
120 lb. at 25¢.
640 lb. at $12\frac{1}{2}$¢.
250 lb. at 40¢.
$33\frac{1}{3}$ lb. at 75¢.
750 lb. at 24¢.
$2\frac{1}{2}$ lb. at 44¢.

(2)

72 yd. at $3\frac{1}{3}$¢.
125 yd. at 10¢.
888 yd. at $8\frac{1}{3}$¢.
250 yd. at 12¢.
$12\frac{1}{2}$ yd. at 48¢.
400 yd. at $66\frac{2}{3}$¢.
78 yd. at $33\frac{1}{3}$¢.

(3)

$6\frac{1}{4}$ gal. at $1.96.
12 gal. at $1.25.
60 gal. at $6\frac{2}{3}$¢.
14 gal. at $14\frac{2}{7}$¢.
$12\frac{1}{2}$ gal. at 88¢.
240 gal. at $\$1.33\frac{1}{3}$.
176 gal. at $6\frac{1}{4}$¢.

(4)

144 bu. at 75¢.
160 bu. at $16\frac{2}{3}$¢.
256 bu. at 25¢.
125 bu. at 20¢.
$12\frac{1}{2}$ bu. at 96¢.
80 bu. at $62\frac{1}{2}$¢.
$37\frac{1}{2}$ bu. at 88¢.

(5)

3682 lb. at $16\frac{2}{3}$¢.
2500 lb. at 40¢.
1250 lb. at 12¢.
900 lb. at $11\frac{1}{9}$¢.
1680 lb. at $16\frac{2}{3}$¢.
160 lb. at $31\frac{1}{4}$¢.
$133\frac{1}{3}$ lb. at 9¢.

(6)

1140 yd. prints at 25¢.
762 yd. cambric at 33⅓¢.
432 yd. linen at 37½¢.
2311 yd. gingham at 12½¢.
250 yd. plaids at 40¢.
133⅓ yd. alpaca at 72¢.

(7)

88 bu. corn at 62½¢.
400 bbl. apples at $3.25.
25 sacks flour at $1.24.
158 bu. onions at $1.12½.
288 lb. rice at 8⅓¢.
166 bu. barley at 75¢.

CASE II

248. To find the price or quantity when the quantity or price is an aliquot part of 100.

Example 1. How many yards of cloth can be bought for $6 at 16⅔¢ per yard?

Solution. Since the price is ⅙ of a dollar, it is evident that $1 will buy 6 yards; therefore $6 will buy 6 times 6 yards, or 36 yards.

Example 2. If 33⅓ yards of cloth cost $4, what will one yard cost?

Explanation. In examples of this kind, to find the price of one when the quantity is an aliquot part of 100, consider the total cost as expressed in dollars such a fractional part of the answer as the quantity is a part of 100. Therefore 4 is ⅓ of 12, or 12¢ the required result.

WRITTEN EXERCISE

1. At 25¢ per pound, how many pounds of butter can be bought for $24?

2. Bought 16⅔ pounds of cheese for $2.50. What was the price per pound?

3. Find the cost of 250 acres of land at $88 per acre.

Solution. 250 acres at $88 per acre is equal to 880 acres at $25 per acre. Annexing two ciphers to 880 and dividing the result by 4, we have the required result. Thus, 88,000 ÷ 4 = $22,000.

4. Find the cost of 33⅓ bushels of oats at 54¢ per bushel.

5. At $1.25 per yard, how many yards of silk can be bought for $35?

6. Find the cost of 56 bushels of wheat at 62½¢ per bushel (62½¢ = $½ + $⅛).

Extend and add the following:

(7)

256 yd. at 31¼¢.
2442 yd. at 10¢.
345 yd. at 13⅓¢.
904 yd. at 62½¢.
816 yd. at 56¼¢.
728 yd. at 87½¢.
972 yd. at 83⅓¢.

(8)

128 lb. at 16⅔¢.
320 lb. at 6¼¢.
816 lb. at 25¢.
840 lb. at 75¢.
240 lb. at 8⅓¢.
16 lb. at 93¾¢.
126 lb. at $88\frac{8}{9}$¢.

(9)

162 bbl. at $12½.
48 bbl. at $16⅔.
24 bbl. at $1.25.
140 bbl. at $14\frac{2}{7}$ dollars.
165 bbl. at $3⅓.
305 bbl. at $1.20.
360 bbl. at $1.33⅓.

(10)

2000 yd. at 21¢.
2500 yd. at 44¢.
2500 yd. at 72¢.
2000 yd. at 19½¢.
366 yd. at 33⅓¢.
1445 yd. at 20¢.
720 yd. at 5¢.

(11)

124 gal. at 66⅔¢.
540 gal. at 11⅑¢.
320 gal. at 18¾¢.
250 gal. at 16¢.
33⅓ gal. at 48¢.
200 gal. at 47¢.
500 gal. at 54¢.

(12)

75 acres at $75.
85 acres at $85.
125 acres at $160.
375 acres at $80.
250 acres at $124.
16⅔ acres at $160.
240 acres at $112½.

Find the total quantity of the following:

(13)

Cost $110.00, price per yd. 20¢.
Cost $ 38.50, price per yd. 25¢.
Cost $ 37.00, price per yd. 50¢.
Cost $100.00, price per yd. 6¼¢.
Cost $ 60.00, price per yd. 8⅓¢.
Cost $ 30.00, price per yd. 3⅓¢.
Cost $300.00, price per yd. 75¢.

(14)

Cost $67.00, price per lb. 12½¢.
Cost $65.00, price per lb. 16⅔¢.
Cost $44.00, price per lb. 66⅔¢.
Cost $80.00, price per lb. $14\frac{2}{7}$¢.
Cost $66.00, price per lb. 37½¢.
Cost $45.00, price per lb. $1.25.
Cost $72.00, price per lb. $1.12½.

CASE III

249. To find the cost of articles sold by the hundred or thousand.

Example 1. Find the cost of 672 pounds of barley at $1.25 per hundred.

Solution. Since 672 lb. equals 6.72 hundred pounds, and 1 hundred lb. cost $1.25, 6.72 hundred pounds will cost $1.25×6.72 = $8.40.

Example 2. Find the cost of 2354 feet of lumber at \$23 per thousand.

Solution. Since 2354 feet equals 2.354 thousand feet, and 1 thousand feet cost \$23, 2.354 thousand will cost \$23×2.354 = \$54.14.

250. Rule. *Reduce the quantity to hundreds by pointing off two places to the right, or to thousands by pointing off three places. Multiply the price by this result and point off the product as in multiplication of decimals.*

Note. If preferable, multiply the price by the quantity and then point off as given in the rule.

WRITTEN EXERCISE

1. At \$20 per M. what will 750 ft. of lumber cost?

2. How many feet of lumber may be bought for \$12, if lumber is worth \$16 per M.?

3. What will be the cost of 18640 feet of timber, at \$4.50 per 100?

4. A cooper paid \$78.32 for 16488 barrel staves. Required the price per M.

Find the cost of:

(5)

12500 shingles at \$6⅞ per M.
527 ft. boards at \$15½ per M.
3080 fence pickets at \$25 per M.
2456 ft. oak lumber at \$22 per M.
12346 lb. nails at 12½¢ per C.
3750 rails at \$3.33⅓ per C.

(6)

1250 lb. at \$14.40 per C.
2687 lb. at 16⅔¢ per C.
4560 lb. at \$14 per M.
4596 ft. at \$13 per M.
8675 ft. at \$12.50 per M.
2008 ft. at \$17.75 per M.

CASE IV

251. To find the cost of articles sold by the ton of 2000 pounds.

Example. Find the cost of 1711 lb. of coal at \$6.50 per ton.

Solution. Since \$6.50 is the cost of 2000 pounds, ½ of \$6.50, or \$3.25 is the cost of 1000 pounds; and 1711 pounds will cost 1.711 times \$3.25, or \$5.56.

252. Rule. *Multiply one-half the price per ton by the quantity expressed in thousands and decimals of a thousand.*

WRITTEN EXERCISE

1. Find the cost of $12\frac{19}{20}$ tons of hay at \$9.95 a ton.

2. What will it cost to transport 1000 pounds of mail matter 1000 miles, at \$1 per 100 pounds per 100 miles?

3. The cost of a load of coal weighing 2376 pounds was \$6.53. Required the price per ton.

4. A wagon loaded with hay weighs 5125 lb.; the wagon alone weighs 1125 lb. Find the cost of the hay at \$15 a ton.

5. Find the cost of 3130 pounds of stove coal at \$5.25 per ton, and 88 pounds of cannel coal at \$6.90 per ton.

6. A coal dealer bought 350 tons of coal, weighing 2240 pounds each, at \$3.50 per ton. He sold the coal at \$4.25 a ton, each weighing 2000 pounds. What was his profit?

7. A man paid \$58.12½ for a car-load of plaster at \$7.50 per ton. How many pounds should there have been?

8. A farmer delivered four loads of hay weighing in the gross 4777, 3860, 3654, 3609 lb. respectively. The weight of his sleigh and hay-rack is 1380 lb.; how much is due him at \$11.75 per ton for his hay?

WRITTEN REVIEW

1. Change (*a*) $\frac{3}{64}$ to a decimal; (*b*) .0275 to a common fraction.

2. Find the difference between:

$$\frac{.26+.2 \text{ of } 3.7}{.48-.014 \text{ of } 20.} \text{ and } \frac{4.8+5.6}{7.4-.2 \text{ of } 11.}$$

3. If 12 pounds of tea are worth 36 pounds of coffee, and 8 pounds of coffee are worth \$1.92, find the value of 24 pounds of tea.

4. A, B, and C enter into a partnership. A puts in \$500 for 4 months, B \$400 for 6 months, and C \$800 for 3 months; they gain \$340. Find each man's share of the gain.

5. A bought a house and lot paying ⅓ the price down, and the second payment was ⅓ the remainder due. If the sum of the two payments thus made was \$2400, what was the cost price of the property?

6. A boat at Buffalo is chartered to take 3500 tons of coal

at 35¢ per ton to Duluth and bring 105,000 bushels of wheat back from Duluth to Buffalo at 3½¢ per bushel. The trip takes 14 days. Give the average earnings of the boat per day.

7. When butter is 4¢ dearer than cheese per pound, a woman purchased 16 lb. of cheese and 25 lb. of butter for $5.92. How much did she pay for each per pound?

8. A stack of hay will keep 36 cows or 27 horses for a week; how many days will it keep 10 cows and 6 horses?

9. A carpenter worked 11⅔ days, and after paying his board and other expenses with 3/7 of his earnings, had $20 left; how much did he receive a day?

BILLS, ACCOUNTS, AND STATEMENTS

253. An **account** in commercial transactions is a record of debits and credits between two persons. A *debtor* is a person who owes another money, goods, or services; and a *creditor* is a person to whom money, goods, or services are due from another.

254. A **bill** or **invoice** is an itemized statement of goods bought or sold, or of services rendered.

1. A *bill* should state the place, the date, the names of the debtor and creditor, the terms of sale, the quantity and price of each article, or the kind and value of services, and the entire cost.
2. The part of the bill or invoice above the head-line is called the *bill head*, and commonly contains more or less of other matter of an advertising nature.

255. A bill is **receipted** when the creditor, or some one acting for him, acknowledges the payment of the bill by signing his name at the bottom, under the words, *Received Payment*, or *Paid.*

1. When a bill is receipted by an authorized agent he should place his initials under the signature of the firm, preceded by the word *per* or *by*.
2. Generally, in law, the signature of the creditor is sufficient without the words *received payment* as it is presumed that the debtor would have the right to write these words.

256. A **statement of account** is a written list of debits and credits of a personal account.

257. Some of the **common accounts** kept in business are personal accounts; cash account; expense account; merchandise account. *Merchandise* consists of goods, wares, or other commodities usually bought or sold in trade.

The *ledger* is the principal book of accounts and contains, in classified form, the debits and credits of all business transactions.

258. The following abbreviations are in common use and most frequently employed in bills and invoices:

A1	first quality
acct.	account
amt.	amount
agt.	agent
bal.	balance
bbl.	barrel
bot.	bought.
bu.	bushel
bx.	box.
bdls.	bundles
Co.	company
ctg.	cartage
chgd.	charged
C. O. D.	collect on delivery
Cr.	creditor
cwt.	hundredweight
com.	commission
c. f. i.	cost, freight, and insurance
do.	ditto, or the same
disc.	discount
doz.	dozen
dft.	draft
Dr.	debtor
da.	day
ea.	each
E. & O. E.	errors and omissions excepted
exch.	exchange
ft.	foot, or feet
frt.	freight
fol.	folio, or page
for'd.	forward
f. o. b.	free on board
gal.	gallon
gro.	gross
hhd.	hogshead
hlf.	half
hr.	hour
in.	inch
ins.	insurance
inst.	present month
int.	interest
lb.	pound
mdse.	merchandise
mem.	memorandum
mo.	month
Messrs.	Gentlemen, or Sirs
net.	without further discount
No.	number
N. B.	take notice
oz.	ounce
p.	page
pp.	pages
pt.	pint
payt.	payment
pd.	paid
pkg.	package
pc.	piece
pr.	pair
per.	by, or by the
qr.	quarter
qt.	quart
recd.	received
R. R.	railroad
ship't.	shipment
sunds.	sundries
sec.	second
stor.	storage
super.	superfine
ult.	last month
wt.	weight
wk.	week
yd.	yard
yr.	year

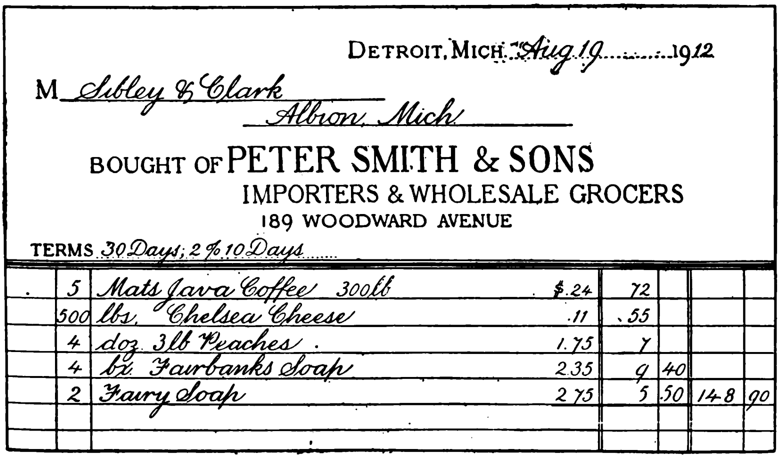
Business Characters

@	at	#	number	C.	by the hundred
℀	account	✓	check mark	M.	by the thousand
℅	care of	"	ditto	2^1	is read $2\frac{1}{4}$
¢	cents	(')	feet	2^2	is read $2\frac{1}{2}$
%	per cent.	(")	inches	2^3	is read $2\frac{3}{4}$
$	dollars	×	by (4×5)	£	pound sterling

Model Bills

259. The model bills, etc., found on the following pages show a variety of current practices and will therefore be found helpful as studies. The pupil should be required to carefully copy the bills and invoices, making all extensions with pen and ink and "foot up" the columns.

1. Groceries



DETROIT, MICH. Aug. 19 1912

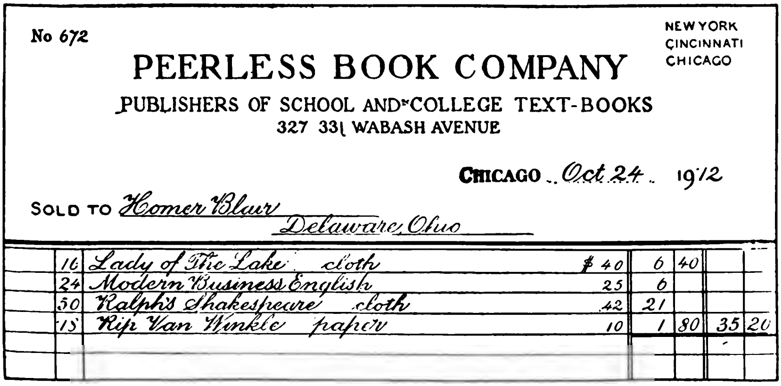
M Sibley & Clark
Albion, Mich.

BOUGHT OF PETER SMITH & SONS
IMPORTERS & WHOLESALE GROCERS
189 WOODWARD AVENUE

TERMS 30 Days; 2% 10 Days

	5	Mats Java Coffee 300 lb	$.24	72			
	500	lbs. Chelsea Cheese	.11	55			
	4	doz. 3 lb Peaches	1.75	7			
	4	bx. Fairbanks Soap	2.35	9	40		
	2	Fairy Soap	2.75	5	50	148	90

2. Books



No 672

NEW YORK
CINCINNATI
CHICAGO

PEERLESS BOOK COMPANY
PUBLISHERS OF SCHOOL AND COLLEGE TEXT-BOOKS
327 331 WABASH AVENUE

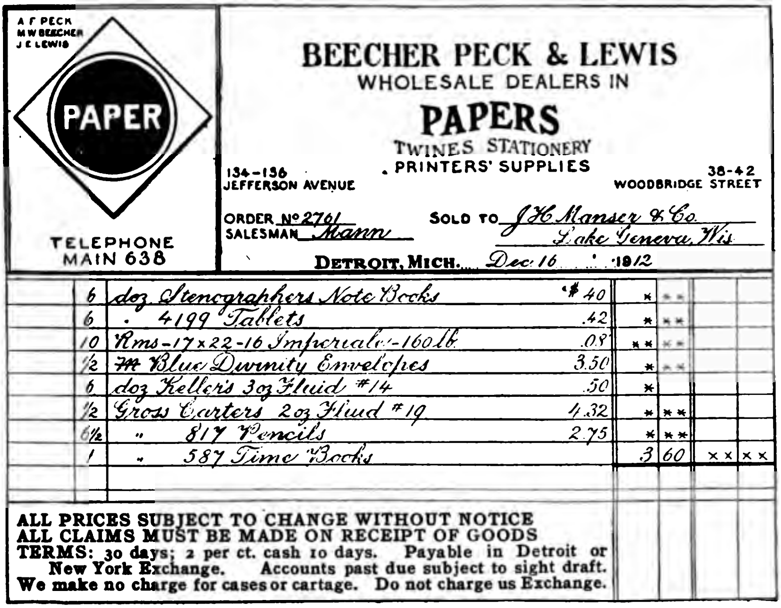
CHICAGO Oct. 24 1912

SOLD TO Homer Blair
Delaware, Ohio

	16	Lady of The Lake cloth	$ 40	6	40		
	24	Modern Business English	25	6			
	50	Ralph's Shakespeare cloth	42	21			
	15	Rip Van Winkle paper	10	1	80	35	20

The above are the simplest forms of invoices but they are forms which are in use in many lines of business. Many firms prefer to place considerable advertising matter on the invoice, especially that pertaining to credits, terms of discount, etc., as is shown on Invoice 3 on the following page.

3. Stationery



A F PECK
M W BEECHER
J E LEWIS

PAPER

TELEPHONE
MAIN 638

BEECHER PECK & LEWIS
WHOLESALE DEALERS IN
PAPERS
TWINES STATIONERY
PRINTERS' SUPPLIES

134-136
JEFFERSON AVENUE

38-42
WOODBRIDGE STREET

ORDER No 2761
SALESMAN Mann

SOLD TO J. H. Manser & Co.
Lake Geneva, Wis.

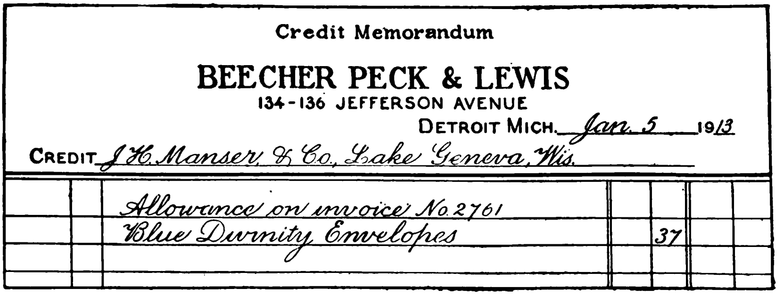
DETROIT, MICH. Dec 16 1912

	6	doz Stenographers Note Books	$.40	*	**		
	6	" 4199 Tablets	.42	*	**		
	10	Rms-17×22-16 Imperiale-160 lb.	.08	**	**		
	½	M Blue Divinity Envelopes	3.50	*	**		
	6	doz Keller's 3 oz Fluid #14	.50	*			
	½	Gross Carters 2 oz Fluid #19	4.32	*	**		
	6½	" 817 Pencils	2.75	*	**		
	1	" 587 Time Books		3	60	××	××

ALL PRICES SUBJECT TO CHANGE WITHOUT NOTICE
ALL CLAIMS MUST BE MADE ON RECEIPT OF GOODS
TERMS: 30 days; 2 per ct. cash 10 days. Payable in Detroit or New York Exchange. Accounts past due subject to sight draft.
We make no charge for cases or cartage. Do not charge us Exchange.

In the above bill the first figures show the quantity bought, and this times the price per each gives the cost of each item, with the exception of the third, which means 10 reams of 160 lb., in all, at 8¢ per lb.

4. Credit Memorandum



Credit Memorandum

BEECHER PECK & LEWIS
134-136 JEFFERSON AVENUE
DETROIT MICH. Jan. 5 1913

CREDIT J. H. Manser & Co., Lake Geneva, Wis.

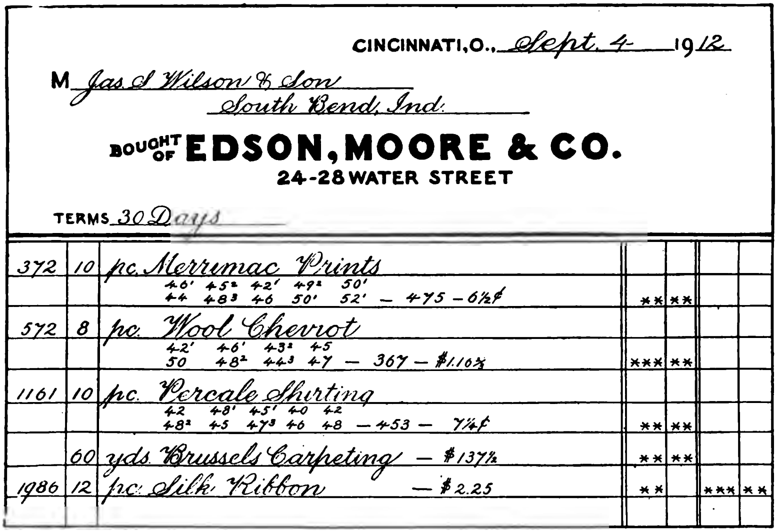
		Allowance on invoice No. 2761				
		Blue Divinity Envelopes		37		

260. A **credit memorandum** is given when a claim against an invoice is allowed for some cause, or for goods returned.

261. Several discounts are sometimes offered on one bill, giving the purchaser the choice of settlement. These discounts are usually made to induce the buyer to pay the bill before it is due.

Terms 30 da., 2% 10 da., means that if the purchaser settles the invoice within 10 days a deduction of 2% is allowed from the face of the bill, otherwise no deduction is made. If the invoice is not paid at the end of the term of credit it is then subject to an interest charge from this date at the legal rate.

5. Dry Goods



CINCINNATI, O., Sept. 4 1912

M Jas. S. Wilson & Son
South Bend, Ind.

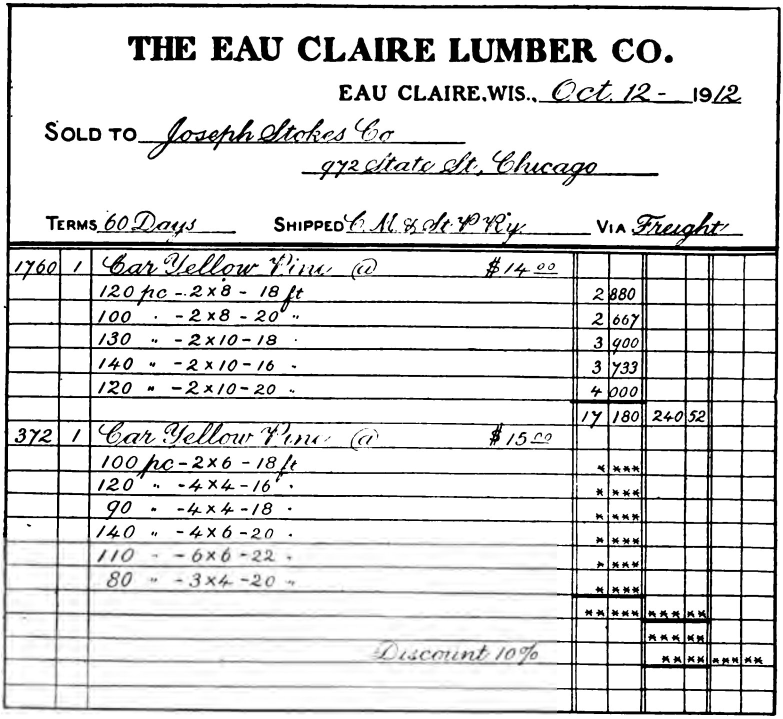
BOUGHT OF **EDSON, MOORE & CO.**
24-28 WATER STREET

TERMS 30 Days

372	10	pc. Merrimac Prints				
		$46'$ 45^2 $42'$ 49^2 $50'$ 44 48^3 46 $50'$ $52'$ — 475 — 6½¢	**	**		
572	8	pc. Wool Cheviot				
		$42'$ $46'$ 43^2 45 50 48^2 44^3 47 — 367 — \$1.16⅔	***	**		
1161	10	pc. Percale Shirting				
		42 $48'$ $45'$ 40 42 48^2 45 47^3 46 48 — 453 — 7¼¢	**	**		
	60	yds. Brussels Carpeting — \$1.37½	**	**		
1986	12	pc. Silk Ribbon — \$2.25	**		***	**

$46'$, 45^2, $42'$, etc., in the first item of the above invoice means the number of yards in each piece as the 10 pieces are not uniform in quantity. The 475 immediately following these numbers is the total number of yards and the price per yard. In finding the total number of yards use horizontal addition.

6. Lumber.



THE EAU CLAIRE LUMBER CO.

EAU CLAIRE, WIS., Oct. 12 - 1912

SOLD TO Joseph Stokes Co
972 State St. Chicago

TERMS 60 Days SHIPPED C. M. & St. P. Ry. VIA Freight

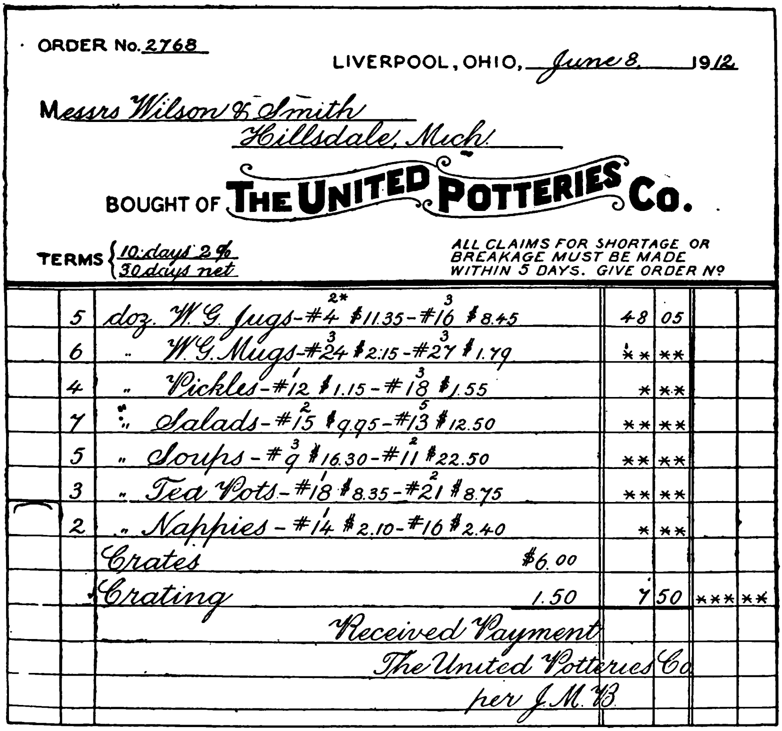
1760	1	Car Yellow Pine @ $14.00						
		120 pc - 2×8 - 18 ft	2	880				
		100 " - 2×8 - 20 "	2	667				
		130 " - 2×10 - 18 "	3	900				
		140 " - 2×10 - 16 "	3	733				
		120 " - 2×10 - 20 "	4	000				
			17	180	240	52		
372	1	Car Yellow Pine @ $15.00						
		100 pc - 2×6 - 18 ft	*	***				
		120 " - 4×4 - 16 "	*	***				
		90 " - 4×4 - 18 "	*	***				
		140 " - 4×6 - 20 "	*	***				
		110 " - 6×6 - 22 "	*	***				
		80 " - 3×4 - 20 "	*	***				
			**	***	***	**		
					***	**		
		Discount 10%			***	**	***	**

Lumber is sold by the thousand feet. In billing lumber, the number of pieces is entered first, then the thickness and width in inches, then the feet in length.

In the first item (2×8×18×120)÷12, the number of cubic inches in a board foot, gives 2880 board feet. 2×8, 2×10, etc., indicates the dimensions in width and thickness of the pieces.

7. China and Glassware

The following illustrates a form of billing used in the china and glassware business. In this case an extra charge is made for packing and crating.



· ORDER No. 2768

LIVERPOOL, OHIO, June 8, 1912

Messrs Wilson & Smith
Hillsdale, Mich.

BOUGHT OF THE UNITED POTTERIES Co.

TERMS { 10 days 2%
30 days net

ALL CLAIMS FOR SHORTAGE OR BREAKAGE MUST BE MADE WITHIN 5 DAYS. GIVE ORDER No

	5	doz. W. G. Jugs-#$\overset{2*}{4}$ \$11.35-#$\overset{3}{16}$ \$8.45	48	05		
	6	" W. G. Mugs-#$\overset{3}{24}$ \$2.15-#$\overset{3}{27}$ \$1.79	**	**		
	4	" Pickles-#$\overset{1}{12}$ \$1.15-#$\overset{3}{18}$ \$1.55	*	**		
	7	" Salads-#$\overset{2}{15}$ \$9.95-#$\overset{5}{13}$ \$12.50	**	**		
	5	" Soups-#$\overset{3}{9}$ \$16.30-#$\overset{2}{11}$ \$22.50	**	**		
	3	" Tea Pots-#$\overset{1}{18}$ \$8.35-#$\overset{2}{21}$ \$8.75	**	**		
	2	" Nappies-#$\overset{1}{14}$ \$2.10-#16 \$2.40	*	**		
		Crates \$6.00				
		Crating 1.50	7	50	***	**
		Received Payment				
		The United Potteries Co				
		per J. M. B.				

* NOTE. The figure 2 indicates the number of dozen at $11.35 per doz.

Allowance for Tare, etc., in Billing.

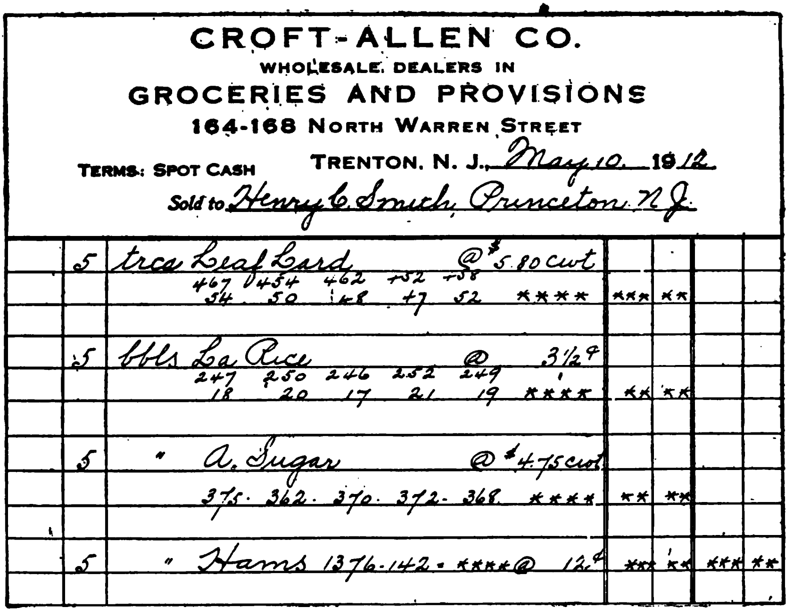
262. Gross weight is the weight before any allowances are made for tare, breakage, leakage, etc.

263. Net weight is the difference between the gross weight and the tare.

Tare is the weight of the box, bag, crate, or other covering of the goods. *Leakage* is an allowance made for the waste of liquids imported in casks, barrels, etc. *Breakage* is an allowance made on account of waste of goods shipped in glass or other breakable material.

8. Provisions.

The following illustrates a form of billing used in the grocery and provision business in which allowances are made for tare, etc.

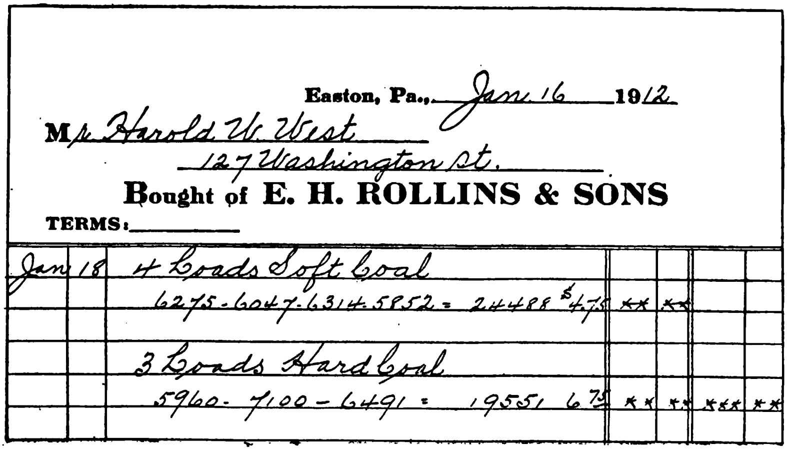


CROFT-ALLEN CO.
WHOLESALE DEALERS IN
GROCERIES AND PROVISIONS
164-168 NORTH WARREN STREET
TERMS: SPOT CASH TRENTON, N. J., May 10. 1912
Sold to Henry C. Smith, Princeton, N. J.

	5	trcs Leaf Lard @ $5.80 cwt				
		467 454 462 452 458				
		54 50 48 47 52 ****	***	**		
	5	bbls La Rice @ 3½¢				
		247 250 246 252 249				
		18 20 17 21 19 ****	**	**		
	5	" A. Sugar @ $4.75 cwt				
		375 · 362 · 370 · 372 · 368 ****	**	**		
	5	" Hams 1376 - 142 = ****@ 12¢	***	**	***	**

In the first items in the above bill the gross weight is shown; the number below, the tare. In the last item the gross weight of the five bbls. is shown, and the tare.

9. Retail Coal.



Easton, Pa., Jan. 16 1912
Mr. Harold W. West
127 Washington St.
Bought of E. H. ROLLINS & SONS
TERMS:

Jan	18	4 Loads Soft Coal				
		6275 - 6047 - 6314 - 5852 = 24488 $4.75	**	**		
		3 Loads Hard Coal				
		5960 - 7100 - 6491 = 19551 6.75	**	**	***	**

10. ~~Sales~~ Customers Ledger.

264. The following is a section of a page from a ledger, showing the account of John W. Bloom and the amount unpaid as exhibited on the books of Gimbel Brothers, Philadelphia.

John W Bloom.

1912 Jan	16	Mdse	70	156	25	1912 Feb	1	Cash	90	250		
"	28	"	75	340		Mar	1		95	500		
Feb	16	"	81	264	50							
Mar	20	"	90	178	25							

11. Statement.

L. F. 275 Philadelphia, Pa., Apr 1 1912.

M John W Bloom
Frenchtown, Pa.

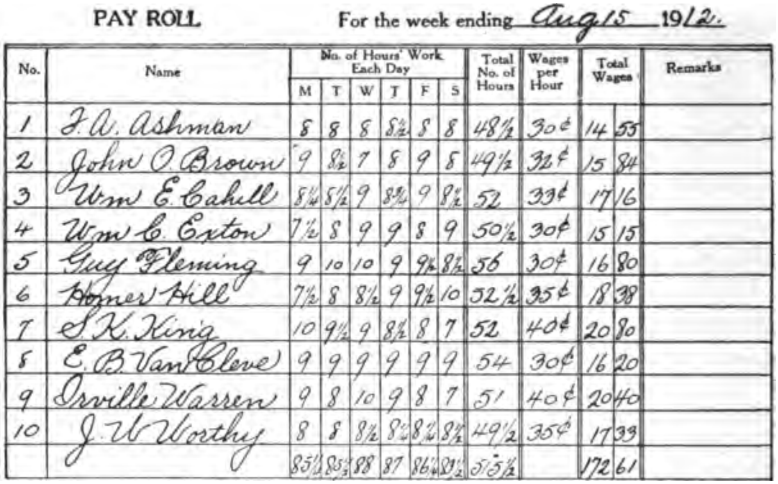
In Account with **GIMBEL BROTHERS**

Jan	16	To Mdse	156	25		
"	28	" "	340			
Feby	16	" "	264	50		
Mar	20	" "	178	25	939	00
		Cr.				
Feby	1	By Cash	250			
Mar	1	" "	500		750	
		Balance due			189	

The above statement is an abstract, as will be seen, of John W. Bloom's account, showing the dates and details of debits and credits and the balance remaining unpaid.

265. Statements are usually rendered at the end of each month. The aggregate of each separate sale is sufficient, inasmuch as an itemized bill has been previously rendered.

PAY ROLLS



PAY ROLL For the week ending Aug 15 1912.

No.	Name	No. of Hours' Work Each Day						Total No. of Hours	Wages per Hour	Total Wages		Remarks
		M	T	W	T	F	S					
1	F. A. Ashman	8	8	8	8½	8	8	48½	30¢	14	55	
2	John O. Brown	9	8½	7	8	9	8	49½	32¢	15	84	
3	Wm E. Cahill	8¼	5½	9	8¾	9	8½	52	33¢	17	16	
4	Wm C. Exton	7½	8	9	9	8	9	50½	30¢	15	15	
5	Guy Fleming	9	10	10	9	9½	8½	56	30¢	16	80	
6	Homer Hill	7½	8	8½	9	9½	10	52½	35¢	18	38	
7	S. K. King	10	9½	9	8½	8	7	52	40¢	20	80	
8	E. B. Van Cleve	9	9	9	9	9	9	54	30¢	16	20	
9	Orville Warren	9	8	10	9	8	7	51	40¢	20	40	
10	J. W. Worthy	8	8	8½	8¼	8¼	8½	49½	35¢	17	33	
		85½	85½	88	87	86¼	81½	515½		172	61	

The above is a common form of *pay roll* among manufacturing companies and other concerns which employ considerable labor.

Wages are usually calculated by the hour on the basis of 8 or 10 hours to a day. Checks are frequently used in paying off employees, but most large concerns prefer the envelope system—that is, paying each person the amount due in currency. In doing this the book-keeper usually draws from the bank the money needed, in the denominations required for each person, as indicated in the *change memorandum* shown below.

The total of the change memorandum should prove with the *pay-roll memorandum* which is shown on the following page.

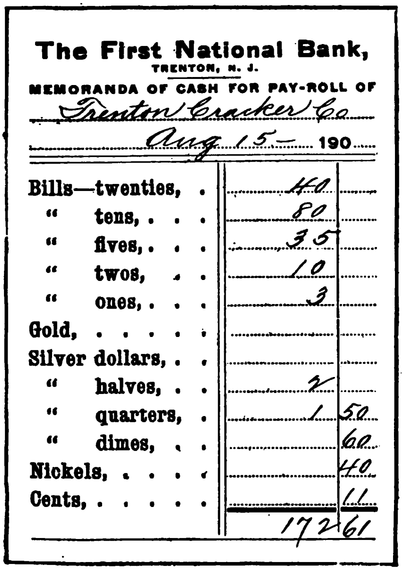
Change Memorandum

No.	Bills					Coin				
	$20	$10	$5	$2	$1	50c.	25c.	10c.	5c.	1c.
1		1		2		1			1	
2		1	1			1	1		1	4
3		1	1	1				1	1	1
4		1	1					1	1	
5		1	1		1	1	1		1	
6		1	1	1	1		1	1		3
7	1					1	1		1	
8		1	1		1			2		
9	1						1	1	1	
10		1	1	1			1		1	3
	2	8	7	5	3	4	6	6	8	11

Pay-Roll Memorandum.

When it is determined how much money is needed of each denomination of bills and coin to meet the requirements of the pay roll it is written on a *pay-roll memorandum slip*, as shown in the accompanying illustration. A check is then drawn payable to *Pay Roll* to which is attached this memorandum slip and both are sent to the bank.

In filling out the *pay-roll memorandum* it is not necessary to indicate how many pieces of each denomination is required, only the total of each. The pay-roll memorandum should "foot up" the same as the pay roll, and is therefore a check upon the correctness of the work.



The First National Bank,
TRENTON, N. J.
MEMORANDA OF CASH FOR PAY-ROLL OF
Trenton Cracker Co
Aug 15— 190

Bills—twenties, .	40	
" tens, . . .	80	
" fives, . . .	35	
" twos, . .	10	
" ones, . . .	3	
Gold, ...		
Silver dollars, . .		
" halves, . .	2	
" quarters, .	1	50
" dimes, . .		60
Nickels, ...		40
Cents, ...		11
	172	61

WRITTEN EXERCISE

1. Jan. 1, W. S. Solan & Co., Jamestown, Va., bought of the Howard Grocery Co., New Orleans, La., 10 bags coffee, weighing 134, 137, 139, 136, 138, 133, 131, 138, 135, and 136 lb., at 13½¢; 15 bbl. molasses containing 43, 47, 45, 46, 40, 39, 45, 48, 42, 43, 44, 47, 46, 45, and 42 gal., at 35¢; 4 bbl. rice each containing 250–19, 247–21, 253–20, and 241–19 lb., at 5¾¢. Terms: 60 da., or 2% discount for cash within 10 da. Write the bill.

2. Sept. 30, Thomas J. Ryan, Louisville, Ky., bought of the J. W. Stokes Lumber Co., Jackson, Tenn., lumber as follows: white pine, 20 pc. 4×4–16, 36 pc. 4×2–12, 18 pc. 2×2–14; lot yellow pine, 72 pc. 2×8–18, 88 pc. 2×8–20, 60 pc. 2×10–18, 100 pc. 2×10–20, 120 pc. 2×12–18; 1 car yellow pine containing 100 pc. 4×4–16, 100 pc. 2×4–16, 96 pc. 2×8–18, 88 pc. 6×6–20, 80 pc. 6×6–22, 68 pc. 6×8–18, 40 pc. 2×4–12. This lumber is all billed at the uniform price of $13 per M. Terms 30 da. Write the bill.

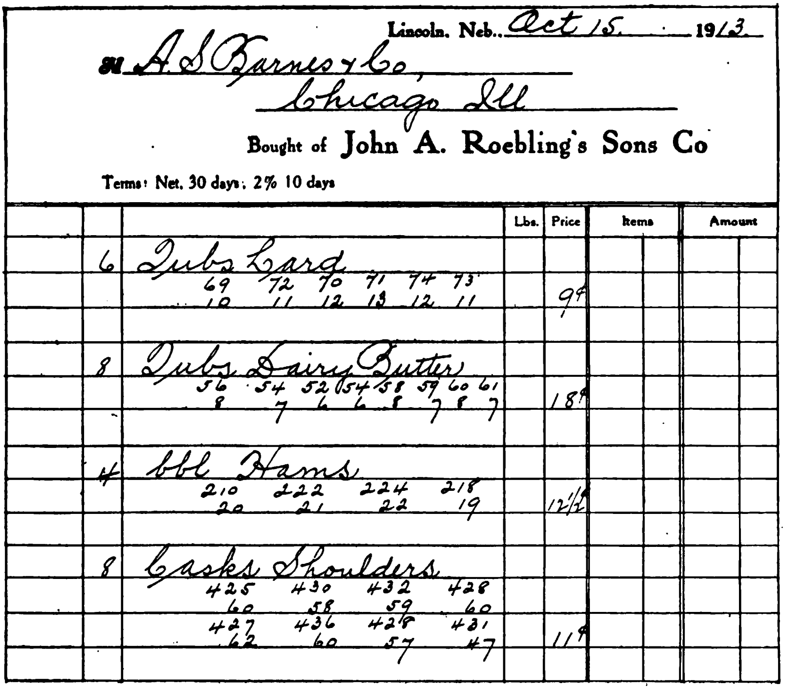
3. Mar. 4, J. S. McCord & Co., Danville, Pa., bought of Strawbridge & Clothier, Philadelphia, trunks as follows: lot 88,

2 each No. 34 $5, No. 36 $4.50, No. 47 $6.25, No. 33 $6.50, No. 45 $7.25; lot 92, 3 each No. 29 $3.25, No. 32 $3.75, No. 36 $4.25, No. 39 $5, No. 41 $5.35. On the first lot there is a discount of 15%, on the second lot a discount of 10%. Terms 10 da. Write the bill.

4. Study the model retail coal bill on page 138. Increase the price of each ton 50¢, then copy and find the amount of the bill.

5. Mrs. Florence Mather bought of Gilmore Brothers, Bowling Green, Ky., June 10, 1912, 10 lb. sugar at $10\frac{1}{2}$¢, $12\frac{1}{2}$ lb. dried beef at 16¢, 4 gal. N. O. molasses at $52\frac{1}{2}$¢, $8\frac{1}{4}$ lb. mackerel at $8\frac{1}{2}$¢, 1 box of starch, 10 lb. at $8\frac{1}{2}$¢. Make out and receipt the bill.

6. Find the amount of the following bill:



Lincoln, Neb., Oct 15. 1913

M A S Barnes & Co,
Chicago Ill

Bought of **John A. Roebling's Sons Co**

Terms: Net. 30 days; 2% 10 days

			Lbs.	Price	Items	Amount
	6	Tubs Lard				
		69 72 70 71 74 73 10 11 12 13 12 11		9¢		
	8	Tubs Dairy Butter				
		56 54 52 54 58 59 60 61 8 7 6 6 8 7 8 7		18¢		
	4	bbl Hams				
		210 222 224 218 20 21 22 19		12½¢		
	8	Casks Shoulders				
		425 430 432 428 60 58 59 60 427 436 428 431 62 60 57 47		11¢		

7. March 1, 1912, Hill & Young, Albion, Minn., bought of the W. L. Douglas Shoe Co., Brockton Mass., Case No. 3245, 15 pairs boys' shoes, sizes from 3 to 5, at 90¢; Case No. 5432, 24 pr. Misses French kid, width E. size 1¼ to 3½, at $1.55; Case No. 3425, 24 pr. pat. lea., button, width E, sizes 5 to 9½, at $2.33⅓; Case No. 2245, 16 pr. French kid, plain lace, width EE, sizes 7 to 10, at $3.10; Case No. 1567, 18 pr. pat. lea. H. S. lace, width D, sizes 5 to 8, at 95¢. Terms 60 days, with a discount of 5% if paid within 10 days. Write the bill.

8. Copy the following pay roll, find the total amount of weekly wages, and make a change memorandum and a pay-roll memorandum:

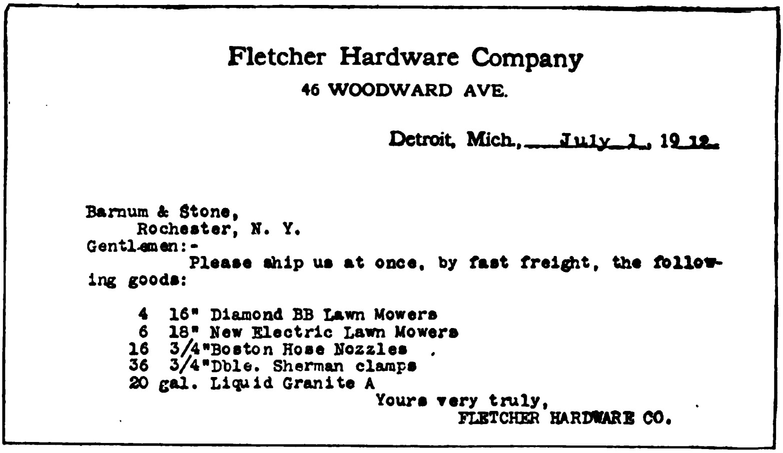
PAY ROLL For the week ending Sept 15 1912.

	Name	No. of Hours' Work Each Day						Total No. of Hours	Wages per Hour	Total Wages	Remarks
		M	T	W	T	F	S				
1	L. H. Stern	7	8	9	6	8	9		48¢		
2	E. H. Blake	7½	9	8	7½	8	8		.51		
3	I. J. Ryan	9	8	6¼	7	7¾	6½		47½		
4	Wm. H. Coleman	9	7	8	6	4	9		.50		
5	J. H. Manser	8	7½	8	9	10	9		.55		
6	C. E. Prest	8	7½	8	4	10	9		.50		
7	E. E. Goodwin	9	6	8	9	8	9		47½		

9. April 1, 1912, the United Cigar Co., bought of Edward-Roe Co., Louisville, Ky., 1 hhd. (No. 2345) leaf tobacco, 1298–155 lb. at $3.25 cwt.; 1 hhd. (5432) leaf tobacco, 1235–153 lb. at $6.25 cwt.; 1 hhd. (1132) leaf tobacco, 1300–155 lb. at $5.55 cwt.; 1 hhd. (1223) leaf tobacco, 1075–148 lb. at $4.50 cwt. Terms 60 days net, or 5% discount for cash within 10 days. Write the bill.

10. Assume the name of the purchaser and date, and with your own name as the seller bill the following. Terms: net, 30 days, with a discount of 2 per cent if paid in 10 days. 10 bbl. Acme flour at $7.25; 40 sacks buckwheat flour at 45¢; 64 sacks rye flour at 52¢; 120 bbl. corn meal at $3.25.

11. Make out a bill for the following order:



Fletcher Hardware Company
46 WOODWARD AVE.

Detroit, Mich., July 1, 1912

Barnum & Stone,
Rochester, N. Y.
Gentlemen:-
Please ship us at once, by fast freight, the following goods:

4 16" Diamond BB Lawn Mowers
6 18" New Electric Lawn Mowers
16 3/4"Boston Hose Nozzles
36 3/4"Dble. Sherman clamps
20 gal. Liquid Granite A

Yours very truly,
FLETCHER HARDWARE CO.

Bill the diamond lawn mowers at $5.25 each; new electric lawn mowers at $4.75 each: Boston hose nozzles at $2.50 per. doz.; the Sherman clamps at 60¢ per. doz.; and the liquid granite at $2.50 per gal.

DENOMINATE NUMBERS

Measures

266. A **denominate number** is a concrete number in which the unit of measure has been established by law or custom; as, 6 pounds, 5 dollars, 3 feet.

267. **Simple denominate numbers** are expressed in units of one denomination only. **Compound denominate numbers** are expressed in units of two or more denominations.

268. A **standard unit** of measure is a unit established by law or custom and from which other units of the same kind may be derived; as,

The yard has been prescribed by the United States Government as the standard *unit of extension;* the Troy pound *for weight;* etc.

269. A **scale** is the ratio by which numbers increase and decrease. It is either *uniform* or *varying.*

United States currency, being decimal, the scale is *uniform.* In measures of capacity it is *varying,* for 2 pints equal 1 quart; 8 quarts, 1 peck, etc.

270. **Denominate numbers** may be divided into six classes, as follows:

1. Measures of Value.
2. Measures of Weight.
3. Measures of Capacity.
4. Measures of Extension.
5. Measures of Time.
6. Circular Measure.

MEASURES OF VALUE

271. **Value** is the worth of one thing as compared with another. *Money* is the measure of value.

United States Money

272. This subject has been treated on pages 57 to 62 inclusive.

273. *Barter*, or the actual exchange of commodities, was originally carried on among primitive people before money came into use. The inconvenience arising from transactions of this kind brought about the adoption of a *medium of exchange*, or money. Gold and silver was used at a very early period in the world's history and seems at first to have been exchanged for commodities by weight. Finally, as in the case of weights and measures, governments adopted definite standards of money value.

U. S. Money

10 mills = 1 cent (ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

English Money

4 farthings = 1 penny (d.)
12 pence = 1 shilling (s.)
20 shillings = 1 pound sterling = $4.8665 (£)

1. The *mill* in United States money is not coined, but the name is frequently used in business calculations. The legal currency of *Canada* has about the same values as United States money. Its *unit* is the dollar, the same as United States money. There are no Canadian gold coins, the larger denominations consist of paper currency and the gold coins of England and the United States.

2. *British* gold coins are $\frac{11}{12}$ pure gold and $\frac{1}{12}$ alloy; the silver coins, $\frac{37}{40}$ pure silver and $\frac{3}{40}$ copper; the penny and half-penny pieces are pure copper.

3. The British *gold coins* are the sovereign and half-sovereign; the *silver coins* are the crown (5s), half-crown, florin (2s.), shilling, and the six- and three-penny pieces; the *copper coins* are the penny, half-penny, and the farthing.

4. £ is an abbreviation of the Latin word *libra* and means pound; *s.* of the word *solidarius*, meaning shilling; and *d.* of the word *denarius*, meaning penny.

5. 1 *far.* = $\frac{50}{99}$¢; 1*d.* = 2$\frac{2}{99}$¢; 1*s.* = 24$\frac{1}{3}$¢.

French Money

10 centimes (c) = 1 decime (dc.)
10 decimes = 1 franc (fr.)

German Money

100 pfennigs (Pf.) = 1 mark (Rm.)

1. *French money* is a decimal currency, and consists of gold, silver, and bronze coins and national bank notes. The *gold coins* are 5, 10, 20, 50, and 100 *franc* pieces; the *silver coins* are 1, 2, and 5 *franc* pieces; also the 25 and 50 *centime* pieces; and the *bronze coins* are the 1, 2, 5, and 10 *centime* pieces.

2. The *unit* of the French money is the *franc*, which is equal to $.193 in United States money.

3. The currency of the *German Empire* consists of gold, silver, and nickel coins, and paper money.

4. The German *gold coins* are the 5, 10, and 20 *mark* pieces; the *silver coins* are the 1 and 2 *mark* pieces, and the 20 and 50 pfennig pieces; the *nickel coins* are the 5 and 10 pfennig pieces; and the copper coins are the 1 and 2 pfennig pieces.

5. The unit of German money is the *mark*, which is equal to $.2385 in United States money.

MEASURES OF WEIGHT

274. Weight is the measure of the force of gravity.

1. The standard unit of weight in the United States is the Troy pound as registered at the mint, and contains 5760 grains.

2. The attractive force of the earth upon 22.794377 cu. in. of distilled water at the temperature of 39.83° Fahrenheit, with the barometer at 30 inches, is the standard unit of weight. This force just equals the Troy pound.

275. Measures of weight are of *four* kinds: *Troy Weight, Diamond Weight, Avoirdupois Weight,* and *Apothecaries' Weight.*

Troy Weight

276. Troy weight is used in weighing gold, silver, and other precious metals; in philosophical experiments; and is the standard at the United States mint.

Table

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

The ounce Troy and the pound Troy must be carefully distinguished from the ounce and pound avoirdupois. The grain, however, is the same throughout.

Diamond Weight

277. Diamond weight is used in weighing diamonds and other precious stones. The *standard unit* is the carat, whose weight is equal to 3.168 Troy grains.

Table

16 parts = 1 carat grain
4 carat grains = 1 carat

The term carat as here employed in diamond weight is an ***actual weight*** and must not be confused with the ***assay carat*** as used at the mint. There it is used, as well as with jewelers, to express the fineness of gold, 24 carats being pure. Thus, "18 carats fine," means that in the mixture there are 18 parts of pure gold and 6 parts alloy, or it is ¾ pure.

Apothecaries' Weight

278. Apothecaries' weight is used by druggists and physicians in compounding medicines and for filling prescriptions.

Table

20 grains = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb. or ℔)

1. The unit of Apothecaries' weight is the pound, containing 5760 Troy grains. The ounces and grains are identical with Troy weight.

2. In wholesaling, most drugs are bought and sold by Avoirdupois weight.

3. Physicians usually write prescriptions in Roman notation, using small letters, substituting *j* for *i* when terminal, and writing the symbol of the denomination before the numeral expressing the quantity. Thus, 6 oz., 3 dr., 1 sc., are written ℥ vj, ʒ iij, ℈ j.

Signs Used in Prescriptions

℞ = recipe
P = small part
P. aeq. = equal parts
aa. = equal quantities
q. p. = as much as you please
ss. = half
misce. = mix
gr. = grain

Avoirdupois Weight

279. Avoirdupois weight is employed in weighing all heavy articles such as merchandise, farm produce, and metals, except the precious metals.

Table

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 cwt., or 2000 lbs. = 1 ton (T.)

1. The ***long***, or gross ton, contains 2240 pounds, and is still used in Great Britain, at the U. S. Custom Houses, in ocean freights, and by wholesale

dealers in coal and iron. The ton of 2000 lb. is sometimes called a ***short ton.***

2. The avoirdupois pound contains 7000 grains.

Comparison of Weights

5760 grains = 1 lb. Troy   480 grains = 1 oz. Troy
5760 " = 1 lb. Apoth.   480 " = 1 oz. Apoth.
7000 " = 1 lb. Avoir.   437½ " = 1 oz. Avoir.

The pound and the ounce are the same in Troy and Apothecaries' weight but differ in the Avoirdupois weight. The grains, however, are uniform in weight.

280. The following table gives the weight of a bushel in pounds according to the established law of most states:

Avoirdupois Pounds in a Bushel

Barley	48	Corn, shelled	56	Peas	60
Beans	60	Corn, ears	70	Potatoes	60
Beets	60	Corn meal	50	Rye	56
Blue Grass seed	14	Flax seed	56	Timothy seed	45
Buckwheat	48	Oats	32	Wheat	60
Clover seed	60	Onions	57	Mineral coal	70

Miscellaneous Weights

1 quintal of fish = 100 lb.   1 barrel of salt = 280 lb.
1 keg of nails = 100 lb.   1 barrel of flour = 196 lb.
1 cental of grain = 100 lb.   1 barrel of pork or beef = 200 lb.
1 firkin of butter = 56 lb.   1 cu. ft. of water = 7½ gal. = 62½ lb.

MEASURES OF CAPACITY

281. Capacity means extent of room, or space.

282. Measures of capacity are divided into two classes—*liquid measure*, and *dry measure*.

283. Dry measure is used in measuring grain, fruit, vegetables, and other dry articles. **Liquid measure** is used in measuring liquids of all kinds.

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

1. The ***unit of liquid measure*** is the gallon, which contains 231 cubic inches.

2. The ***unit of dry measure*** is the ***Winchester bushel***, which is 8 inches deep, 18½ inches in diameter, and contains 2150.42 cubic inches.

3. A ***heaped bushel***, used for measuring apples, corn in the ear, etc., equals 2747.71 cu. in. A ***dry gallon*** (½ pk.) contains 268⅘ cu. in.

4. The barrel and hogshead are not fixed measures, but the following are in common use: 31½ gallons = 1 barrel; 63 gallons = 1 hogshead.

5. The tierce, hogshead, pipe, and tun are the names of casks, and do not express any fixed measures. They are usually ***gauged***, and have their capacities marked on them in gallons. Any measure under 31½ gallons is usually called a ***keg;*** over 31½ gallons, a ***barrel***.

Apothecaries' Fluid Measure

284. **Apothecaries' fluid measure** is used in prescribing and compounding liquid medicines.

Table

60 minims (♏) = 1 fluid drachm (f ʒ)
8 fluid drachms = 1 fluid ounce (f ℥)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gallon (Cong.)

1. *Cong.* is for the Latin *congius* meaning gallon; *O.*, for the Latin *octarius*, meaning one-eighth,—a pint being one-eighth of a gallon.

2. The ***minim*** is equivalent to one drop of water. The gallon is the same as in liquid measure.

MEASURES OF EXTENSION

285. **Extension** is that which has one or more of the dimensions, *length*, *breadth*, or *thickness*.

286. A **line** has only one dimension—*length*. A **surface** has two dimensions—*length* and *breadth*. A **solid,** or body, has three dimensions—*length*, *breadth*, and *thickness*.

287. The **standard unit** of measures of extension in the United States is the *yard*.

The *standard yard* is the length of a metal bar preserved in the office of standard weights and measures. Under the reign of William IV. it was declared to be fixed by dividing a pendulum which vibrates seconds in a vacuum, at the level of the sea, at 62° Fahrenheit, in the latitude of London, into 391,393 equal parts, and taking 360,000 of these parts for the yard. A copy of the standard is kept in each state capitol.

Long Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards, or 16½ ft. = 1 rod (rd.)
320 rods, or 5280 ft. = 1 mile (mi.)

Miscellaneous Long Measures

4 inches = 1 hand
6 feet = 1 fathom
120 fathoms = 1 cable length
1.152⅔ miles = 1 knot

1. The *fathom* and *cable length* are used by sailors for measuring depths at sea. The *knot* is used for measuring distances at sea, and hence is sometimes called a *nautical* or *geographical* mile. Three knots are called a *league*.

2. The hand is used in measuring the height of horses at the shoulder.

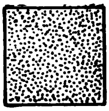
3. The ancients usually derived their units of length from some part of the human body. Thus, we find the *fathom*, the distance of the outstretched hands; the *cubit* (18 in.), the length of the forearm; the *foot*, the length of the human foot; the *span* (9 in.), the distance between the ends of the thumb and little finger when outstretched; the *hand*, the width of the hand; a *pace*, three feet. The Roman *pace* was five feet, and the *mille passus*, a thousand paces, is the origin of the modern *mile*.

4. The *furlong*, or 40 rods, the length of a furrow, is seldom used.

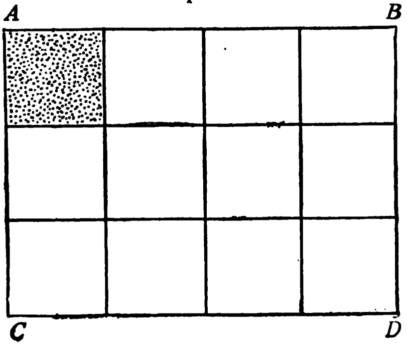
Square Measure

288. **Square** or **surface measure** is used in computing the areas of plain surfaces.

The *unit of measure* of any surface is a square whose length and breadth are some known unit. Thus, 1 sq. in. is the unit of measure for square inches; 1 sq. ft. for square feet; etc.



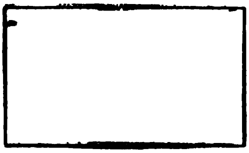
Square



289. The **area** of a surface is expressed by the product of its two dimensions.

Thus, in the figure ABCD there are 3 times 4, or 12 squares in all; and since these make up the entire surface, the measure of the surface called its *area*, is 12 square units.

290. A **square** is a plain surface bounded by four equal sides and having four square corners.



Rectangle

291. A **rectangle** is a plain surface, or figure, bounded by four straight lines, and having four square corners.

Table

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
36 square miles = 1 township (Tp.)

1. The term *perch* or *pole* is sometimes used instead of rod. **The *rood*,** 40 perches, is found in old title-deeds and surveys.

Surveyors' Measure

292. **Surveyors' measure** is used by surveyors in computing the area of land, laying out roads, establishing boundaries, etc.

293. The *unit* of long measure is the *Gunter's chain*, which is composed of 100 links, each 7.92 inches long. The *acre* is the *unit* of land measure.

A steel measuring-tape 100 feet long, each foot being divided into *tenths* and *hundredths*, is now frequently used by surveyors instead of the chain.

Long Measure

7.92 inches = 1 link (l.)
25 links = 1 rod (rd.)
4 rods, or 100 l. = 1 chain (ch.)
80 chains = 1 mile (mi.)

Square Measure.

625 square links = 1 sq. rod
16 square rods = 1 sq. chain
10 square chains = 1 acre
640 square acres = 1 sq. mi.

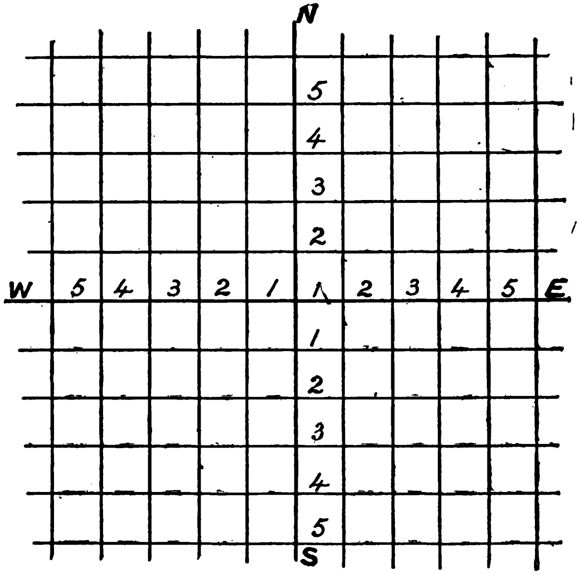
1. In the government surveys, a square mile, or 640 acres, is called a *section*.
2. City lots are usually measured by feet and decimal fractions of a foot; farms, by rods or chains.

United States Public Lands

294. In the Eastern States, the land was divided, as convenient, when settled, and the description of tracts of land refer

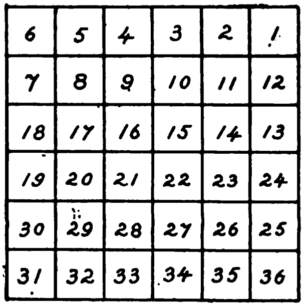
to such natural objects as near-by boulders, ponds, established roads, etc. But all states whose lands have been surveyed since 1802 are divided by a system of meridians and parallels. Ohio, the first state laid out on this plan, was modeled by Col. Jared Mansfield when he was Surveyor-General of the North West Territory. The plan was adopted by the general Government in the early part of Jefferson's administration.

The system provides for the establishment of a north and south line called the ***principal meridian*** and an east and west line intersecting this as a ***base line.*** Other lines are then run, at regular intervals of 6 miles each, both east and west of the principal meridian, and north and south of the base line. In this manner the territory is divided into tracts 6 miles square, called ***townships.*** The lines of townships extending north and south are called ***ranges,*** which are numbered from the principal meridian east and west as shown in the accompanying illustration. A ***township*** is therefore designated by its number and the number of its range.

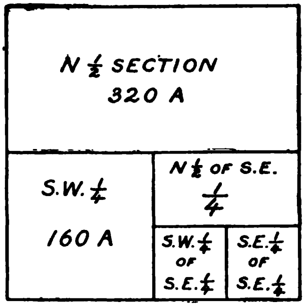


295. Each township is divided into 36 squares, or tracts, each 1 mile square, called sections. The numbering of sections begins at the northeast corner and runs to the west as shown in the diagram below, then to the east on the second line, continuing back and forth, until all are numbered. The corners of all sections are permanently marked by stones, or otherwise.

The divisions of sections into half sections, quarter sections, etc., are shown in the diagram.



A Township



A Section

Thus, the S. E. ¼ of the S. E. ¼, section 4, means the southeast quarter of the southeast quarter of section 4.

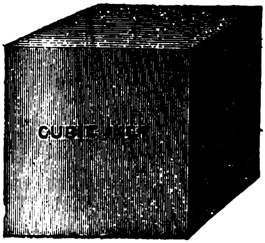
1. Since the earth's surface is convex, the principal meridians converge as they proceed northward. This tends to throw the townships and sections out of square, and necessitates occasional lines of offset, called "*correction lines.*"

2. City and village plats are usually sub-divided into *blocks*, and these into *lots.*

3. The old French *arpent* is still used as the unit of land measure in the vicinity of St. Louis, and in other parts of the Mississippi valley that were settled by the French; while in Texas, New Mexico, and other Spanish sections of the United States, the Spanish land measure is still in use. The unit of length is the *vara*, equal in Texas to 33⅓ inches, in California to 33 inches, and in Mexico to 32.9927 inches.

296. The advantages of the United States survey over all others are its simplicity, uniformity, and official character.

Cubic Measure



297. **Cubic** or **solid measure** is used in measuring solids, or bodies which have length, breadth, and thickness.

298. A **cube** is a regular solid body with six equal square sides or faces.

The unit of cubic measure is a cube,

each of whose edge is a unit of some known length. Thus, a cubic foot is a cube, etc.

299. The **volume,** or solid contents, of a cubical body is the number of cubic units it contains. This is expressed by the product of the three dimensions.

TABLE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
1 cubic yard = 1 load (of earth, etc.)
16 cubic feet = 1 cord foot (cd. ft.)
8 cord feet, or 128 cu. ft. = 1 cord (C.)
24¾ cubic feet = 1 perch

1. A *perch* of stone or masonry is 16½ ft. long, 1½ ft. thick, and 1 ft. high, and contains 24¾ cu. ft.

2. A cord of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. Usually, however, a pile 8 ft. long and 4 ft. high is considered a cord without reference to the length of the stick.

3. One hundred cu. ft. of the internal capacity of a vessel is registered a ton.

4. In estimating weight, 40 feet of square timber, or 50 feet of round timber is reckoned as a ton.

MEASURES OF TIME

300. **Time** is a measure of duration.

The *unit* of measure is the *mean solar day* though there are three natural time units—the year, the month, and the day.

301. The *natural day,* from midnight to midnight, is not of uniform length. It includes one revolution of the earth on its axis. The *mean solar day* is the average of all the days in the year.

302. The *month,* the period from one new moon to the next, averages 29½ days nearly. The year, the period between two successive vernal equinoxes, equals 365.2422 days, or 365 da., 5 hr., 48 min., 49.7 sec.

303. A **calendar** is an adjustment of these natural time units for civil purposes, by making the number of days in each calendar month and calendar year *integral,*

1. The common year contains 52 weeks and 1 day, the leap year 52 weeks and 2 days.

2. The *lunar calendar* makes the months alternately 30 and 29 days and the year 12 months, or 354 days. Since the lunar year is 11 days too short, extra months must be added from time to time. The lunar year is still used in Mohammedan countries. It was used by most ancient peoples and by the Romans until 46 B.C.

3. The Julian calendar, established by Julius Cæsar in 46 B.C., provided a common year of 365 days, and every fourth year a leap year of 366 days. The months, beginning with March, were made alternately 31 and 30 days, February receiving 29 days in ordinary years. In the time of Augstus Cæsar, the eighth month was named in his honor and given an additional day at the expense of February. October and December were made months of 31 days instead of September and November. The average Julian year exceeds the true solar year by .0078 of a day, or $\frac{1}{128}$ of a day. By 1582 the error amounted to 13 days. Pope Gregory XIII corrected as much of the error as had accumulated since A.D. 325, the date of the Council of Nice, by decreeing that in 1582 the 5th of October should be called the 15th. He then instituted the Gregorian calendar, in which every year divisible by 4 and not by 100 is a leap year. Of the century years only those divisible by 400 are leap years.

4. Most Catholic countries adopted the Gregorian calendar soon after it was established. Great Britain did not adopt until 1752, when the error amounted to 11 days. By act of Parliament, the 3rd of September was called the 14th. The civil year by the same act was made to commence on the 1st of January, instead of the 25th of March, as was previously the case.

5. Dates reckoned by the Julian calendar are called Old Style (O. S.), and those reckoned by the Gregorian calendar are called New Style (N. S.).

Table

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 com'l month

Table

12 months (mo.) = 1 year (yr.)
360 days = 1 commercial year
365 days = 1 common year
366 days = 1 leap year
100 years = 1 century

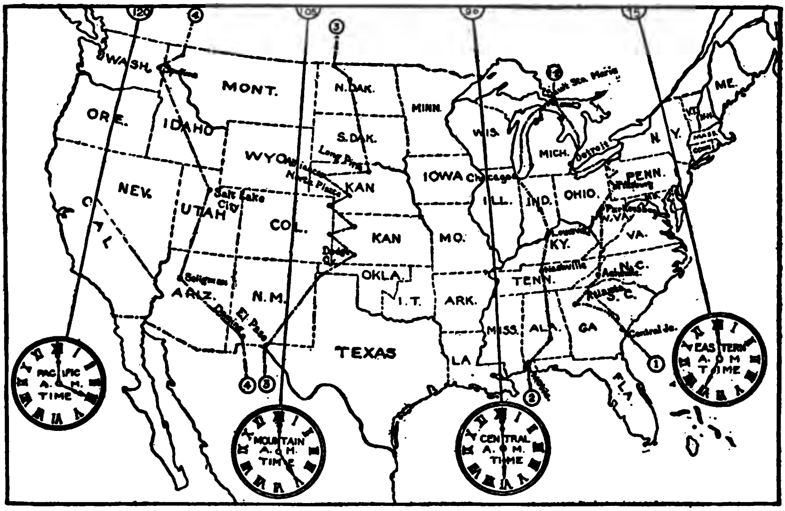
304. The months vary in length from 28 to 31 days. The following table gives the order, name, and length of each:

January (Jan.)	31 da.	May	31 da.	September (Sept.)	30 da.
February (Feb.)	28–9 da.	June	30 da.	October (Oct.)	31 da.
March (Mar.)	31 da.	July	31 da.	November (Nov.)	30 da.
April (Apr.)	30 da.	August (Aug.)	31 da.	December (Dec.)	31 da.

305. The number of days in each month may be kept in mind by memorizing the following rhyme:

Thirty days hath September,
  April, June, and November;
All the rest have thirty-one,
  Save February, which alone
Hath twenty-eight, and one day more
  We add to it one year in four.

306. Standard Time. In 1883, the principal railroad companies throughout the United States and Canada decided to adopt standard time They divided the country into four time belts each of approximately 15° of longitude in width. The time in the various belts therefore differs by hours, while the minute and second hand of all timepieces remain the same. The correct time is distributed by telegraph throughout the United States from the Naval Observatory at Washington each day.



The region of *Eastern Time* lies approximately 7½° each side of the 75th meridian and includes all territory between the Atlantic Ocean and an irregular line drawn from Detroit, Mich., to Charleston, S. C. The time throughout this belt is the same as the local time of the 75th meridian. Similarly, the *Central*, *Mountain*, and *Pacific* time belts lie approximately 7½° each side of the 90th, 105th, and 120th meridians, and the time througout in each belt is determined by the local time of the 90th, 105th, and 120th meridians.

Central Time includes all the territory between the above-named line and an irregular line drawn from Bismark, N. Dak., to the mouth of the Rio Grande. *Mountain Time* includes all territory between the last-named line and nearly all the west borders of Idaho, Utah, and Arizona. *Pacific Time* includes all territory between the last-named line and the Pacific Ocean.

The time of each section differs from that next to it by exactly one hour; thus at 12 o'clock noon, New York (Eastern time) it is 11 a.m. at Chicago (Central time), 10 a.m. at Denver (Mountain time), and 9 a.m., San Francisco (Pacific time). Standard time is 16 minutes slower at Boston than the true local time, 4 minutes slower at New York, 8 minutes faster at Washington, 19 minutes faster at Charleston, 28 minutes slower at Detroit, 8 minutes faster at Kansas City, 10 minutes slower at Chicago, 1 minute faster at St. Louis, 28 minutes faster at Salt Lake City, and 10 minutes faster at San Francisco.

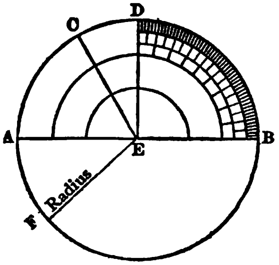
Standard time has been adopted by most of the leading governments of the world.

CIRCULAR MEASURE

307. **Circular** or **angular measure** is used in measuring angles and arcs of circles, in determining latitude and longitude, the location of places and vessels, etc.

The *unit* of this measure is the *degree*, which is $\frac{1}{360}$ of the circumference of any circle.

308. A **circle** is a plane figure bounded by a curved line every point of which is equally distant from a point within called the center.



309. The **circumference** of a circle is the line that bounds it.

310. An **arc** is any part of the circumference of a circle; as AC, CD, etc.

311. The **diameter** of a circle is a straight line passing through the center and terminating in its circumference.

312. The **radius** of a circle is a straight line passing from the center to any point in the circumference.

313. The **measure of an angle** is the arc of the circle included between its sides. Thus, the arc AC is the measure of the angle AEC.

TABLE

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circle (Cir.)

1. A *quadrant* is $\frac{1}{4}$ of a circumference, or 90°; a *sextant* is $\frac{1}{6}$ of a circumference, or 60°; a *sign* is $\frac{1}{12}$ of a circumference, or 30°.

2. A degree varies with the size of the circle; thus, a degree of longitude at the equator is 69.16 statute miles, but diminishes in length as we approach the poles.

MISCELLANEOUS MEASURES

Counting

12 units or things = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross
20 units or things = 1 score (scr.)

Paper

24 sheets = 1 quire (qr.)
20 quires = 1 ream (rm.)
2 reams = 1 bundle (bdl.)
5 bundles = 1 bale (bl.)

Manufacturers and wholesalers usually sell paper by the pound.

Printers' Measure

314. **Composition** is the term applied to type when set for printing, also the act of setting type.

1. There are two standard units of measurement, the *point* and the *pica*. A point = $\frac{1}{72}$ of an inch in width, a pica ≐ $\frac{1}{6}$ of an inch.

2. Each piece of type when in position for printing stands $\frac{15}{16}$ of an inch high:

3. Sizes and corresponding names for ordinary type as used in book and common newspaper work are as follows:

4½ = diamond
5 = pearl
5½ = agate
6 = nonpareil
7 = minion
8 = brevier
9 = bourgeois
10 = long primer
11 = small pica
12 = pica
14 = English
16 = columbian
18 = great primer
20 = paragon

315. An **em** is a piece of type, of any size, exactly square.

1. A *pica em* is 12 point, or $\frac{1}{6}$ of an inch square; a *nonpareil em* is 6 point, or $\frac{1}{12}$ of an inch square, etc.

2. A *quad* is a piece of metal shorter than regular faced-type and is used for spacing and filling out incomplete lines.

316. **Newspaper columns** are generally 13 pica ems wide, or 2⅙ inches.

1. Composition work is usually estimated by the 1000 ems.

2. From the above data a newspaper column 22 inches deep and 13 ems wide would contain 13×6×22=1716 pica ems.

317. In **printing** and **binding books,** the term folio, quarto, etc., are used to denote the number of leaves into which a sheet of paper is folded.

When a sheet is folded into 2 leaves the book is called a folio, and 1 sheet of paper makes 4 pages; when into 4 leaves, a quarto; when into 8 leaves, an octavo; when into 12 leaves, a duodecimo, etc.

DRILL EXERCISE

1. What is a decimal fraction? What are denominate numbers?

2. How do you reduce a decimal to a fraction? A fraction to a decimal? Give three underlying principles in the division of decimals.

3. Write the abbreviations or conventions used respectively (*a*) for miles, rods, yards, feet, and inches; (*b*) for per cent.; (*c*) for degrees; (*d*) for links; (*e*) for hogsheads; (*f*) for square yards.

4. How many ½-inch cubes in an 8-inch cube?

5. Write the standard unit of dry measure in cubic inches; the standard unit of weight in the United States in grains; the standard unit of extension.

6. Draw a township map and locate section 16. Indicate the N. E. ¼ of S. E. ¼ and find its value at $40 an acre.

7. How many cubic inches in a 3-inch cube, and how many square inches in its surface?

8. Explain how Washington was born Feb. 11, 1732 (O. S).

9. Which is heavier, a Troy ounce or an Avoirdupois ounce? A Troy pound or an Avoirdupois pound?

10. How many miles around the boundary of a government section of land? Around a quarter section? How many farms of 40 acres each can be made from 1 square mile of land?

11. What is the difference between a simple and compound number?

12. When Alaska was bought from Russia in 1867, the

United States authorities made a change of 11 days in the reckoning. Why not 12?

13. A man has 2 English sovereigns, 3 half-dollars, 3 francs, and 1 mark. What is their value in United States money?

14. How many quarts of milk in 693 cubic inches?

15. How many grains of silver in a dollar? feet in a mile? cubic inches in a bushel? degrees in a circle? cubic inches in a gallon?

16. What is the difference in weight, expressed in Avoirdupois pounds, between 100 pounds Troy and 100 pounds Avoirdupois?

17. Write a number expressing volume; distance; surface.

REDUCTION OF DENOMINATE NUMBERS

318. Reduction of denominate numbers is the process of changing them from one denomination to another without altering their value.

319. The process of changing from a higher to a lower denomination is called *reduction descending;* and from a lower to a higher, *reduction ascending.*

1. In reducing from a higher to a *lower* denomination, whether the quantity be an integer, a fraction, or a decimal, *multiply* by the number of units of the lower denomination to reduce to the denomination required.

2. In changing from a lower to a *higher* denomination, whether the quantity be an integer, a fraction, or a decimal, *divide* by the number of units of the higher denomination to reduce to the denomination required.

Reduction Descending

320. Reduction of a denominate number to a required lower denomination.

Example 1. Reduce 24 rd. 5 yd. 1 ft. 8 in. to inches.

$$\begin{array}{l} 24 \text{ rd. } 5 \text{ yd. } 1 \text{ ft. } 8 \text{ in.} \\ \underline{\quad 5\frac{1}{2} \quad} \\ 137 \text{ yd.} \\ \underline{\quad 3 \quad} \\ 412 \text{ ft.} \\ \underline{\quad 12 \quad} \\ 4952 \text{ in.} \end{array}$$

Solution. Since 1 rd. $= 5\frac{1}{2}$ yd., 24 rd. $= 24$ times $5\frac{1}{2}$ yd., or 132 yd., and $132 + 5$ yd. $= 137$ yd.

Since 1 yd. $= 3$ ft., 137 yd. $= 137$ times 3 ft., or 411 ft., and $411 + 1$ ft. $= 412$ ft.

Since 1 ft. $= 12$ in., 412 ft. $= 412$ times 12 in., or 4944 in., and $4944 + 8$ in. $=$ 4952 in., the required result.

Example 2. Reduce $\frac{2}{3}$ of a mile to integers of lower denominations.

$\frac{2}{3}\times 320=\frac{640}{3}=213\frac{1}{3}$ rd.
$\frac{1}{3}\times 5\frac{1}{2}=\frac{11}{6}=1\frac{5}{6}$ yd.
$\frac{5}{6}\times 3=\frac{15}{6}=2\frac{1}{2}$ ft.
$\frac{1}{2}\times 12=\frac{12}{2}=6$ in.

Solution. To reduce $\frac{2}{3}$ of a mile to lower denominations, the successive multipliers are 320, $5\frac{1}{2}$, 3, and 12. Multiplying $\frac{2}{3}$ mi. by $320=213\frac{1}{3}$ rd. Multiplying $\frac{1}{3}$ rd. by $5\frac{1}{2}=1\frac{5}{6}$ yd. Multiplying $\frac{5}{6}$ yd. by $3=2\frac{1}{2}$ ft. Multiplying $\frac{1}{2}$ ft. by $12=6$ in. Therefore, $\frac{2}{3}$ of a mile is equal to 213 rd. 1 yd. 2 ft. 6 in.

NOTE. As far as possible, the work of reduction should be performed mentally.

Example 3. Reduce .645 of a £ to integers of lower denominations.

$$\begin{array}{r} \pounds.645 \\ \underline{20} \\ 12.900\ s. \\ \underline{12} \\ 10.8\ d. \\ \underline{4} \\ 3.2\ \text{far.} \end{array}$$

Solution. Since there are 20 shillings in £1, in £.645 there are .645×20 shillings, or 12.9 shillings. Since there are 12 pence in 1 shilling, in .9 there are .9×12 pence, or 10.8 pence. Since there are 4 farthings in 1 penny, in .8 pence there are .8×4, or 3.2 pence. Therefore .645 of a £=12*s.* 10*d.* 3.2 far.

321. **General Rule.** *To reduce a denominate number from a higher to a lower denomination, whether it be an integer, a fraction, or a decimal, multiply by the number of units in the scale to reduce to the denomination, or denominations required.*

WRITTEN EXERCISE

1. Reduce £8 13*s.* 10*d.* to pence.
2. Reduce 2 yr. 8 mo. 21 da. to days.
3. In 3 T. 6 cwt. 21 lb. 12 oz., how many ounces?
4. How many francs in $2123 United States money?
5. Change £16.467 to integers of lower denominations.
6. Reduce $\frac{3}{14}$ of a Troy pound to grains.

Solution. $\frac{3}{\underset{7}{\cancel{14}}}\times\frac{\overset{6}{\cancel{12}}}{1}\times\frac{20}{1}\times\frac{24}{1}=\frac{8640}{7}=1234\frac{2}{7}$ grains.

7. Reduce $\frac{5}{12}$ of a £ to pence.
8. If 1 pound sterling can be bought for $4.8665, how many pounds can be bought for $800?

9. Reduce 3.548 yr. to integers of lower denominations.

10. Change $\frac{7}{8}$ of a cord to cubic feet.

11. Which has the greater intrinsic value, £15, 300 marks, or 375 francs, and by how much in United States money?

12. If I buy the N. E. $\frac{1}{4}$ and the E. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of a section of land, how many acres do I buy? Draw a diagram of a section and locate the above parts.

13. Reduce $\frac{3}{7}$ of a mile to integers of lower denominations.

14. A locomotive wheel is 180 inches in circumference. How many revolutions will it make in going from Trenton to Jersey City, a distance of 56.8 miles?

WRITTEN EXERCISE

Reduce:

1. $\frac{3}{7}$ of a mile to feet.
2. $\frac{15}{17}$ of a bushel to pints.
3. $\frac{5}{12}$ of a £ to pence.
4. 365 miles to feet.
5. 4 sq. mi. to acres.
6. .85 cords to cubic feet.
7. .375 miles to feet.
8. $2\frac{1}{2}$ T. to ounces.
9. 5 lb. 7 oz. to pennyweights.
10. Reduce 29120 lb. to long tons.

Reduction Ascending

322. Reduction of denominate numbers to higher denominations.

Example 1. Reduce 1674 pence to higher denominations.

12)1674*d*.
 20)139*s*.+6*d*.
£6 +19*s*.
1674*d*.=£6 19*s*. 6*d*.

Solution. Since 12*d*. make 1 shilling, in 1674*d*. there are 139*s*. and 6*d*. more.

Since 20*s*. make £1 in 139*s*. there are £6 and 19*s*. more. Therefore 1674*d*.=£6 19*s*. 6*d*.

Example 2. Reduce $\frac{5}{7}$ of a penny to the fraction of a £.

$$\frac{\overset{1}{\cancel{5}}}{7}\times\frac{1}{12}\times\frac{1}{\underset{4}{\cancel{20}}}=\pounds\frac{1}{336}.$$

Solution. Divide successively by 12, and 20, the numbers in the ascending scale, required to reduce pence to pounds, and we have £$\frac{1}{336}$.

NOTE. If the answer is required in the form of a decimal, reduce the resulting fraction to a decimal by Sec. 232.

Example 3. Reduce .75 of a pence to the decimal of a pound.

12).75*d*.
20).0625*s*.
£.003125

Solution. Divide the given decimal by the numbers of the scale required to reduce pence to pounds, and we have £.003125.

NOTE. If the answer is required in the form of a fraction, reduce the resulting decimal to a fraction by sec. 230.

Example 4. Reduce 8*s*. 6*d*. 2 far. to the decimal of a £.

4)2 far.
12)6.5*d*.
20)8.54166+*s*.
£.427083+

Solution. Divide the lowest denomination by the number in the scale which reduces it to the next higher. Putting with this the 6*d*., the result is 6.5*d*. Dividing 6.5*d*. by 12, and putting with this the 8*s*., the result is 8.54166+*s*. Then dividing by 20, the result is .427083+ pounds sterling.

323. **General Rule.** *To reduce a denominate number from a lower to a higher denomination, whether it be an integer, a fraction, or a decimal, divide by the number of units in the scale to reduce to the denomination, or denominations required.*

WRITTEN EXERCISE

Reduce to higher denominations:

1. 17345 farthings.
2. 3456 feet.
3. 16630 seconds.
4. 3000 square inches.
5. 4236 gills.
6. 22346 links.
7. 1234 lb. avoirdupois.
8. 4320 oz. Troy.
9. 56780 cu. in.
10. 4688 hours.

WRITTEN EXERCISE

1. Reduce 12*s*. 7*d*. 3 far. to the decimal of a pound sterling.
2. What decimal of a mile are 28 rd. 2 yd. 1 ft. 11.04 in.?
3. What decimal of a quart is .0175 of a bushel?
4. What fraction of a ton is 17 cwt. 72 lb. 4 oz.?
5. Change $\frac{7}{8}$ of a shilling to the fraction of a pound sterling.
6. If a cubic foot of limestone weighs 175 lb., what is the weight of a cubic yard?
7. How much time will a person gain in 40 yr., by rising 25

min. earlier and retiring 20 min. later every day, counting 9 leap years in the time?

8. What fraction of an hour is $\frac{5}{1176}$ of a week?
9. Reduce £10 18*s.* 6*d.* to United States money.
10. How many carats fine is a piece of gold $\frac{5}{8}$ pure?

Addition of Denominate Numbers

324. Denominate numbers are *added*, *subtracted*, *multiplied*, and *divided* by the same general methods as are employed in similar operations in simple numbers. The only modification of the rule needed is that which is required by a *varying scale* instead of a *uniform* scale of 10.

1. The principles will be made sufficiently plain in the operations and analyses to render special rules unnecessary.

Example 1. Find the sum of £12 11*s.* 6*d.*, £8 5*s.* 10*d.*, £6 6*s.* 6*d.*, £13 4*s.*

£	*s.*	*d.*
12	11	6
8	5	10
6	6	6
13	4	0
40	7	10

Solution. Write the numbers so that units of the same denomination stand in the same column, and begin at the right to add. The sum of the pence is 22*d.* = 1*s.* 10*d.* Write the 10*d.* under the column of pence, and add the 1*s.* to the column of shillings, obtaining for the sum 27*s.* = £1 7*s.* Write the 7*s.* under the column of shillings and add the £1 to the column of pounds, obtaining for the sum £40, which write under the column of pounds. Hence the sum is £40 7*s.* 10*d.*

Example 2. Add $\frac{3}{7}$ bbl., $\frac{7}{8}$ gal., and $\frac{1}{2}$ qt.

	gal.	qt.	pt.
$\frac{3}{7}$ bbl. =	13	2	0
$\frac{7}{8}$ gal. =		3	1
$\frac{1}{2}$ qt. =			1
	14	2	0

Solution. Find the value of each denominate fraction in integers of lower denominations (320), and then add as in the example above. If a fractional answer is required, reduce the given fractions to fractions of the same denomination, then add the results.

ORAL EXERCISE

1. Add 5 rd. 2 ft., and 6 rd. 7 ft.
2. Add 10 lb. 4 oz., 6 lb. 6 oz., and 4 lb. 2 oz.

3. Add £12 4*s*., £10 8*s*., and £6 8.

4. Add 4 T. 6 cwt. 50 lb., and 6 T. 6 cwt. 50 lb.

Name the sum of:

(5)	(6)	(7)	(8)
30 yd. 2 ft.	6 lb. 6 oz.	12 ft. 6 in.	12 mo. 20 da.
8 1	5 6	8 7	11 10

(9)	(10)	(11)	(12)
80 mi. 40 rd.	9 gal. 2 qt.	18 bu. 3 pk.	22 mo. 22 da.
20 80	1 3	44 2	11 16

WRITTEN EXERCISE

1. Find the sum of 2 mi. 6 rd. 4 yd. 3 ft., 3 mi. 7 yd. 6 ft., 4 mi. 12 rd. 3 ft. 6 in., 8 mi. 8 rd. 5 yd. 2 ft. 4 in., 7 mi. 6 rd. 7 yd. 2 ft. 8 in.

2. Add $3\frac{1}{2}$ hhd. 24 gal. 3 qt. 1 pt., $\frac{3}{4}$ bbl., 12 bbl. 4 gal. 3 pt., $\frac{3}{4}$ gal. 10 qt. 1 pt., 3 hhd. 6 gal.

3. Bought three loads of hay at $15 a ton. The first weighed 1.125 T., the second $1\frac{3}{5}$ T., and the third 2750 lb. What did the whole cost?

4. Find the sum of 142 cu. yd. 18 cu. ft. 1234 cu. in., 324 cu. yd. 23 cu. ft. 1276 cu. in., 346 cu. yd. 16 cu. ft. 1274 cu. in., 143 cu. yd. 30 cu. ft. 1645 cu. in.

5. What is the sum of $\frac{3}{7}$ mi., $\frac{3}{8}$ rd., and $\frac{2}{3}$ yd.?

6. To .005 of a ton add $\frac{3}{64}$ cwt. $+\frac{9}{10}$ of a pound.

Subtraction of Denominate Numbers

Example. From £175 6*s*. 8*d*. take £87 12*s*. 6*d*.

£	*s*.	*d*.
175	6	8
87	12	6
87	14	2

Solution. Write the numbers so that units of the same denomination stand in the same column, and begin to subtract at the right. 6*d*. from 8*d*. leaves 2*d*., which write under the column of pence. Since 12*s*. cannot be subtracted from 6*s*., take £1 = 20*s*. from £175, leaving £174, and add it to the 6*s*., making 26*s*. Subtracting 12*s*. from 26*s*. leaves 14*s*., which write under the column of shillings. £87 from £174 leaves £87. Hence, the difference is £87 14*s*. 2*d*.

ORAL EXERCISE

Name the difference between:

(1)	(2)	(3)	(4)
20 gal. 3 qt.	34 bu. 3 pk.	12 cwt. 20 lb.	75 ft. 10 in.
16 5	14 2	10 80	39 12

(5)	(6)	(7)	(8)
16 rd. 12 ft.	44 mi. 80 rd.	40 yd. 2 ft.	11 mo. 25 da.
11 15	23 75	12 $1\frac{1}{2}$	8 28

9. A man owned a farm of 140 A. and sold 50 A. 100 sq. rd. How much had he remaining?

WRITTEN EXERCISE

1. From 28 rd. 2 yd. 1 ft. 8 in., take 16 rd. 5 yd. 2 ft. 3 in.
2. Find the difference between $8\frac{9}{10}$ cwt., and $48\frac{3}{8}$ lb.
3. From a farm containing 250 A., two lots were reserved, one containing 75 A. 136.4 sq. rd., and the other 65 A. 116.3 sq. rd. What was the remainder of the farm worth at $78.50 an acre?
4. Find the difference in yards of $\frac{5}{8}$ yd. and $2\frac{1}{4}$ ft.
5. $\frac{1}{7}$ yd. $-\frac{1}{7}$ ft. $-\frac{1}{7}$ in. $=$?
6. A cycling party rode $75\frac{1}{4}$ miles at the average rate of $10\frac{1}{2}$ miles per hour. If they started at 9:55 A.M., at what hour did they complete the trip?

Finding the Interval of Time between Two Dates

325. There are two methods employed in finding the interval of time between two dates: *compound subtraction*, and *counting the actual number of days*.

1. In finding the difference of time by compound subtraction, 12 mo. are considered a year, and 30 da. a month.
2. In finding the date of maturity of a note, draft, or other contract, the date on which it is written is omitted in the calculation.
3. In banking circles and business transactions generally, involving short periods of time, the difference is generally found by counting the exact number of days.

Example. Find the difference of time, by compound subtraction, between January 15, 1910, and March 25, 1912.

yr.	mo.	da.
1912	3	25
1910	1	15
2	2	10

Solution. The method employed in subtracting dates by compound subtraction is the same as employed in other denominate numbers. In arranging the work the expression for a date requires that the *numbers* of the months be written instead of their *names*, as shown in the illustration.

WRITTEN EXERCISE

Find the exact number of days between:

1. Mar. 2, and Nov. 6.
2. Feb. 4, and Oct. 20.
3. Aug. 23, and Dec. 28.
4. Feb. 20, and Apr. 30.
5. Oct. 1, and Nov. 30.
6. Jan. 1, 1911, and Oct. 5, 1911.
7. June 3, 1910, and Dec. 24, 1910.
8. Nov. 23, 1910, and Feb. 23, 1911.
9. June 25, 1911, and Mar. 16, 1912.
10. Sept. 6, 1912, and Mar. 1, 1913.

Find the difference, by compound subtraction, between:

1. Jan. 5, 1910, Sept. 23, 1912.
2. Feb. 23, 1909, Aug. 16, 1912.
3. Mar. 12, 1910, Oct. 4, 1910.
4. Mar. 5, 1909, Feb. 28, 1912.
5. June 2, 1909, Dec. 31, 1915.
6. July 4, 1912, Aug. 3, 1914.
7. Sept. 6, 1909, Dec. 23, 1912.
8. May 25, 1908, July 3, 1911.
9. Apr. 23, 1912, June 16, 1915.
10. Aug. 23, 1910, Nov. 2, 1914.

Multiplication of Denominate Numbers

Example. Multiply 14 rd. 3 yd. 2 ft. by 8.

rd.	yd.	ft.	
14	3	2	
		8	
117	1(½)	1	
	½ =	1	6 in.
117	1	2	6

Solution. Write the multiplier under the multiplicand as indicated. 8 times 2 ft. = 16 ft. 16 ft. = 5 yd. 1 ft. Write the 1 ft. under feet, and add the 5 yd. to the product of yards. 8 times 3 yd. + 5 yd. = 29 yd. = 5 rd. 1½ yd. Write 1½ yd. under the yards, and add the 5 rd. to the product of rods. 8 times 14 rd. + 5 rd. = 117 rd., which write under the rods.

The ½ yd. = 1 ft. 6 in., which added to the product, gives 117 rd. 1 yd. 2 ft. 6 in. for the entire product.

WRITTEN EXERCISE

1. Multiply 5 rd. 4 yd. 2 ft. 4 in. by 9.

2. Multiply 7 T. 15 cwt. 10.5 lb. by 1.7.

3. Find the difference between ⅜ of 17 bu. 1 pk. 4 qt., and 21 bu. 3 qt.

4. Multiply £17 10*s*. 8*d*. by 9; by 11; by 15.

Suggestion. Reduce the multiplicand to pence according to Sec. 320, perform the required multiplication, and reduce the result to pounds, shillings, and pence according to Sec. 322.

5. Required the cost of one dozen silver spoons, each weighing 18 pwt. 18 gr., at $1.15 per ounce.

6. A watch case of gold, 14 carats fine, weighs 32 pwt. Find the value of the gold in the case if fine gold is worth $20.25 per ounce Troy.

7. A farmer sold 5 loads of oats, averaging 41 bu. 3 pk. each at 45¢ a bushel. What did he receive for the whole?

8. From a farm of 45 A. 40 sq. rd. there were sold 16 building lots, each ¾ A. 30⅛ sq. rd. How much remained?

Division of Denominate Numbers

Example. Divide £14 16*s*. 8*d*. by 8.

£	*s*.	*d*.
8)14	16	8
1	17	1

Solution. Write the divisor at the left of the dividend as in simple numbers. ⅛ of £14 is £1 and £6 remaining. Write the £1 in the quotient, and reduce the £6 to shillings. £6 = 120*s*. + 16*s*. = 136*s*. ⅛ of 136*s*. is 17*s*. Write the 17*s*. in the quotient. ⅛ of 8*d*. is 1*d*. Write the 1*d*. in the quotient. Hence, the quotient is £1 17s. 1*d*.

WRITTEN EXERCISE

1. Divide 52 rd. 4 yd. 1 ft. 6 in. by 9.

2. How many miles of road, three rods wide, will contain 8 acres of land?

3. A rectangular field is 7 ch. 75 l. long and 4 ch. 87½ l. wide. How many rods of fence are required to enclose it?

4. If in building a railroad, rails weighing 80 pounds to the yard are used, how many tons of rails will be required to build a mile of single track road?

5. Divide 16*s.* 3*d.* by .03; by .11; by .09; by .35.

Suggestion. 16*s.* 3*d.* = 195*d.*, and 195*d.* ÷ .03 = 6500*d.*
6500*d.* = £27 1*s.* 8*d.*

6. Reduce $154.25 to equivalent English money.

Suggestion. Reduce $154.25 to pounds and decimal of a pound by dividing by $4.8665. Then reduce the resulting quotient as illustrated in Sec. 320.

7. Reduce $232 to equivalent English money.

8. How many times is 3 bu. 2 pk. 5 qt. contained in 62 bu. 5 qt.?

9. How many boxes, each holding $\frac{3}{4}$ qt., will be required to hold 12 bu. 3 pk. 1 gal. 2 qt. of strawberries?

WRITTEN REVIEW

1. What is the total surface of a cube, the edge of which measures $4\frac{1}{2}$ inches?

2. Bought $1\frac{1}{2}$ acres of land at $3200 an acre and sold it at 12¢ a square foot. Find the entire gain.

3. If a silver dollar weighs $412\frac{1}{2}$ grains Troy, how many pounds avoirdupois do $1,000,000 weigh?

4. A printer used 5 rm. 10 qr. 10 sh. of paper for half-sheet posters. How many did he print? What did they cost at $2.50 per C.?

5. A druggist bought 6 pounds of quinine at $11 per pound, avoirdupios weight, and sold it in 2-grain capsules at 10¢ per dozen. What was his profit?

6. Reduce (a) $\frac{32}{165}$ mi. to integers of lower denominations; (b) $3\frac{3}{7}$ qt. to the fraction of a bushel.

7. If a grocer's scales give $\frac{1}{2}$ oz. short of true weight on every pound, of how much money does he defraud his customers in the sale of 2 bbl. of sugar, each weighing 2 cwt. 10 lb. at 6¢ a pound?

8. If I buy 120 quarts of berries by dry measure, and when selling them use a liquid pint measure, what is my gain if I buy them at 8¢ a quart and sell them at 5¢ a pint?

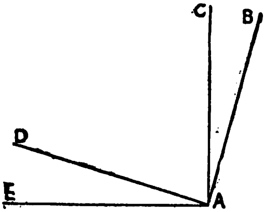
PRACTICAL MEASUREMENTS

Angles and Surfaces

326. An **angle** is an opening between two lines in the same plane that meet in a point.

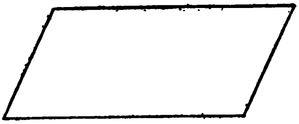
NOTE. A *straight line* is the shortest distance between two points.

327. If one straight line meets another straight line so that the two angles formed are equal, the angles are called **right angles** and the two lines are said to be perpendicular to each other. An **obtuse angle** is one greater than a right angle; an **acute angle** is one less than a right angle.



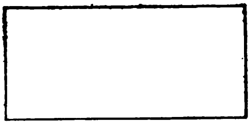
CAE, right angle
CAD, acute angle
BAE, obtuse angle

328. A **parallelogram** is a figure having four straight sides and whose opposite sides are parallel, and consequently equal.

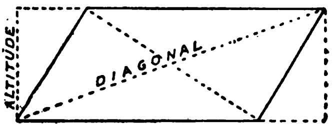


Parallelogram

1. When the angles of the parallelogram form right angles it is called a *rectangle.*
2. The side upon which a figure is supposed to rest is called the *base.*
3. The perpendicular distance between the base line and the highest point opposite is the *altitude.*
4. The straight line joining the opposite angles of a parallelogram is called its *diagonal.*



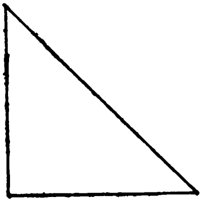
Rectangle



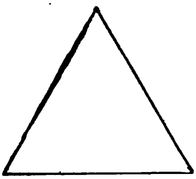
329. A **triangle** is a plane figure bounded by three sides and having three angles.

An *equilateral triangle* has three sides equal; *isosceles* has only two of its sides equal; *scalene* has all of its sides unequal.

330. A **right-angled triangle** is a triangle having one right angle.



An *acute-angled triangle* has three acute angles; an *obtuse-angled triangle* has one obtuse angle.

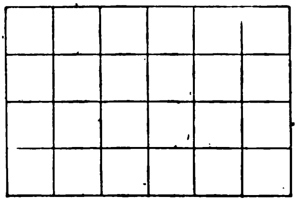


Surface Measure

331. The **area** of a surface is the number of square units it contains.

1. A *surface* has two dimensions, length and breadth.

2. The *measuring unit* of a surface may be any square. Thus, a rectangle 4 inches wide and 6 inches long contains four rows of square inches with six squares in each row, or 24 square inches in all.



332. **To find the area of a rectangle.**

333. **Rule.** *Multiply the length by the breadth and the product will be the area required.*

WRITTEN EXERCISE

1. How many square yards of plastering in the sides of a room 18 feet long, 15 ft. 6 in. wide, and 9 ft. 6 in. high?

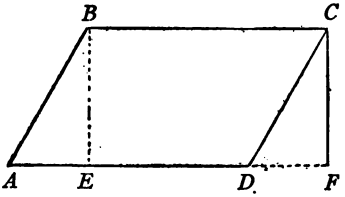
2. A railroad ran the full length of a man's farm $1\frac{1}{4}$ miles and took a strip 4 rods wide, for which it paid at the rate of $150 per acre. How much did it pay him?

3. Find the cost of painting both sides of a fence 6 ft. high, enclosing a field 20 rods long and 300 feet wide, at 27¢ a square yard.

4. How many yards of silk $\frac{5}{8}$ of a yard wide will be required to line 24 yards of satin $\frac{3}{4}$ of a yard wide?

5. A school-room is 30 ft. long, 24 ft. wide, and 10 ft. high above the wainscotting. The trustees pay $20 per thousand for a new floor; $15 a thousand for a new board ceiling; 10¢ a square yard for painting the ceiling; 4¢ a square yard for tinting the walls, and $2 per day for 6 days' labor. Find the total cost.

334. To find the area of a parallelogram.



It will be seen that any parallelogram has the same area as a rectangle with the same base and altitude. Hence, ABCD = the rectangle EFCB; that the base AD = the base EF, and that the altitude of each is BE.

335. Rule. *The area of a parallelogram is equal to the product of its base by its altitude.*

NOTE. The base and altitude must be expressed in units of the same denomination.

WRITTEN EXERCISE

1. The area of a parallelogram is 1620 sq. ft. Its length is 72 ft. What is its altitude?

2. A farm is in the form of a parallelogram, the sides being 125 rods and the altitude 1320 feet. How many acres does it contain?

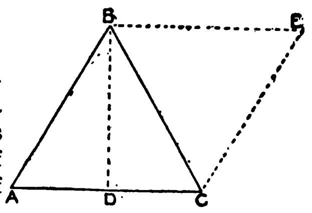
3. A lady has a flower-bed in the form of a parallelogram; if the area is 30 sq. ft. and the base 8 ft., what is the altitude?

4. Find the cost of fencing a rectangular lot of 6 rods frontage, and 10 rods in depth, at a cost of 55¢ per yard for the front fence and 20¢ per yard for the sides and rear.

5. A parlor 18 feet wide requires 64 yards of Brussels carpet, ¾ yard wide. How long is the parlor?

6. One blackboard in the school is 4 ft. wide and 16 ft. long; the other is 3½ ft. wide and 24 ft. long. How many feet must I cut off the length of the larger blackboard so that the remainder will have the same area as the smaller one?

336. To find the area of a triangle.



It will be seen that the triangle ABC is one-half of the parallelogram ABEC. But the area of a parallelogram is equal to the product of its base by its altitude (335); hence, the area of the triangle is equal to one-half the product of the base by the altitude.

337. *Multiply the altitude by one-half the base, or multiply the base by one-half the altitude.*

The point in any figure opposite to, and farthest from the base is called the *vertex*.

WRITTEN EXERCISE

1. How many square feet in a triangle whose base is 18 ft. 3 in. and altitude 8 ft. 6 in.?

2. How many square feet in the two gable ends of a house 42 ft. wide, the ridge being 16 ft. higher than the top of the walls?

3. At 10¢ a square foot, what will it cost to lay sod on a triangular yard whose base is 36 ft. and altitude 21 ft.?

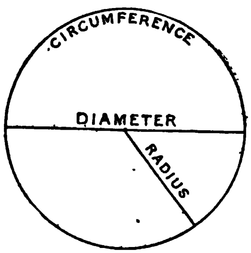
4. What will be the cost of a triangular piece of land whose base is 22.45 ch., and the altitude 12.25 ch., at $85 per acre?

The Circle

338. A **circle** is a plane figure bounded by a curved line, called the circumference, every point of which is equally distant from a point within called the center.

339. The **perimeter** of any plane figure is the distance around it.

The *diameter* of a circle is a line passing through the center and terminating in the circumference; the *radius* is half the diameter.



340. **To find the diameter or circumference of a circle when one of the dimensions is given.**

It is proved in geometry that the circumference of a circle is 3.1416 times the diameter.

341. To find the circumference of a circle when the diameter is given, *multiply the diameter by 3.1416.*

342. To find the diameter of a circle when the circumference is given, *divide the circumference by 3.1416.*

WRITTEN EXERCISE

1. What is the diameter of a circle whose circumference is 40.8408 ft.?

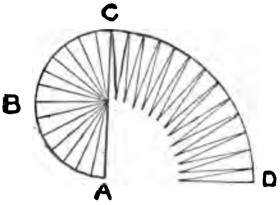
2. The distance across a circular reservoir is 75 feet; what is the distance around it?

3. If it takes 380 steps, $2\frac{3}{4}$ ft. each, to walk around a circular pond, what is the diameter of the pond?

4. Find the circumference of the greatest circle that can be drawn with a string 15 inches long, used as a radius.

343. To find the area of a circle.

The circle may be divided into a number of equal figures that are essentially triangles, the sum of whose bases form the circumference of the circle, and whose altitudes are radii of the circle. Hence:



344. Rule. *The area of a circle is equal to the circumference multiplied by half the radius; or square the diameter and multiply by .7854 ($\frac{1}{4}$ of 3.1416).*

WRITTEN EXERCISE

1. The distance around a circular park is $1\frac{1}{2}$ miles. How many acres does it contain?

2. What part of the area of a sphere 8 in. in diameter is the area of a sphere 4 in. in diameter?

3. A man rolling a wheel-barrow takes 96 steps, each 2 ft. 9 in. long, in a minute. How often will the wheel of the barrow, which is 14 in. in diameter, revolve in an hour?

4. How many square yards are there in a circular walk, the radius of the inner edge of the walk being 10 feet, and that of the outer edge being 15 feet?

5. Find the area and the circumference of a circle whose radius is 8 feet. If the radius of another circle be 3 times as great, or 24 feet, how would the areas of the two compare?

6. What is the area of a semicircle, if the radius of the whole circle is 75 ft.?

SQUARE ROOT AND ITS APPLICATIONS

ORAL EXERCISE

1. Of what number are 3 and 3 the factors? 5 and 5? 7 and 7? 5, 5, and 5? 10, 10, and 10?

2. What is the second power of 6? Of 4? Of 8? Of 10? Of 12?

3. How much is 7^2, 15^2, 25^2, 35^2, 75^2?

4. Name *one* of the two equal factors of 4, 9, 16, 25, 36, 64, 100, 144?

5. What is the *second* power of $\frac{3}{4}$? Of $\frac{4}{5}$? Of $_7$?

345. The **power** of a number is the product arising from using it a certain number of times as a factor.

Thus, 16 is the *second* power of 4; 27 is the *third* power of 3.

346. The **square** of a number is its second power. **Square root** is the finding of two equal factors which multiplied together produce the number.

The number of times a number is used as a factor is indicated by a small figure, called an *exponent*, written a little above and at the right of the number. Thus, $4^2=16$; $3^3=27$; $5^2=25$.

347. A **radical sign** $\sqrt{\ }$ when prefixed to a number indicates that some root is to be found.

Thus, $\sqrt{25}$ denotes the *square root* of 25; $\sqrt[3]{64}$ denotes the cube root of 64. Or, they may be written thus, $25^{\frac{1}{2}}$, $64^{\frac{1}{3}}$.

348. The small figure placed above the radical sign to denote what root is to be found is called the **index** of the root. When no index is written, 2 is understood.

349. The extraction of square root can be better understood by a careful understanding of unit relations as they combine in multiplication.

Example. What is the square of 24?

$$\begin{array}{ll} 24= & 20+\ 4 \\ 24= & \underline{20+\ 4} \\ & 80+16 \\ & \underline{400+\ 80\qquad} \\ & 400+160+16=576. \end{array}$$

Solution. The product of 20+4 by 4 = 20 ×4+4², or 80+16. The product of 20+4 by 20 = 20²+(20×4), or 400+80; hence, the sum of these partial products is 400+160+16, or 576.

350. It will be seen from the above that *the square of a number consisting of tens and units is equal to the sum of the squares of the tens and units increased by twice their product.*

351. Principles of Square Root

1. The square of any number contains twice as many figures as the number itself, or one less than twice as many.

2. If any perfect square be separated into periods of two figures each, beginning with units' place, the number of periods will be equal to the number of figures in the square root of that number.

Example 1. What is the square root of 576?

$$\begin{array}{lr} & 5'76(24 \\ & \underline{4\quad\ } \\ 20\times2=40 & 176 \\ \underline{40+4=44} & \underline{176} \end{array}$$

Solution. Beginning at the right separate the number into periods of two figures each. Then we find that the greatest square in the left-hand period is 4, and the root is 2, which write in the quotient as the tens' figure of the root. Squaring the root figure and subtracting we have 1 for a remainder; to this annex the next period 76, and the dividend is 176. This remainder is equal to twice the product of the tens by the units, plus the square of the units (350). Hence, 2 times 20 = 40, which forms the trial divisor, and which is contained in the dividend 176, 4 times. Write 4 in the quotient, and also add it to the trial divisor, forming the complete divisor, 44. Multiplying the complete divisor 44 by 4, the result is 176. Therefore, 24 is the required root.

Example 2. What is the square root of 5329?
Example 3. What is the square root of 6.5536?

(2)

```
        53'29(73
         49
140     429
143     429
```

(3)

```
        6.55'36(2.56
        4
40      255
45      225
500      3036
506      3036
```

352. **Rule.** *Separate the given number into periods of two figures each, beginning at the units' place.*

Find the greatest number whose square is contained in the left-hand period, and write its root for the first figure of the required root. Subtract this square from the left-hand period, and annex the remainder to the next period for a dividend.

Double the part of the root already found, considered as tens. Find how many times this divisor is contained in the dividend, and write the quotient as the next figure of the root. Add this quotient to the trial divisor to form the complete divisor. Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend.

If there are more periods to be brought down, continue the operation as before.

1. If a cipher occurs in the root, annex a cipher to the trial divisor, and another period to the dividend, and proceed as before.

2. When the given number is a decimal, separate the number into periods by proceeding in both directions from the decimal point.

3. If there is a remainder after the root of the last period is found, annex ciphers and continue the root to as many decimal places as are required.

4. The square root of a fraction may be found by extracting the square root of the numerator and denominator separately. If the terms of the fraction are not perfect squares, reduce the fraction to a decimal before extracting the root.

WRITTEN EXERCISE

Find the square root of:

1. 1089
2. 3025
3. 9604
4. 5476
5. 8836
6. 2209
7. 11881
8. 97344
9. 41616
10. 97.8121
11. .001225
12. 196.2801
13. $\frac{169}{196}$
14. $4.1820\frac{1}{4}$
15. $.000\frac{4}{9}$
16. $\frac{1024}{1600}$

17. A square field contains 1016064 square feet. What is the length of each side?

18. Separate the number 6765201 into four equal factors.

19. What must be taken from 39721 to make it a perfect square?

Applications of Square Root

353. There are many problems pertaining to geometrical figures which involve the applications of square root.

354. It has been shown that the area of a square is equal to the product of its two sides; therefore, the sides of a square must be equal to the square root of its area.

1. A field is 208 rd. long and 13 rd. wide. What is the length of the side of a square containing an equal area?

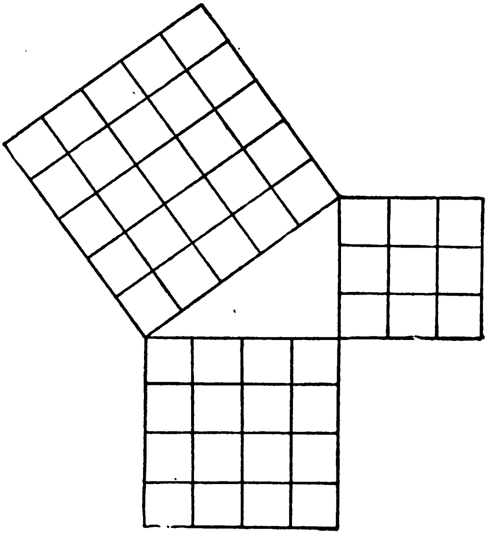
2. A square field contains 2½ acres. How many feet in its perimeter?

3. At $72 an acre, a farm is worth $12,240. It is in the form of a rectangle 160 rods wide. What will it cost to put a fence around it at 75¢ a rod?

4. The length of a rectangular field is 2½ times its breadth, and its perimeter is 308 yards. Find the area of the field.

355. It is a geometrical conclusion that the square formed on the hypotenuse, or side opposite the right angle, of a right-angled triangle is equal to the sum of the squares upon the other two sides. Then it is evident:

1. That the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.



2. *The square of the base, or of the perpendicular, of a right-angled triangle is equal to the square of the hypotenuse less the square of the other side.*

The relation that the base, perpendicular, and hypotenuse bear to each other is shown by the illustration on the preceding page. It was discovered by Pythagoras, a Greek philosopher, and is sometimes known as the 47th proposition of the first book of Euclid's Elements.

WRITTEN EXERCISE

1. The base of a right-angled triangle is 40 ft. and the hypotenuse is 50 ft. What is the perpendicular?

2. The base and perpendicular of a right-angled triangle are 42 ft. and 440 ft., respectively. Required the hypotenuse.

3. What is the area of a rectangle whose diagonal is 75 ft., and whose sides are as 9 to 16?

4. Draw a diagram of a section of land; draw lines to divide the same into quarter sections, and show a square 40 acres in the N. E. corner of the S. E. quarter. Find the cost of a ditch diagonally across said 40 acres, at 85¢ a rod.

5. A square and triangle contain equivalent areas. The base of the triangle is 36.1 inches, altitude 5 in. What is the side of the square?

6. The boundaries of a square and circle are each 40 feet. Which has the greater area?

7. How many rods of fencing will be required to enclose an acre lot in the form of a right-angled triangle whose altitude is twice its base?

8. If a boy is standing in the center of a rectangular field 300 rods long, and 150 rods wide, what is the shortest distance he can travel in going to each corner of the field and terminating his journey at his starting point?

9. A has a circular garden 11 feet in diameter and wishes to lay out another 16 times as large; what must be the diameter of the latter?

356. The three sides of a triangle being given to find its area.

Example. Find the area of a triangle whose sides are 20, 30, and 40 feet.

Solution. $(20+30+40)\div 2=45$; $45-20=25$; $45-30=15$; $45-40=5$. $\sqrt{45\times 25\times 15\times 5}=290.4+$ ft.

357. **Rule.** *From half the sum of the three sides, subtract each side separately. Multiply the half sum and the three remainders together, and extract the square root of the product.*

1. How many acres in a triangular piece of land 40 rods on one side, 50 rods on the second, and 60 on the third?

2. At $90 an acre, what is the cost of a piece of ground in the form of an equilateral triangle whose perimeter is 150 chains?

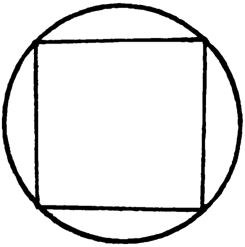
3. How many square feet of boards in the two gable ends of a barn 45 ft. wide, the rafters being 30 ft. long?

358. To find the side of the largest square that can be inscribed in a circle.

Example. What is the side of a square inscribed in a circle whose diameter is 6 feet?

Solution. $6^2\div 2=18$. $\sqrt{18}=4.24$ ft., side of square.

359. **Rule.** *Extract the square root of half the square of the diameter.*



WRITTEN EXERCISE

1. The circumference of a circle is 120 rds. Find the side of the inscribed square.

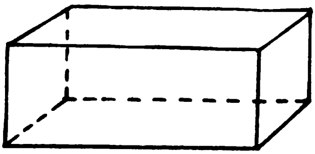
2. The diameter of the smaller end of a log 20 feet long is 1 ft. 6 in. What is the size of the largest square stick of timber that can be cut from it?

3. How large a square can be cut out of a circular board whose circumference is 75.3984 inches?

VOLUMES OR SOLIDS

360. A **solid** or **body** is anything that has length, breadth, and thickness.

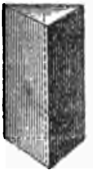
361. A **rectangular solid** is a solid bounded by six rectangles. The bounding rectangles are called *faces.*



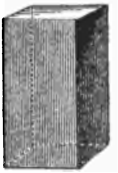
362. A **cube** is a rectangular solid having six square faces.

363. A **cylinder** is a body bounded by a uniformly curved surface, its ends being equal and parallel circles.

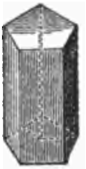
The lines joining the centers of the bases, or ends, of the cylinder are its *altitude*.



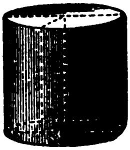
Triangular Prism.



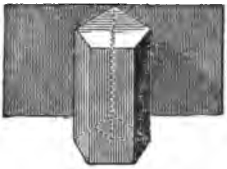
Quadrangular Prism.



Pentagonal Prism.

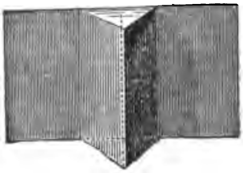


Cylinder.



364. A **prism** is a volume whose upper and lower bases are equal polygons, and its sides parallelograms.

The *unit of measure* for solids is the *cube*, the edge of which is a unit of some known length.



365. A **sphere** is a body bounded by a uniformly curved surface, all the points of which are equally distant from a point within, called the center.

366. **To find the convex surface of any prism or cylinder.**

Rule. *Multiply the perimeter of the base by the altitude.*

367. **To find the volume of any prism or cylinder.**

Rule. *Multiply the area of the base by the altitude.*

368. **To find the area of a sphere.**

Rule. *Square the diameter and multiply by 3.1416.*

369. **To find the volume of a sphere.**

Rule. *Multiply the surface by $\frac{1}{6}$ of the diameter, or $\frac{1}{3}$ of the radius.*

WRITTEN EXERCISE

1. Find the solid contents of a cube whose length is 28 inches.

2. Find the area of the convex surface of a triangular prism, whose altitude is 9½ feet, and the sides of its base 5, 6, and 7 feet, respectively.

3. Required the surface of a sphere whose diameter is 22 inches.

4. The diameter of the earth is 8000 miles, what is the area of its surface?

5. A mining shaft was 250 ft. deep and 6½ ft. square. What was the cost of excavating and timbering, excavations costing $2.50 per cu. yd., and timbering 35¢ per sq. ft. of lateral area?

6. Find the solid contents of a sphere whose diameter is 6 feet.

7. A state capitol building has a dome 50 feet in diameter. What would be the cost to gild it at $3.25 per square foot if it is a perfect hemisphere?

8. What is the volume of the largest sphere that can be made from a 16-inch cube?

9. What will be the cost of a sheet-iron smokestack 50 ft. high and 2½ ft. in diameter, at 20¢ per square foot?

10. Required the contents of a cylindrical log 16 feet long and 3 feet in diameter.

Wood Measure

370. A **cord** of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high and contains 8 cord feet, or 128 cubic feet.

In actual practice a pile of wood 8 ft. long and 4 ft. high is usually considered a cord, the price depending on the length of the stick.

371. To find the number of cords of wood in any pile.

Example. How many cords of wood are there in a pile 100 ft. long, 4 ft. wide, and 5 ft. high?

Solution. $\frac{\overset{25}{\cancel{100}} \times \overset{1}{\cancel{4}} \times 5}{\underset{\underset{8}{\cancel{32}}}{\cancel{128}}} = \frac{125}{8} = 15\frac{5}{8}$ cords.

372. **Rule.** *Divide the product of the length, width, and height as expressed in cubic feet, by 128.*

WRITTEN EXERCISE

1. Find the value of a pile of wood 24 ft. long, 16 ft. wide, and 4 ft. high at $4.25 per cord.

2. Which is more profitable to buy, 4-foot wood at $4.50 per cord, or 20-inch wood at $1.60 per cord?

3. A man agreed to cut wood for a farmer for 85¢ a cord. The farmer charged him $2.75 a week for meals, to be deducted from the wood money. In 11 weeks he cut 6 piles, each 32 ft. long, 12 ft. wide, and 9 ft. high. How much money should the chopper receive?

4. What must be the width of a shed 25 ft. long and 8 ft. high to contain 20 cords of wood?

Boards and Timber.

373. **Boards and timber** are usually estimated in square feet, called board feet, instead of cubic feet.

374. A **board foot** is 1 ft. long, 1 ft. wide, and 1 in. thick, or its equivalent.

1. In board measure all boards are assumed to be 1 in. thick. Boards more than 1 in. in thickness, planks, joists, beams, scantling, and sawed timber are generally measured by the board foot; hewn and round timber are sometimes measured by cubic measure.

2. The average width of a board that tapers regularly from one end to the other is found by taking half the sum of the two ends. Thus, a board 10 ft. long, 9 in. wide at one end and 7 in. wide at the other and 1 in. thick, averages 8 in. wide and contains $6\frac{2}{3}$ ft. of lumber.

3. In some special kinds of choice lumber the price charged is by the board foot, and the width of the board is reckoned only to the ***next smaller half inch.*** Thus, a board $9\frac{3}{4}$ inches in width is reckoned 10 inches; a board $9\frac{1}{4}$ inches in width is reckoned 9 inches; etc.

375. *When lumber is more than 1 inch in thickness the number*

of feet, board measure, is found by multiplying the length in feet by the width and thickness in inches, and dividing the product by 12.

Example. How many feet of lumber in 5 planks 18 ft. long, 3 in. thick, and 10 inches wide?

$$\frac{5\times\overset{9}{\cancel{18}}\times\cancel{3}\times\overset{5}{\cancel{10}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{12}}}=225\text{ ft.}$$

Solution. Arranging the problem in the form for cancellation, as shown in the illustration, and reducing according to the above rule, we have as the result 225 board feet.

376. In *charging* or *billing lumber* the number of pieces is entered first, then the thickness and width in inches and the length in feet.

Thus, 6 pc. pine, 2 in. thick, 8 in. wide, 14 ft. long, the form would be: 6 pc. 2″ ×8″ ×14′. (See p. 134.)

ORAL EXERCISE

By inspection determine the number of feet in the following boards:

1. 12 ft. long, 15 in. wide.
2. 18 ft. long, 14 in. wide.
3. 16 ft. long, 18 in. wide.
4. 24 ft. long, 10 in. wide.
5. 25 ft. long, 14 in. wide.
6. 6 pc., 2″×6″, 20′.
7. 12 pc., 2″×8″, 14′.
8. 10 pc., 2″×8″, 18′.
9. 24 ft. long and 9 in. square.
10. 20 ft. long and 6 in. square.

WRITTEN EXERCISE

In each of the following columns of figures the widths of ten boards are given. The length of the boards is indicated at the top of each column. How many feet of lumber in each lot?

(1)	(2)	(3)	(4)
12-ft. bds.	14-ft. bds.	16-ft. bds.	10-ft. bds.
11 in.	12 in.	14 in.	13 in.
13 in.	13 in.	9 in.	11 in.
15 in.	14 in.	12 in.	12 in.
16 in.	12 in.	10 in.	9 in.
10 in.	9 in.	13 in.	12 in.
12 in.	15 in.	12 in.	13 in.

12-ft. bds.	14-ft. bds.	16-ft. bds.	10-ft. bds.
14 in.	11 in.	11 in.	11 in.
9 in.	14 in.	12 in.	13 in.
13 in.	12 in.	10 in.	14 in.
16 in.	10 in.	13 in.	15 in.

5. At $20 per thousand feet, what is the cost of 40 scantling, each 16 feet long, 4 inches wide, and 3 inches thick?

6. Find the cost at $24 per M., of 15 planks 18 ft. long, 3 in. thick, 12 in. wide at one end, and 16 in. wide at the other end?

7. How many feet of flooring an inch and a quarter thick will be required for a room 18 ft. 6 in. long and 15 ft. 3. in. wide, allowing one-fifth for matching?

8. Making no allowance for corners, how many feet of boards would be required to make a box 6 ft.×4 ft.×3 ft.?

9. A rectangular field, 18 ch. long and 10 ch. wide, is enclosed by a board fence; the posts are set 8¼ ft. apart, and the boards are 16½ feet long, and the fence is 5 boards high. The bottom board is 12 in. wide, the top board 6 in., and the other three each 9 in. wide. The posts cost $30 per C., and the boards $18.50 per M. Required the number of posts, the amount of lumber, and the cost of both.

10. What will be the cost of a board 18 feet long, 20 inches wide at one end and 15 inches at the other, and 1½ inches thick, at $25 per M.?

MEASUREMENT OF CAPACITIES

377. The method of finding the contents of any regular vessel in gallons, bushels, barrels, etc., is called **guaging.**

378. The *capacity of tanks, cisterns,* etc., is usually expressed in gallons or barrels. In every liquid gallon there are 231 cu. in.

379. *To find the exact number of gallons in any vessel, divide the number of cubic inches in the vessel by 231.*

1. To find the number of gallons in a *cylindrical vessel,* multiply the square of the diameter by the height, and this product by $5\frac{7}{8}$.

2. To find the *approximate number* of gallons in a cistern, multiply the number of cubic feet by $7\frac{1}{2}$ and from the product subtract $\frac{1}{480}$ of the product.

380. The *capacity of bins,* etc., is usually expressed in bushels. The standard bushel of the United States is a measure 8 inches deep, $18\frac{1}{2}$ inches in diameter, and contains 2150.42 cubic inches. Hence,

381. *To find the number of bushels in any bin, divide the number of cubic inches in the bin by 2150.42.*

1. The *stricken bushel* is used in measuring grain, but grain, seeds, etc., are usually sold by weight. For table of weights see sec. 280.

2. Potatoes, corn in the ear, large fruits, coal, and other bulky articles are sold by *heaped measure.* In every bushel, heaped measure, there are 2747.71 cubic inches.

3. To find the *approximate* number of stricken bushels in any number of cubic feet, multiply by .8.

4. To find the *approximate* number of heaped bushels in any number of cubic feet, multiply by .63.

5. The half-peck, or *dry gallon,* contains 268.8 cu. in.

Example 1. Find the contents in gallons of a tank 4 ft. square and 5 ft. deep.

$$\frac{1728\times4\times4\times5}{231}=598.4+\text{ gal.}$$

Solution. 1728, the number of cu. in. in a cubic foot multiplied by the number of cu. ft. in the tank and this divided by 231 equals 598.4+ gallons.

Example 2. How many bushels of wheat may be put in a bin 6 ft. by 4 ft., and 5 ft. deep?

$$\frac{1728\times6\times4\times5}{2150.42}=96.4+\text{ bu.}$$

Solution. 1728, the number of cu. inches in a cu. ft. multiplied by the number of cu. ft. in the bin and this divided by 2150.42 equals 96.4+ bu.

Coal. Coal is bought and sold in large quantities by the ton; in small quantities by the bushel, 28 heaped bushels, or about 43.5 cu. ft., being considered equal to a ton. Ordinary *anthracite coal* measures from 36 to 40 cu. ft. to the ton; *bituminous coal,* from 36 to 45 cu. ft. to the ton. A bushel of coal is estimated to weigh 70 pounds.

Hay. To find the amount of hay in a mow, allow 512 cu. ft. for a ton. Some allowance should be made for the condition of hay. The above is for dry, compact, but not pressed hay. 700 cu. ft. of new-mown hay will make a ton.

ORAL EXERCISE

1. About how many bushels in 4301 cu. in.? How many gallons in 693 cu. in.?

2. About how many bushels of grain will a bin contain that is 10 ft. long, 8 ft. wide, and 5 ft. deep?

3. About how many cu. ft. in a space that will hold 10 liquid gallons?

4. About how many gallons in a vat 5 ft. square and 4 ft. high?

WRITTEN EXERCISE

1. The water in a cistern 8 ft. square is 2 ft. deep, how many gallons does it contain?

2. A bin 8 ft. long, 5 ft. wide, and $4\frac{1}{2}$ ft. high, will hold how many bushels of grain?

3. A bin 8 ft. by 4 ft. by (?) contains 90 bushels of grain. Find the missing dimension.

4. A cylindrical cistern 5 ft. in diameter, 15 ft. $8\frac{1}{4}$ in. in cir-

cumference, and 5 ft. 6 in. high, will hold how many barrels of water?

5. A reservoir 24 ft. 8 in. long by 12 ft. 9 in. wide is full of water. How many cubic feet must be drawn off to sink the surface 1 foot?

6. A bought a bushel of chestnuts at 10¢ a quart, dry measure, and sold them at 10¢ a quart liquid measure. How much did he gain or lose?

7. How many tons of water will fill a tank 11 ft. 8 in., by 3 ft. 6 in. by 2 ft. 3 in., if the weight of a cubic foot of water is 1000 ounces?

8. A bin 18 ft. long and 7 ft. wide must be how deep to contain 21,654 pounds of coal, allowing 35 cu. ft. to the ton?

9. What must be the width of a wagon-bed 9 ft. long and 20 in. deep, to contain 40 bushels of apples?

Painting and Plastering

382. Painting and plastering is usually estimated by the square yard.

Allowance is sometimes made for openings in the walls, as doors and windows, but custom varies so much in different localities that no general rule can be given. It is generally considered as expensive to paint the window sills and sashes as to paint the surface area of the entire window.

WRITTEN EXERCISE

1. How many square yards in the ceiling, walls, and floor of a room 28 ft. by 24 ft., and 18 ft. high?

2. Find the cost of plastering the walls and ceiling of a room 14 ft. by 10 ft. and 9 ft. high at 35¢ a square yard, allowing 8 square yards for openings?

3. At 18¢ a square yard, what will it cost to plaster the walls and ceiling of a room 16 ft. long, 12 ft. wide, 14 ft. high, deducting for two doors, each 8 ft. by 4 ft., and for three windows, each 7 ft. by 3 ft.?

4. How much will it cost, at 30¢ per square yard to plaster the bottom and sides of a cubical cistern whose edges are 8 feet?

5. What will it cost to lath and plaster the four walls and

ceiling of a room 25 ft. long, 15 ft. wide, and 14 ft. high at 30¢ a square yard, allowance being made for ½ of three windows 3 ft. 6 in. by 6 ft.?

6. How many square feet of painting surface, excepting the bottom, on the outside of a car, 30 ft. long, 9 ft. wide, and 8 ft high?

Flooring and Roofing

383. **Flooring and roofing** are generally estimated by the *square* which consists of 100 square feet.

384. **Shingles** are estimated by the *thousand* or *bundle*, and commonly measure 16 in. by 4 in. Shingles are usually laid about 4½ in. to the weather, or 1000 for each square of 100 ft., allowing some for waste that is usual in shingling.

1. There are 250 shingles in a bundle.
2. The *size of slates* used in roofing varies in size from 6 in. by 12 in. to 16 in. by 24 in.
3. Flooring is often laid by the square, the price being from 75¢ to $1.50.

WRITTEN EXERCISE

1. At $5.75 per 100 square feet, how much will it cost to tin the roof of a house 45 feet long if the rafters are 23 ft. long?

2. Each side of the roof of a barn is 62 feet by 25 feet; how many shingles, each covering 16 square inches of the roof, will it require?

3. Find the cost of laying a pine floor which is 16 ft. by 14 ft., if the labor and other incidentals amount to $22, the price of the lumber is $45 per thousand, and an allowance of 32 square feet is made for waste.

4. Allowing 900 shingles to the square, how many will be required to shingle a roof 70 ft. by 28 ft. on each side at a cost of $3.75 per M.?

Paving

385. **Paving** is estimated by the *square foot* or the *square yard.*

WRITTEN EXERCISE

1. What will be the expense of paving a sidewalk 324 ft. long and 6 ft. wide, at $1.35 a square yard?

2. A rectangular plot 120 ft. long and 50 ft. wide is surrounded by a walk 4 ft. wide; find the cost of paving this walk at 30¢ a square yard.

3. What will it cost to build a sidewalk a quarter of a mile in length, 8 ft. wide, and 2 inches thick, supported by three continuous lines of scantling 4 inches square, timber costing $17.50 per M.?

4. My house is on a corner lot, 90 ft. on one street and 50 ft. on the other, surrounded by a walk 4 ft. wide. How many cubic feet of snow do I shovel in clearing my walk after a 15-inch snowstorm?

5. Of the cost of paving a street 36 ft. wide at $3.85 a square yard, the city pays one-third and the remainder is charged to the property on both sides of the street in proportion to the frontage on the street. Find the cost of the improvement to the owner of a lot of 50 feet front.

6. How many paving bricks 3½ by 8¾ inches will be required to pave a mile of roadway 35 feet in width?

Papering

386. Wall paper is usually 18 inches wide and is sold in single rolls 8 yards long, or in double rolls 16 yards long.

There is considerable waste in cutting and matching paper and for this reason it is usually more economical to buy double rolls.

It is not an easy task to determine in advance the exact number of rolls of paper required for the walls of any room, but paperhangers usually calculate the number of full strips necessary for the regular surface of the walls, and divide this number by the number of whole strips that can be cut from one roll. The waste pieces, if any, are utilized in covering the surface above the doors, above and below the windows, etc.

Example. How many double rolls of paper will be required to cover the walls of a room 24 ft. long, 18 ft. wide, and 9 ft. high above the baseboard, allowing for one door and two windows, each 3½ ft. wide?

$2(24+18)=84$ ft. perimeter.
$84-(3\frac{1}{2}\times 3)=73\frac{1}{2}$ ft., surface to be covered.
$48\div 9=5$ strips to each roll.
$73\frac{1}{2}\div(5\times 1\frac{1}{2})=10$ rolls.

Solution. The distance around the room is $2\times(24+18)=84$ ft. From this subtract $3\frac{1}{2}\times 3=10\frac{1}{2}$ ft., the width of the door and windows, or $84-10\frac{1}{2}=73\frac{1}{2}$ ft., the length of the regular surface of the wall. Since the height to be covered is 9 ft., we can cut 5 full strips from a double roll. Hence, the number of rolls required will equal $73\frac{1}{2}\div(5\times 1\frac{1}{2})=10$ rolls.

387. From the above illustrative example we may deduce the following:

RULE. *1. From the perimeter of the room deduct the width of the openings.*

2. Calculate the number of strips necessary for the regular surface of the walls and divide this number by the number of whole strips that can be cut from each roll of paper, the quotient is the required number of rolls.

1. Any whole rolls left over after papering may be returned to the dealer, but part of a roll will not usually be taken back.

2. Borders or friezes are sold by the yard, and vary in width from 3 in. upward.

WRITTEN EXERCISE

1. Find the number of double rolls of paper required to paper the walls of a room 16 ft. by 14 ft. and having a height of 8 ft. from the baseboard, allowing for one door 3 ft. by 7 ft., and for two windows, each 3 ft. by 6 ft.

2. What will be the entire cost at 35¢ a roll for paper and a gilt moulding around the top of the walls at 6¢ a foot, to decorate the walls of a room 30 ft. long, 22½ ft. wide, and 10 ft. 8 in. high, allowing for 4 windows 3 ft. 3 in. by 6 ft., and 2 doors 3 ft. by 7 ft.? (Single rolls.)

3. How many double rolls of paper, 18 in. wide will be required for the sides and ceiling of a room, 20 ft. long, 16 ft. wide, and 8 ft. 6 in. high, allowing for 3 windows 39 in. by 6 ft., and 2 doors 3 ft. by 7 ft.? The ceiling paper to run lengthwise of the room.

4. Find the cost of papering the walls and ceiling of a hall 36′×23′, and 10′ high, there being 2 doors 4′×8′ and 9 windows

3′×7′, with paper costing 50¢ a double roll. The ceiling paper to run crosswise of the room.

5. Estimate the cost of paper for a room 16′×15′ and 10 ft. high from the floor, with a baseboard 8″ wide, allowing for 4 windows and 2 doors, each 39″ wide including facing. An 11-inch border is desired worth 7¢ a foot. The paper loses 8″ in matching, and costs 60¢ a double roll.

Carpeting

388. Carpeting is usually sold by the linear yard, regardless of the width. It is cut up into *breadths*, and these breadths are *matched* and sewed together. As the same figure is repeated at intervals varying from 1 inch to 6 feet, according to the pattern, few carpets can be matched without loss.

1. Merchants will sell fractional lengths but not fractional widths of carpet.
2. Such floor covering as oilcloth and linoleum is frequently sold by the *square yard.*

389. The cheaper grades of carpet are usually 1 yd. wide, and the more expensive, such as Brussels, Wilton, etc., are $\frac{3}{4}$ of a yard wide.

1. Carpets may often be laid with less waste one way of the room than the other; hence, it is sometimes best to compute the cost with strips running both ways, and by comparison determine which involves the smaller waste.
2. When borders are put around carpets the corners must be counted twice because one half of each corner is wasted in making.
3. In computing the cost of carpets the number of strips must be found as they are sometimes necessarily made a little too wide and are *turned under.*

ORAL EXERCISE.

1. A room 14 ft. by 13 ft. is to be carpeted, how many breadths will be required if they run lengthwise? How many if they run crosswise?

2. If there is no loss in matching, how many yards, 1 yd. wide, should be purchased, the breadths running lengthwise? How many if the breadths run crosswise?

3. How many yards of carpet $\frac{3}{4}$ of a yard wide will be required for a parlor floor 18 ft. by 15 ft. if the strips run crosswise and there is no waste in matching?

4. If the strips in problem 3 are run lengthwise of the room, how many yards will be required? What part of the strip must be cut off or *turned under* in this case?

Example. How many yards of carpeting $\frac{3}{4}$ yard wide will carpet a room $18\frac{2}{3}$ ft. long and $16\frac{1}{3}$ ft. wide, if the strips run lengthwise, and there is a loss of 7 in. on each breadth for matching?

$16\frac{1}{3}$ ft. $= \frac{49}{3} \div 3 = 5\frac{4}{9}$ yd.
$5\frac{4}{9} \div \frac{3}{4} = 7\frac{7}{27}$, or 8 strips
$18\frac{2}{3}$ ft. $+ 7$ in. $= 19\frac{1}{4}$ ft., length of each strip.
$19\frac{1}{4} \times 8 = 154$ ft., or $51\frac{1}{3}$ yd.

Solution. Since the strips run lengthwise of the room, the width of the room divided by the width of the carpet equals the number of strips required.

$16\frac{1}{3} = 5\frac{4}{9}$ yd., and $5\frac{4}{9} \div \frac{3}{4} = 7\frac{7}{27}$, or 8 whole strips, since an even number of strips must be purchased.

To the length of the room, $18\frac{2}{3}$ ft. add 7 in. waste and we have $19\frac{1}{4}$ ft., the length of each strip. Then 8 times $19\frac{1}{4}$ ft. equals 154 ft., or $51\frac{1}{3}$ yd., the required result.

WRITTEN EXERCISE

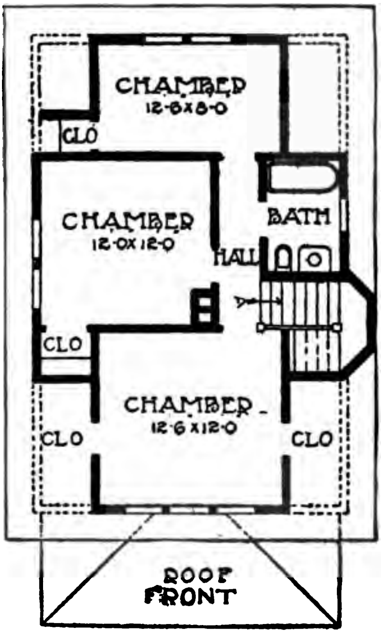
1. What will it cost, with carpet $\frac{3}{4}$ of a yard wide, at $1 per linear yard, to carpet a room 18 ft. square?

2. Find the cost of carpeting a room 12 ft. by 16 ft. with material $\frac{3}{4}$ of a yard wide, at 60¢ a yard. Carpet to run lengthwise.

3. A parlor is 18 ft. long, 15 ft. wide. Find the cost of carpeting the room at 95¢ per running yard for carpet, 5¢ per square yard for lining, and 10¢ per running yard for sewing and laying.

4. The volume of a room is 2295 cubic feet, the height 9 feet, and the width 15 feet. Which way of the room should carpet one yard wide be laid to prevent waste? What would be the loss by laying it the other way, carpet costing 90¢ a yard?

5. At $1.12½ a yard, what will it cost to carpet a flight of stairs of 18 steps, 12 inches tread and 6 inches rise, allowing 2 inches for each step for turning, and $\frac{3}{4}$ yard each for top and bottom?



6. The accompanying diagram represents the plan of the second floor of a two-story house. The dimensions of the *rooms* are given in the diagram. The *height* from floor to ceiling is 9 feet, with a baseboard 8 in. wide. The *openings* for windows, doors, etc., will average 3 ft. each, in width. If the two rear chamber rooms are carpeted in the most economical way, what will be the cost at $1.10 per yard, by using carpet ¾ yard wide, and there is no waste in matching the pattern?

7. In the accompanying diagram in the front chamber a rug 10′×10′ is placed centrally on the floor and filling is used to cover the remainder of the space, what will be the entire cost for the room, if the rug is worth $25.75 and the filling, ½ yard wide, costs 60¢ per linear yard?

8. A certain hall 60 ft. long is to be carpeted. It is found that by stretching the carpet lengthwise any one of the four pieces, width, respectively, ¾ yd., 1 yd., 1¼ yd., and 1½ yd., will exactly fit the hall without cutting anything from the width of the carpet. If the narrowest piece, worth $1.10 per yard, be chosen, what will it cost to carpet the hall?

Brick and Stone Work

390. **Masonry** is usually estimated by the *cubic yard* or the *perch.*

391. A *perch of stone* or *masonry* is 16½ ft. long, 1½ ft. wide, and 1 ft. high, and contains 24¾ cubic feet.

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1. A common brick is 8 in. by 4 in. by 2 in. Bricks are usually estimated

by the thousand, sometimes by the cubic foot, 22 bricks laid in mortar being taken as a cubic foot.

2. In estimating *labor*, bricklayers and masons measure the length of the wall on the outside. The corners are thus measured twice, but this is considered an allowance for the greater difficulty in building them. It is also generally considered that the work around openings is more difficult, so that allowance is frequently made here.

3. In estimating *material*, allowance is made for doors, windows, and corners.

4. In some localities the *perch* contains 16½ cu. ft.

392. To find the number of bricks in a wall.

Example. How many ordinary bricks are required for the walls of a building 45 ft. by 32 ft. and 20 ft. high, if the walls are 18 in. thick and 225 sq. ft. are allowed for openings?

Solution:

2(45+32) = 154 ft., perimeter of the wall.
154 − (1½×4) deduction for ends = 148 ft., correct length of wall.
148 ft.×20 = 2960 sq. ft. of exposed surface.
2960 sq. ft. − 225 sq. ft. for deductions = 2735 sq. ft., net exposed surface.
2735 sq. ft.×1½ = 4102½ cu. ft. of wall.
22, number of brick to 1 cu. ft.×4102½ = 90255, the number of bricks required for the wall.

393. Rule. *Multiply the number of bricks required for one cubic foot of wall by the number of cubic feet in the wall.*

394. To find the number of perches of stone or masonry in a wall.

Example. How many perches in the foundation walls of a building, mason's measure, 40′×32′, if the walls are 2 ft. wide and 8 ft. high?

Solution.

2(40+32) = 144 ft., perimeter of the wall.
144×2×8 = 2304 cu. ft. in the entire wall.
2304 ÷ 24¾ = $93\frac{1}{11}$ perches in the wall.

Note. Mason's measure exceeds the *true length* by 4 times the thickness of the wall. Hence, to find the *actual* length, subtract 4 times the width of the wall from the *girth* or gross outside measurement.

395. Rule. *Divide the contents of the wall in cubic feet by 24¾, the number of cubic feet in one perch.*

WRITTEN EXERCISE

1. How many perches of stone, actual measure, will be required to build the walls of a cellar 36 ft. long and 24 ft. wide, the walls to be 8 ft. high and 18 in. thick, deducting 96 cu. ft. for openings?

2. If 150,000 bricks are used for a house whose walls average 1½ ft. thick, 30 ft. high, and 216 ft. long, how many bricks will build one with walls 2 ft. thick, 24 ft. high, and 324 ft. long?

3. How many bricks 9 in. long, 4½ in. wide, and 4 in. thick, will be required for a wall 60 ft. long, 17 ft. high, and 4 ft. thick, allowing that the mortar increases the bulk of each brick $\frac{1}{16}$?

4. There is a solid pile of bricks which is 36 ft. long, 16 ft. 6 in. wide, and 14 ft. 6 in. high and contains 122,496 bricks of uniform size; each brick is 9 in. long and 4½ in. wide. Find its thickness.

5. How many cubic yards of masonry in the walls of a cellar that is 24 ft. long and 18 ft. wide, measured on the outside, the walls being 7 ft. 3 in. high and 18 in. thick?

6. A piece of land is surrounded by a stone wall 8 ft. high and 2 ft. thick; the land inside the wall is 100 ft. long and 50 ft. wide, how many cubic feet of stone does the wall contain?

7. How many perches of stone, actual measure, will be required to enclose a field 40 rods long and 32 rods wide, with a wall 5 ft. high and 2½ ft. thick, counting 25 cu. ft. to the perch?

EXAMINATION TEST

(From New York State Department of Public Instruction)

1. Write (a) in Arabic notation, five thousand seven hundred-thousandths; (b) in words, 800.003⅓; in words, MCDLXXXIX.

2. The factors of the dividend are 18½, 15, 24¾, and 7⅓, while those of the divisor are 17⅗, 8$\frac{2}{9}$, and 19⅘. Find the quotient by cancellation.

3. A coal bin 16½ ft. long and 8 ft. 9 in. wide must be how deep to contain 10 tons of coal, if one ton occupies 40 cu. ft. of space?

4. A rectangular field 12 ch. 19 l. by 8 ch. 42 l. is worth how much, if valued at $70 per acre?

5. Find the number from which, if $\frac{3}{7}$ of itself be taken, and to the remainder 40 be added, the sum will be $\frac{4}{5}$ of the number.

6. Find the square root of .000071, correct to four decimal places.

7. To what common unit of measure may (a) Troy weight and avoirdupois weight be reduced? (b) Dry measure and liquid measure? (c) Linear measure and surveyors' long measure?

8. A bicycle wheel 88 inches in circumference must make how many revolutions a minute to run 18 miles an hour?

9. The gable of a barn is 27 ft. wide and 9 ft. high. Find the value of the lumber in it at $18.50 per M.

RATIO AND PROPORTION

Ratio

396. **Ratio** is the relation which one quantity has to another of the same kind. The *value* of the ratio is the quotient of the first divided by the second. Thus, the ratio of 12 to 3 is $12 \div 3$, or 4; therefore, the terms bear the same relation as dividend and divisor, or as the terms of a fraction. Hence, the ratio of 3 to 4 may be written $3 \div 4$, or $\frac{3}{4}$.

The ratio is always an *abstract* number, since it is the relation of one number to another of the same kind. There is no ratio between quantities of different kinds; as, 6 hr. and 5 bu. But there can be a ratio between 5 ft. and 9 in., since the quantities are of the same kind. Both terms, however, must be reduced to the same unit. Thus, 5 ft. $=$ 60 in., and 60 in.: 9 in. $= \frac{60}{9} = 6\frac{2}{3}$.

397. The **sign** of ratio is the colon (:), or the sign of division with the line omitted. Thus the ratio of 5 to 10 is expressed 5 : 10.

398. The **terms of a ratio** are the numbers compared. The two terms form a *couplet*, the first of which is the *antecedent*, and the second, the *consequent*.

1. A *simple ratio* is the ratio between numbers; as, 6 : 8.
2. A *compound ratio* is the ratio of the products of the corresponding terms of two or more simple ratios; as, $\left\{\begin{matrix} 3:4 \\ 5:10 \end{matrix}\right\}$, and is equivalent to $3 \times 5 : 4 \times 10$, or $15 : 40$.
3. *Reverse*, *indirect*, or *reciprocal ratios* are formed by reversing the position or order of the terms.
4. The ratio of two *fractions* is obtained by reducing them to a common denominator, when they are to each other as their numerators.

399. Since the antecedent is a dividend, and the consequent a divisor, any change in either or both of the terms of a ratio will affect its value according to the laws of division or of fractions (160), which laws become the

General Principles of Ratio

1. A ratio is multiplied by multiplying the antecedent, or dividing the consequent.

2. A ratio is divided by dividing the antecedent, or multiplying the consequent.

3. A ratio is not changed by multiplying both terms by the same number, or dividing both terms by the same number.

WRITTEN EXERCISE

1. M has 2025 sheep, and N has 1350. Required the ratio of N's sheep to M's.

2. The antecedent is 60, the ratio 7; what is the consequent?

3. What is the ratio of $\frac{1}{2}$ foot to a yard? Of a pint to a peck? Of a square rod to 2 acres? Of 2 rods to $\frac{1}{2}$ a mile?

4. Change the couplet $\frac{25}{48} : \frac{5}{96}$ to the integral form in its lowest terms.

5. The consequent is 6.12\frac{1}{2}$, the ratio is $\frac{13}{28}$; what is the antecedent?

6. Divide 1043 into three parts that shall be to one another as $\frac{3}{8} : \frac{2}{3} : \frac{1}{5}$.

7. Eggs are sold at the rate of 35 for 54¢, and the gain at this rate is $\frac{1}{5}$ of the cost. Find at what rate per dozen they were purchased.

8. Reduce the couplet $\left\{\begin{matrix} 16\frac{1}{2} : 5 \\ 1\frac{1}{3} : 9\frac{3}{4} \\ 52 : 3.2 \end{matrix}\right\}$ to a simple couplet in it lowest terms.

Proportion

400. Proportion is an equality of ratios. The equality is indicated by the sign =, or by the double colon (::) between the ratios. Thus, 6 : 8 :: 3 : 4.

The method of solving problems by proportion is often called the ***Rule of***

Three, since problems which give three quantities so related that two of them sustain the same ratio to each other as the third to the quantity required, can readily be solved by proportion. Thus, if any three of the four terms of a proportion are known, the other one can be found.

401. The *first* and *fourth* terms of a proportion are called the **extremes,** and the *second* and *third,* the **means.**

402. The *test of proportion* is that the product of the extremes is equal to the product of the means. Hence,

The Principles of Proportion

1. A missing mean may be found by dividing the product of the extremes by the given mean.

2. A missing extreme may be found by dividing the product of the means by the given extreme.

ORAL EXERCISE

Find the missing term of the following proportions:

1. 16 : 4 :: 8 : ?
2. 3 : 9 :: ? : 24.
3. 5 : ? :: 8 : 4.
4. ? : 32 :: 2 : 16.
5. ? : 21 :: 7 : 49.
6. 3 : 9 :: ? : 57.
7. \$5 : \$17 :: 25 : ?
8. 13 : 39 :: \$5 : ?
9. \$4 : ? :: 21 : 84.

Simple Proportion

403. Simple proportion is an equality of two simple ratios. It is used to solve problems of which *three* terms are given, and the *fourth* is required.

Of the three given numbers, two must always be of the same kind; and the *third,* of the same kind as the *required term.*

404. A statement is the arrangement of the terms in the form of a proportion.

WRITTEN EXERCISE

Find the missing term:

1. 12 : ? :: 1 : 144.
2. 8 : 52 :: 20 : ?
3. $\frac{5}{16}$: $\frac{3}{8}$ :: ? : $\frac{2}{5}$.
4. 32 : 27 :: ? : 135.
5. 4½ yd. : ? :: \$9¾ : \$27.25.
6. 144 yd. : 175 yd. :: \$18 : ?
7. $\frac{3}{5}$ rd. : ? :: \$$\frac{7}{8}$: \$59.0625.
8. 7.5 : 18 :: ? oz. : 7$\frac{1}{16}$ oz.

9. ? : 4 :: $\frac{9}{32}$: $\frac{3}{8}$.
10. 16 : ? :: 24 : 18.
11. ? : 33 :: 7 : 49.
12. 48 : 75 :: 32 : ?
13. 23 : 92 :: $\frac{3}{8}$: ?
14. 7 da. : ? da. :: 98 bu. : 56 bu.
15. \$375 : \$144 :: 625 lb. : ?
16. ? : 9.01 :: 16.05 : 5.35.
17. $\left.\begin{array}{l}\text{6 men} : ?\\ \text{10 hr.} : \text{9 hr.}\end{array}\right\} :: \left\{\begin{array}{l}\\ \text{15 A.} : \text{45 A.}\end{array}\right.$

Compound Proportion

405. A **compound proportion** is a proportion in which one or both the ratios are compound.

The principles of compound proportion are the same as those which apply to simple proportion. Hence, a statement in compound proportion may be simplified in practically the same manner as a statement in simple ratio, or in simple proportion.

Cause and Effect

406. A **cause** is that which produces or effects a result. An **effect** is the result or that which is produced by an agent or cause.

407. The terms of every proportion have not only the relations of magnitude, but also the relations of *cause* and *effect*.

Thus, men working, work done for pay, grain growing, horses drawing, money at interest, etc., are all a *cause;* the result of the labor, what the grain produces, what the horses draw, etc., is the *effect*.

Causes are *like* quantities. Effects are *like* quantities. Effects usually include the object and all its qualities and measurements.

408. As there are two conditions involved in every problem in proportion, whether simple or compound, we may resolve the problem then into a comparison of *two causes* and *two effects*.

Thus, if 2 horses as a *cause* draw 4 tons as an *effect*, 6 horses as a *cause* will draw 12 tons as an *effect*.

It will be seen from the above that the problem naturally resolves itself into two conditions involving a *first cause*, *first effect*, *second cause*, and *second effect*.

409. Since like causes produce like effects, the ratio of two like causes equals the ratio of two like effects produced by these causes. Hence,

1st cause : 2d cause :: 1st effect : 2d effect.

410. The *Single Rule of Three*, or *Simple Proportion*, is included under compound proportion, and needs no illustration.

The method of solving problems in proportion by cause and effect will be readily understood from an example or two.

Example 1. In how many days, working 9 hours a day, will 20 men dig a trench 420 yards long, 5 yards wide, and 3 yards deep, if 240 men, working 11 hours a day, in 5 days dug a trench 220 yards long, 3 yards wide, and 2 yards deep?

$$\begin{array}{ccccccc} \textit{1st cause} & & \textit{2d cause} & & \textit{1st effect} & & \textit{2d effect} \\ \left.\begin{array}{r}240\\11\\5\end{array}\right\} & : & \left.\begin{array}{r}20\\9\\(?)\end{array}\right\} & :: & \left.\begin{array}{r}220\\3\\2\end{array}\right\} & : & \begin{array}{r}420\\5\\3\end{array} \end{array}$$

$$\frac{\overset{\overset{\overset{10}{\cancel{30}}}{\cancel{120}}}{\cancel{240}} \times \cancel{11} \times \cancel{5} \times \overset{\overset{7}{\cancel{21}}}{\cancel{420}} \times 5 \times \cancel{3}}{\underset{\cancel{4}}{\cancel{20}} \times \underset{\cancel{3}}{\cancel{9}} \times \underset{\cancel{20}}{\cancel{220}} \times \cancel{3} \times \cancel{2}} = 350 \text{ days.}$$

Explanation. An examination of the question at once suggests that the work done in either case constitutes the *effect;* and the corresponding men and time its *cause.* Thus, one of the conditions gives the first cause and the corresponding first effect. The other condition gives the second cause and the second effect; which are completed by letting (?) take the place of the required number of days. We now see that the *required* term is an element of the second cause and a *mean.* Hence, divide the product of the extremes by the product of the given means, and the quotient is the required factor or term, 350 days (402, 1).

Example 2. If 2 yards of linen 5 quarters wide require 1 pound of thread, then how many pounds of thread will 50 yards of linen 3 quarters wide require?

$$\begin{array}{ccccccc} \textit{1st cause} & & \textit{2d cause} & & \textit{1st effect} & & \textit{2d effect} \\ \left.\begin{array}{l}2 \text{ yards}\\5 \text{ qr.}\end{array}\right\} & : & \left.\begin{array}{l}50 \text{ yards}\\3 \text{ qr.}\end{array}\right\} & :: & 1 \text{ pound} & : & (?) \text{ pounds} \end{array}$$

$$\frac{\overset{\overset{5}{\cancel{10}}}{\cancel{50}} \times 3 \times 1}{\cancel{2} \times \cancel{5}} = 15 \text{ pounds.}$$

NOTE. Generally, upon reading the conditions of a problem, the most natural sequence of cause and effect will at once present itself.

WRITTEN EXERCISE

1. Arrange the numbers, \$51, 87, \$255, and 435 in the form of a proportion. Prove that the proportion is true.

2. If 40 men can do a piece of work in 60 days, how long will 15 men take to do half as much?

3. If 44 cannons, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 400 barrels last 66 cannons, firing 40 rounds an hour for 5 hours a day?

4. A ditch 80 yards long, 10 ft. deep, and 9 ft. wide was dug by 20 men in 12½ days of 10 hours each; and a ditch 76 yards long and 12 ft. wide was dug by 30 men in 7½ days of 9½ hours each. How deep was the latter ditch?

5. If 32 men have enough provisions to last them 40 days, how long will twice the quantity last 75 men?

6. If 2 pounds of sugar cost 25¢, and 8 pounds of sugar are worth 5 pounds of coffee, what will 100 pounds of coffee cost?

7. If 3 lb. of yarn will make 10 yd. of cloth 1¼ yd. wide, how many pounds will be required to make a piece 100 yd. long and 1½ yd. wide?

8. If a Troy ounce of silver is worth 75¢, what is the value of an avoirdupois ounce?

9. Wishing to ascertain the height of a certain tree, I set up a pole 10 feet long, measured the shadows cast at the same time by both tree and pole, and found them to be respectively 36 ft. 10 in. and 5 ft. 8 in. Required the height of the tree.

10. If a 2-cent loaf weighs 6 ounces when wheat is 80¢ per bushel, what should a 10-cent loaf weigh when wheat is $1.20 per bushel?

11. If the gas for 5 burners 5 hours each evening for 10 days cost $1.02, what will be the cost of 75 burners which are lighted 4 hours every evening for 15 days, if 5 of the first set of burners consume as much gas as 6 of the second set?

12. Three men did a piece of work for which they were paid $54. If one worked 6 hr. per day for 11 days, another 9 hr. per day for 10 days, and the third 12 hr. per day for 5 days, how much should each receive?

13. If 234 long tons of coal last a manufacturer 18 months, how long would 234 short tons last him?

14. A certain number of men, twice as many women, and three times as many boys, earn $123.80 in 5 days; each man

earned $1.20, each woman $66\frac{1}{3}$¢, and each boy $53\frac{1}{3}$¢ per day. How many were there of each?

15. A house and lot are worth $12,400, and the value of the lot is to the value of the house as 5 to 11. Find the value of the lot.

16. How many hours a day must 42 boys work to do in 45 days what 27 men can do in 28 days of 10 hours each, if the work of a boy is half that of a man?

17. I paid $675 for a building lot $3\frac{3}{8}$ rd. front by 11 rd. deep. Afterward I bought another lot $3\frac{1}{2}$ rd. front by $14\frac{1}{7}$ rd. deep, at the same rate. How much did I pay for the second lot?

WRITTEN REVIEW

1. Explain to class why we have leap years in our calendar.

2. After diminishing a certain number by $\frac{2}{3}$ of itself, and the remainder by $\frac{2}{15}$ of itself, the remainder is 65. Find the number.

3. A cubic foot of water weighs 1000 ounces avoirdupois, and the specific gravity of gold is 19.34 (*i.e.*, gold weighs 19.34 times as much as water). Find the weight of a piece of gold 8″×4″×2″.

4. 3 A. 80 sq. rd. are what fractional part of 8.4 A.?

5. Reduce 2 yr. 8 mo. 21 da. to years and decimal of a year; .09625 bbl. to integers of lower denominations.

6. When the scale is $\frac{1}{4}$ inch to a mile, find the value of the lands of a township represented by a rectangle $\frac{3}{4}$ inch by $1\frac{1}{2}$ inch, average price of the land being $32 per acre.

7. Abraham Lincoln died at the age of 56 years, 2 months, 3 days, after serving as president 4 years, 1 month, 11 days. Give the date of his birth, the date of his inauguration being March 4, 1861.

8. A cubic foot of ice weighs $56\frac{1}{2}$ lb. Find the weight of ice on a pond covering $1\frac{1}{2}$ acres. The ice is $1\frac{1}{4}$ feet thick.

9. A certain bin is 6 feet long and 5 feet wide, how deep must it be to hold 180 bushels? (Use 2150.4.)

10. An orchard is $24\frac{2}{3}$ rd. long, and $15\frac{1}{4}$ rd. wide. At $1\frac{3}{4}$¢

per cubic foot what will it cost to dig a ditch around it 3 ft. 9 in. wide and 4 ft. deep?

11. If 1000 laths cover 74 yd. of surface, and 11 pounds of lath nails nail them on, what will it cost to lath the walls and ceiling of a schoolhouse 42 ft. long, 30 ft. wide, and 14 ft. high, at $2 a thousand for laths, 6¢ a pound for nails, and $3\frac{1}{3}$¢ a yard for labor?

12. Divide $541 among A, B, and C, giving A twice as much as B, and C $25 more than three times as much as B.

13. A rectangular field is 40 rd. by 28 rd. Find one side of a square field containing the same area.

14. At $1.20 per yard what would be the cost of carpeting a room 26 ft. 4 in. by 13 ft. 6 in. with 27-inch carpet, the strips being laid lengthwise of the room, and 8 inches per strip being lost in matching?

15. A lot 60 ft. wide and 120 ft. long is to be enclosed on the two sides and the back by a tight board fence 6 feet high. The posts are to be placed 6 ft. apart and to cost 15¢ each. There are to be two string pieces of scantling, 2 inches thick and 4 inches wide, from post to post on which to nail the boards. Lumber is $25 per M. feet. Find the entire cost of building the fence.

16. Simplify $\frac{(\frac{1}{4}\div\frac{1}{5}\text{ of }\frac{1}{7})-(\frac{1}{4}\div\frac{1}{5}\times\frac{1}{7})}{\frac{8}{7}\text{ of }.0003\div .25}$.

17. The number of children of school age in a certain state is in round numbers 824,000. If marching in ranks of four, $3\frac{1}{2}$ feet apart, how long a column would they make?

18. A's money is to B's as 4 : 3; but A's money is in joint business only $\frac{5}{6}$ as long as B's. How should a profit of $3000 be divided?

19. Add $20\frac{}{5}$ sq. rd., $16\frac{1}{2}$ sq. yd., and $11\frac{}{16}$ sq. ft.

20. Making no allowance for corners, how many feet of boards would be required to make a box 8 ft. × 4 ft. × 5 ft.?

21. A farmer sold 250 animals, sheep and pigs, for $1720. He received $8 for a sheep and $6 for a pig. How many were there of each?

22. A man has a garden 12 rods square surrounded by a

fence. How many trees may be planted in it, if no tree is nearer than $\frac{1}{2}$ a rod to the fence, and the trees shall be one rod apart?

23. Find the value of a diamond weighing $\frac{11}{16}$ of a carat, at $35 per grain.

24. The breadth of a room is twice its height, and the length is three times the height. It cost $86.40 to paint the walls at 6¢ a square foot. Find the dimensions of the room. Find the cost of oilcloth for the floor, at $1.75 per square yard.

25. Bought by avoirdupois weight, 20 pounds of opium at 40¢ an ounce, and sold the same by Troy weight at 50¢ an ounce; did I gain or lose, and how much?

26. If $377 be divided among A, B, and C, so that A's share is $7 more than B's share, and B's share $5 more than C's share, find A's share.

THE EQUATION

411. An **equation** is a statement of equality of two numbers or expressions. Thus, $7=5+2$, $8+6\times 2=20$, and $x=6-2$ are equations.

The two expressions are called the ***members*** of the equation. The expression at the left of the sign of equality is called the ***first member***, the one at the right is called the ***second member***.

412. The equation can be used to decided advantage in the solution of many arithmetical problems as it is one of the most powerful instruments used in mathematics.

413. A letter to which a particular value must be given in order that the statement contained in an equation may be true is called an *unknown quantity*. To **solve** an equation is to find the value of the unknown quantity.

Only *simple equations*, or equations which contain only the ***first power*** of the unknown quantity, will be treated here.

414. **Some axioms and their use.** The following principles, usually called axioms, are useful in solving equations:

1. If equals be added to or subtracted from equals, the results will be equal.

2. If equals be multiplied or divided by equals, the results will be equal.

3. A term may be transposed from one member of an equation to the other by merely reversing its sign. Thus, $x+6=24$, may be changed to $x=24-6$, or $x=18$.

Example 1. Find a number such that when 15 is added to its double the sum shall be 31.

Let x = the number.
Then $2x$ = its double,
and $2x+15$ = double the number increased by 15.
But 31 = double the number increased by 15.

$$\therefore 2x+15=31;$$
$$2x=31-15;$$
$$2x=16;$$
$$x=8.$$

Example 2. The sum of two numbers is 20 and their difference is 4. What are the numbers?

Let x = the larger number.
Then $x-4$ = the smaller number.

$$\therefore x+x-4=20;$$
$$2x=24;$$
$x=12$, the larger number.

415. When an expression within a parenthesis is preceded by a *minus sign*, the parenthesis may be removed if the sign of every term within the parenthesis be changed. Thus:

(1) $a+(b-c)=a+b-c$, but (2) $a-(b-c)=a-b+c$.

Example 3. What two numbers are those whose difference is 14, and whose sum is 48?

Let x = the larger number.
Then $48-x$ = the smaller number.
and $x-(48-x)$ = the difference of the numbers.
But 14 = the difference of the numbers.

$$x-(48-x)=14;$$
$$x-48+x=14;$$
$$2x=62;$$
$$x=31.$$

Therefore, the two numbers are 31 and 17.

NOTE. $\therefore$ is read *therefore.*

WRITTEN EXERCISE

1. To four times a certain number I add 16, and obtain as a result 188. What is the number?

2. A man having $100 spent a part of it; he afterwards received five times as much as he spent, and then his money was double what it was at first. How much did he spend?

3. A farmer had two flocks of sheep, each containing the same number. He sold 21 sheep from one flock and 70 from the other, and then found that he had left in one flock twice as many as in the other. How many had he in each?

4. Divide 100 into two such parts that a fourth of one part diminished by a third of the other part may be equal to 11.

5. Find the area of a square field whose diagonal is 50 rods.

6. A father wishes to divide $28,000 among his two sons and a daughter so that the elder son shall receive twice as much as the younger, and the younger son twice as much as the daughter. Find the share of each.

PERCENTAGE AND ITS APPLICATIONS

PERCENTAGE

416. **Percentage** is the process of **computation in which the** basis of comparison is a *hundred.*

417. **Per cent** is a contraction of the Latin words "per centum," and means *by the hundred.*

418. The commercial **sign** of **per cent** is %, and may be read *hundredths* or *per cent.*

Percentage is only an application of the decimal fraction and not a separate department of arithmetic. There are different ways of denoting the same fractional part. Thus,

$\frac{1}{4} = \frac{25}{100} = 0.25 = 25\% = 25$ per cent.

$\frac{1}{6} = \frac{16\frac{2}{3}}{100} = 0.16\frac{2}{3} = 16\frac{2}{3}\% = 16\frac{2}{3}$ per cent.

ORAL EXERCISE

1. Express as common fractions: 5%, 10%, 25%, 8%, 45%, $\frac{1}{2}\%$, $\frac{1}{5}\%$, 125%, $87\frac{1}{2}\%$, 250%, $133\frac{1}{3}\%$, $33\frac{1}{3}\%$, 325%.

2. Express as per cents: .25, .16, .5, .8, $.28\frac{4}{7}$, $.02\frac{1}{2}$, $.62\frac{1}{2}$, $.12\frac{1}{2}$, $16\frac{2}{3}$, .0025, $.00\frac{1}{4}$, .005, .06, $\frac{2}{3}$, $\frac{5}{8}$, 46.25.

3. Express as decimal fractions: 25%, 35%, 28%, $\frac{1}{2}\%$, $\frac{1}{4}\%$, 125%, 250%, $\frac{1}{25}\%$, $\frac{1}{50}\%$, 375%, $.00\frac{1}{4}$, $.00\frac{1}{40}$.

Important Per Cents and their Fractional Equivalents

$1\frac{2}{3}\% = \frac{1}{60}$	$8\% = \frac{1}{12}$	$16\frac{2}{3}\% = \frac{1}{6}$	$42\frac{6}{7}\% = \frac{3}{7}$
$2\% = \frac{1}{50}$	$9\frac{1}{11}\% = \frac{1}{11}$	$20\% = \frac{1}{5}$	$50\% = \frac{1}{2}$
$2\frac{1}{2}\% = \frac{1}{40}$	$10\% = \frac{1}{10}$	$25\% = \frac{1}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$
$3\frac{1}{3}\% = \frac{1}{30}$	$11\frac{1}{9}\% = \frac{1}{9}$	$28\frac{4}{7}\% = \frac{2}{7}$	$66\frac{2}{3}\% = \frac{2}{3}$
$6\frac{1}{4}\% = \frac{1}{16}$	$12\frac{1}{2}\% = \frac{1}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$	$75\% = \frac{3}{4}$
$6\frac{2}{3}\% = \frac{1}{15}$	$14\frac{2}{7}\% = \frac{1}{7}$	$37\frac{1}{2}\% = \frac{3}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$

Since every rate per cent, expressed decimally, is a corresponding number of hundredths, it is evident that neither more nor less than ***two decimal places*** can occur; hence any fractional part of one per cent must be expressed as a common fraction. 16½% written as a simple decimal, .165, expresses ***thousandths***, which is contrary to the meaning of the term ***per cent***.

419. Problems in percentage involve the following elements: *base*, *rate*, *percentage*, *amount* or *difference*.

420. The **base** is the number on which the percentage is computed; the **rate**, the number of hundredths of the base to be taken; the **percentage**, the result obtained by taking a certain per cent of the base.

In the statement "6% of $200 is $12," $200 is the ***base***, 6% the ***rate***, and $12, the ***percentage***.

421. The **amount** is the *sum* of the base and percentage; the **difference**, the base *less* the percentage.

422. The same relationship exists between base, rate, and percentage that exists between multiplicand, multiplier, and product, and if any two of these terms are given the third may be found by the application of the principles governing simple multiplication and division.

423. **General principles.** The *base* may be either concrete or abstract. The *rate* is always abstract; and the *percentage*, *amount*, and *difference* are always of the same name as the base.

424. The subject of percentage is conveniently treated under four distinct cases:

CASE I

425. **To find the percentage when the base and rate are given.**

Example. What is 12½% of $750.40?

$750.40×.12½=$93.80.
or
⅛ of $750.40=$93.80.

Solution. Since 12½% of a number is .12½ of it, 12½% of $750.40 is .12½ of $750.40 or $93.80.

426. **Rule.** *Multiply the base by the rate, expressed decimally, or take such a part of the base as the rate is of 100%.*

Formula. Base×Rate=Percentage.

ORAL EXERCISE

1. State a short method of finding 12½% of a number; 50%; 25%; 16⅔%; 33⅓%; 20%; 6⅔%; 5%; 11⅑%; 66⅔%; 8⅓%; 14$_{7}$%.

2. Find 40% of 250 tons; 25% of 300 miles; 33⅓% of 375 gal.; 80% of 120 sheep; 45% of 200 bricks; 90% of 500 days; 16⅔% of 600 yd.

3. Express as a rate per cent: ¼ of a number; ⅓ of a number; $\frac{1}{16}$ of a number; $\frac{1}{12}$ of a number; ⅝ of a number; ⅔ of a number; ⅜ of a number; ⅐ of a number; $\frac{1}{11}$ of a number.

4. How much is 10 times 80? $\frac{1}{10}$ of 80? .10 of 80? 10% of 80? 8×64? $\frac{8}{100}$ or $\frac{2}{25}$ of 64? .08 of 64? 8% of 64?

5. What is 37½% of 64? 112½% of 32? 325% of 160?

6. A man had 240 sheep and lost 12½% of them; how many had he left?

Short Methods

1. What is 12% of $33⅓?

Solution. Since 12 times 33⅓ will give the same product as 33⅓ times 12, 12% of $33⅓ will give the same result as 33⅓% of $12. 33⅓% is ⅓ of 100%; therefore, ⅓ of $12 is $4, the required result.

2. What is 44% of 250 tons?

Solution. 44% of 250 T. = 25% of 440 T., or ¼ of 440 T. = 110 T., the required result.

3. What is 32% of 125 bu.? 16⅔% of 660 gal.? 75% of 333⅓ da.? 125% of 720? 88% of 1250 horses? 48% of $3750?

4. By inspection find: 32% of 2500; 360% of 7500; 55% of 11⅑.

5. By inspection find: 3⅓% of 3600; 1⅔% of 660; 6¼% of 640.

6. From a hhd. of molasses 33⅓% was drawn. How many gallons remained?

7. A grocer bought 360 dozen eggs, and found 16⅔% of them bad or broken. How many were salable?

8. A capitalist is worth $125,000, of which 64% is invested in stocks. What has he invested in stocks?

WRITTEN EXERCISE

1. A grocer bought 150 boxes of oranges at $1.50 a box. He found 16% spoiled, but sold the remainder at $3 a box. Did he gain or lose, and how much?

2. A man receives a salary of $1800 a year. He pays 18% of it for board, 8⅓% for clothing, and 20% for incidentals. How much does he save?

3. For a certain year a clerk's salary was $1242, and his expenses were 87½% of that sum, how much did he save?

4. I paid ⅔% commission to an agent for selling a farm for $12,500. How much money did I receive?

5. A merchant bought a stock of goods costing him $800, one-half of which he sold for $650, and the remaining half he sold at a loss of 25% on cost. Did he make or lose, and how much?

6. A cellar measures 42 ft. by 30 ft. by 6 ft. in depth. Find the cost of digging at 8¢ a cubic yard for the first foot in depth, the price increasing 25% a cubic yard for the next two feet in depth, and then 20% more for the remaining feet.

7. A captain owned ⅝ of a vessel, and sold 37½% of his share for $10,500. What was his remaining share worth?

8. March 8, a merchant buys a bill of goods amounting to $850.25 on the following terms: 3 months, or less 5% if paid in 10 days. How much would settle the bill March 17?

CASE II

427. To find the rate when the base and percentage are given.

Example. Out of 420 words a student spelled 399 correctly. What per cent of the words were spelled correctly?

$399 \div 420 = .95.$

or

$\frac{399}{420} = \frac{19}{20}$

$\frac{19}{20}$ of $100\% = 95\%.$

Solution. Since the percentage is the product of the base and rate, the rate is the quotient found by dividing the percentage by the base. Therefore, $399 \div 420 = .95$, the rate. Or,

Since 399, the percentage, is $\frac{19}{20}$ of the base, the rate is $\frac{19}{20}$ of 100%, or 95%.

428. Rule. *Divide the percentage by the base, or take such a part of 100% as the pecrentage is of the base.*

Formula. Percentage ÷ Base = Rate.

ORAL EXERCISE

1. 24 is how many times 12? 12 is what fractional part of 24? 12 is what per cent of 24?

2. What per cent of 50 is 5? 50 is what per cent of 5? 9% of 30 is what per cent of 54?

3. What per cent of 34 bu. are 68 bu.? 2.4 are 3.6? 48 hr. are 36 hr.? $240 are $60? 2 mi. are 80 rd.?

4. What per cent of $\frac{1}{2}$ is $\frac{2}{3}$? 12 is $\frac{2}{3}$ of what number? What per cent do I gain by selling $\frac{1}{2}$ of a bu. of grain for what the whole cost?

5. What per cent of 5 is 25% of 10?

6. Cost is 5¢, selling price 7¢. What is the per cent of gain?

WRITTEN EXERCISE

1. $33\frac{1}{3}$ is $\frac{1}{2}$ per cent of what number?

2. A harness that cost $28 was sold at a loss of $4. What per cent was lost?

3. A farmer sold 75 sheep that cost him $2.50 at $1.40 a head. How much and what per cent did he lose on each sheep?

4. A man sold $\frac{3}{8}$ of his interest in a real estate syndicate for $\frac{5}{12}$ of what his interest cost him. Did he gain or lose, and what per cent?

5. If $\frac{5}{6}$ of an article is sold for what $\frac{5}{8}$ of it cost, what is the loss or gain per cent?

6. A grocer sold from a barrel of sugar containing 220 lb. $\frac{1}{4}$ of it at one time, and $\frac{1}{3}$ of the remainder at another time. What per cent of the whole remained?

7. A merchant sells $\frac{2}{3}$ of an article for what it all cost, but is able to collect only 75% of the debt. Does he make or lose, and what per cent?

8. What per cent more is $\frac{4}{5}$ than $\frac{7}{16}$?

9. What per cent less is $\frac{3}{8}$ than $\frac{7}{8}$?

10. What per cent of 110 sq. rd. is 18 sq. rd. 10 sq. yd. 108 sq. in.?

11. If water expands 10% when it turns to ice, what per cent does ice contract when it changes to water?

12. The assets of an insolvent firm, after paying all expenses, are $9872.40, and its liabilities are $22,980.75. How much should A receive on his claim of $1250?

13. Express decimally $\frac{7}{160}$; $\frac{5}{8}$%; $9\frac{1}{16}$%; five sixty-fourths.

14. A built a house costing $5000 upon a lot which cost $800. The house being burned, the insurance company paid him 60% of the cost of the house. He then sold the land for $1200; did he gain or lose by the transaction, and what per cent?

CASE III

429. To find the base when the percentage and rate are given.

Example. 144 is $33\frac{1}{3}$% of what number?

$144 \div .33\frac{1}{3} = 432$.
or
$33\frac{1}{3}$%, or $\frac{1}{3}$ of the number equals 144.
$\frac{3}{3} = 3$ times 144, or 432.

Solution. Since the percentage is the product of the base by the rate, the base is equal to $144 \div .33\frac{1}{3} = 432$. Or,

$33\frac{1}{3}$% of a number is $\frac{1}{3}$ of it. If 144 is $\frac{1}{3}$ of some number, that number must be 3 times 144, or 432.

430. Rule. *Divide the percentage by the rate expressed decimally.*

Formula. Percentage ÷ Rate = Base.

ORAL EXERCISE

1. 18 is 3 times what number?
2. 18 is $\frac{3}{4}$ of what number?
3. 18 is 6% of what number?
4. 12 is $\frac{1}{12}$ of what number?
5. 12 is $8\frac{1}{3}$% of what number?
6. 48 is 8% of what number?
7. 15 bu. is 30% of how many bu.?
8. $40 is 5% of how many $?
9. 1.2 is 6% of what number?
10. 32 is 8% of what number?
11. $18 \div \frac{3}{4} =$ what?
12. $32 $\div \frac{1}{5} =$ how much?
13. $32 ÷ 20% = how much?
14. 15 is $6\frac{2}{3}$% of what?
15. 6% of 12 is 3% of what?
16. $\frac{1}{4}$ is $\frac{1}{2}$ of what?
17. 25 is 50% of what?
18. 16 doz. is 12% of what?
19. 30 lb. is 20% of what?
20. 125 is $62\frac{1}{2}$% of what?

WRITTEN EXERCISE

1. 25% of 800 bu. is 2½% of how many bushels?

2. A clerk spends $1250 and has 25% of his money left. What was the amount of his money?

3. A house and lot was sold for $7030, at a loss of 16⅔% on its cost. What did it cost?

4. 1¾% of a certain number is 16⅔% of 84. Find the number.

5. If milk yields 18% of cream, and cream yields 25% of butter, and a quart of milk weighs 1 lb., how many quarts of milk will yield 470 lb. of butter?

6. A merchant sells goods for 2½¢ per yard more than they cost, and makes a profit of 8%. What was the cost per yard?

7. A certain sum of money is to be divided among five persons: A is to have 25%, B 20%, C $\frac{1}{10}$, D 5%, and E the remainder, which is $20. What is the whole sum to be divided?

8. The surplus fund of a bank is 250% of its circulation; its circulation is 25% of its capital stock, and its capital stock is 222$\frac{2}{9}$% of the cash on hand. If the surplus fund is $250,000, what is the cash on hand?

9. A man spent 60% of his salary in board for himself and family, and 10% of it in clothing for himself. The clothing of his wife and child cost 87½% as much as his own. At the end of the year $187 remained. What was his salary?

10. In selling a barrel of sugar at retail there was a waste of 3½ lb., which was 1⅔% of its total weight. How many pounds were sold?

CASE IV

431. To find the base when the amount or difference and the rate are given.

Example 1. What number increased by 12½% of itself is equal to 720?

$1 + .12\frac{1}{2} = 1.12\frac{1}{2}$.
$720 \div 1.12\frac{1}{2} = 640$.
or
$\frac{9}{8}$ of the number $= 720$.
$\therefore$ the number $= 720$
divided by $\frac{9}{8} = 640$.

Solution. Since the number is increased $12\frac{1}{2}\%$, or by $.12\frac{1}{2}$ of itself, 720 is $112\frac{1}{2}\%$, or $1.12\frac{1}{2}$ of the number. Hence, $720 \div 1.12\frac{1}{2} = 640$, the base or required number. Or,

$\frac{9}{8}$ of the number $= 720$, $\frac{8}{8}$ or the number is equal to $720 \div \frac{9}{8} = 640$, the required number.

Example 2. What number diminished by $12\frac{1}{2}\%$ of itself equals 861?

$1 - .12\frac{1}{2} = .87\frac{1}{2}$
$861 \div .87\frac{1}{2} = 984$.
or
$\frac{7}{8}$ of the number $= 861$.
$\therefore$ the number $= 861$
divided by $\frac{7}{8} = 984$.

Solution. Since the number is diminished by $12\frac{1}{2}\%$, or by $.12\frac{1}{2}$ of itself, 861 is $87\frac{1}{2}\%$, or $.87\frac{1}{2}$ of the number. Hence $861 \div .87\frac{1}{2} = 984$, the required number. Or,

$\frac{7}{8}$ of the number $= 861$, $\frac{8}{8}$ or the number is equal to $861 \div \frac{7}{8} = 984$, the required number.

432. Rule. *Divide the amount by 100% + the rate, or divide the difference by 100% − the rate.*

$$\textit{Formula:}\begin{cases}\text{Amount} \div (100\% + \text{Rate}) = \text{Base.}\\ \text{Difference} \div (100\% - \text{Rate}) = \text{Base.}\end{cases}$$

ORAL EXERCISE

What number increased by:

1. 25% of itself is 125?
2. 10% of itself is 880?
3. $16\frac{2}{3}\%$ of itself is 77?
4. 40% of itself is 140?
5. $6\frac{2}{3}\%$ of itself is 160?
6. $66\frac{2}{3}\%$ of itself is 100?
7. $14\frac{2}{7}\%$ of itself is 160?
8. 200% of itself is 1800?
9. 500% of itself is 1800?
10. 190% of itself is 5800?

What number diminished by:

1. 10% of itself is 90?
2. 25% of itself is 750?
3. $33\frac{1}{3}\%$ of itself is 400?
4. 50% of itself is 1800?
5. $8\frac{1}{3}\%$ of itself is 88?
6. $\frac{2}{3}$ of itself is 3000?
7. 1% of itself is 990?
8. 75% of itself is 225?
9. $37\frac{1}{2}\%$ of itself is 500?
10. 90% of itself is 1440?

WRITTEN EXERCISE

1. A speculator bought 48 bales of cotton, and afterward sold the whole for $2256, losing 6%, what was the cost of each bale?

2. How much grain must a farmer take to mill that he may have 11.2 bushels after the miller has taken 8% for grinding?

3. If 20% is lost in selling a house for $1200, for what should it have been sold to gain 20%?

4. A bought 63 sheep and sold 4/9 of them at a profit of 15%, 1/7 at a profit of 50%, and the remainder at a loss of 25%. What did he pay for the sheep if his gain was $19.25 on the whole?

5. When pork is selling at $4.50 per hundredweight, I lose 10%. What will be my gain per cent if I sell at $6 per hundredweight?

6. A's income is 16% of his capital; he is taxed 2½% of his income, and pays $26.04. What is his capital?

7. A man sold a house at 20% profit, and with the proceeds purchased another which he sold at 30% profit, realizing a total gain of $3136 on both. What did he pay for each?

8. Bought 120 tons of coal at $4.75 long ton, and sold at $5.62½ per ton of 2000 lbs. The delivery cost 10% of receipts. What is the profit?

9. In selling an article the manufacturer makes a profit of 25%, the wholesale dealer a profit of 12%, and the retail dealer 15%. If the customer paid $6.44, what was the cost to manufacture?

10. What number increased by 16⅔% of itself equals 500 diminished by 16% of itself?

11. A man loaned two equal sums of money, one at 4% and the other at 4½%, receiving $8.50 interest per month. How much did he loan?

12. A, B, and C have together $250. B has 66⅔% as much as A, and C has 25% as much as A and B together. How much has each?

ORAL REVIEW

1. A sold B a buggy, gaining 25% of the cost. B sold it for $75, losing 25%. What did it cost A?

2. By selling oranges at 5¢ I gained 25% of the cost. What per cent would I have gained by selling 2 for 11¢?

3. James had twice as much money as Henry, but after he gave Henry $10 their money was equal. How much had each at first?

4. Sold a piano at a gain of 20%. Had it cost $250 more, I should have lost 20%. What did I receive for it?

5. ¾ of ½ of 16 is 10% of what number?

6. A company reduced the pay of their men 20%; but owing to dissatisfaction, increased their pay 20%. The men then received 8¢ per day less than at first. What did they then receive?

7. A man gains 33⅓% by selling 3 pecks of potatoes for 50¢. What did he pay per bushel?

8. A's gain lacks 4% of being twice B's gain, and the difference in their profits is $24. Find the gain of each.

9. A, B, and C together earned $100. A's share was 37½% of B's, and C's 12½% more than B's. How much did each earn?

10. Two men travel toward each other, one at the rate of 3½ miles per hour, the other 40% faster. If the distance was 60 miles, in what time did they meet, and how much farther did one travel than the other?

WRITTEN REVIEW

1. Reduce .0018¾ to the form of per cent; to a common fraction in its lowest terms; to a fraction whose denominator is 8000.

2. To pay $9\frac{1}{11}$¢ a pound for sugar is how much per cent better than to pay one dollar for ten pounds?

3. A man bought a store and contents for $4720, and sold the same for 12½% less than he gave, losing 15% of the selling price in bad debts. Find his entire loss.

4. A mixture of 50 gallons of liquid is 70% alcohol. How much alcohol must be added to make it 80% alcohol?

5. A grocer bought 120 bbl. apples, each containing 2 bu. 1 pk., at $1.50 per bbl., and paid $14.40 to have them brought to his store. If the loss from shrinkage and decay amounts to 20%, at what price per peck must he sell them in order to gain 10% on his outlay?

6. Find $\frac{1}{8}$% of $17\frac{5}{8} \div 3$.

7. In seven years, at a certain rate per cent per annum, simple interest, the sum of $620 amounted to $793.60. Find the rate per cent.

8. For the first three years in business a trader gained $1200.25 a year; for the next three, he gained $1800.62 a year, and for the next two he lost $950.87 a year. Supposing his capital at the beginning of the trade to have been $5000, what was he worth at the end of the eighth year?

9. A druggist buys quinine by Troy weight and marks it to sell at 50% profit, but by mistake sells a pound by avoirdupois weight. What percentage does he really gain on the pound?

10. An agent sold two farms for $12,000 each; on one he gained $14\frac{2}{7}$%, and on the other he lost $14\frac{2}{7}$%. Did he gain or lose and how much?

11. Store fixtures amounting to $250 were inventoried the second year at 20% below cost, and the third year $12\frac{1}{2}$% below inventory. What was the loss the third year?

12. I find that I owe A 50% more than I owe C, and B $33\frac{1}{3}$% more than I owe A. If I owe B $800 more than I do C, how much is my indebtedness to each?

13. July 24, a merchant buys a bill of goods amounting to $1875 on the following terms: 3 months, or less 5% if paid in 10 days. July 30, he makes a payment of $1281.50 with the understanding that he is to have the benefit of the discount of 5%. With what amount should he be credited on the books of the seller?

14. To produce 100 articles involves the following expenditure:

labor \$150, material \$350, other expenses \$200; they are sold at 25% profit. How much would it raise the price of each article to give the laborers 15% better wages?

15. A bankrupt who is paying 60% of his debts, divides among his creditors \$7524. What do his debts amount to, and how much does he pay a creditor whom he owes \$2568?

16. How much per cent is a quantity increased if it is doubled and then increased by ½ of the resulting quantity?

17. I sold a horse costing \$120 at 25% profit and took in exchange horse-blankets and whips, there being twice as many whips as blankets, the blankets being worth \$5 each; but upon examination I find one blanket and six whips to be worthless. Find cost of whips, number of blankets, and per cent gain or loss on first cost of horse.

18. Sold a horse at a gain of 33⅓%. With the money bought another, and lost 20% by selling it for \$96. What was my entire gain or loss?

19. I sold two lots of merchandise at the same price. On one I gained 50%, and on the other I lost 50%. If my entire loss was \$200, what was the cost of each?

20. 66⅔% of the sum of two numbers is 1000. Their difference is 500. What are the numbers?

21. A half-foot square is what per cent of one square foot?

22. In an examination, twenty questions were submitted in each subject, and a candidate answered correctly 18 questions in physiology, 12 in drawing, 16 in civil government, 13 in American history, and 19 in current topics. Find the average per cent passed in these subjects.

23. A piece of cloth on being sponged shrunk 10% in length and 10% in width. What per cent did it shrink?

24. My horse cost 50% more than my carriage. I sold them for \$526.50, which was a gain of 20% on the horse and 15% on the carriage. What was the cost of each?

25. The sum paid for two horses was \$440. 62½% of the sum paid for one equals 75% of the sum paid for the other. Find the price of each.

26. R purchased a house and lot for $3300, paid $975 for repairs, and now rents the premises for $30 a month. If he expends annually for taxes $48.70 and for incidental repairs $35, what is his per cent of annual income on his investment?

27. Bought 2000 bu. wheat for $1250. I find 15% of it worthless. For how much per bushel must I sell the remainder of it that I may gain 20% on the whole cost?

28. A man left his property to his three sons, to A $33\frac{1}{3}$% wanting $180, to B 25%, and to C the remainder, which was $590 less than A and B received. What was the value of the estate?

29. The following tables show the receipts and disbursements for a certain year of three of the largest life-insurance companies in the United States:

TOTAL INCOME	EXPENSES AND TAXES	DEATH CLAIMS	OTHER DISBURSEMENTS
$107363369.84	$1834852.17	$2526179.38	$3165415.43
7824248.35	1563293.28	1636634.66	1857022.74
3306511.36	544032.89	604597.93	690567.22

Find the nearest even per cent of the total income remaining in the hands of each company at the end of the year.

30. A sells 50 bu. of rye, and B sells 60 bu. Together they receive $150, but A's rye is 20% better than B's. How shall they share the money?

31. I bought a drug store and a grocery store for $7000, and I get $550 rent per annum for the two. I make 7% on the cost of the drug store and 9% on the cost of the grocery store. Find the cost of each.

Suggestion. Let x = cost of the drug store; $\$7000-x$ = cost of grocery. Then $.07x+.09(\$7000-x)=\$550$.

32. Find 7% of 54 T. 17 cwt. 60 lb.

33. A carpet large enough to cover a room 75 ft. long and 50 ft. wide was sold for $540, thereby losing 28%. What was the cost of the carpet per square yard?

34. For six years my property increased each year on the previous 100%, and became worth $100,000. What was it worth at first?

35. A farm cost $4000; 60% of this was 40% of three times the value of the house that was built on the farm. What was the cost of the house?

36. Twice $\frac{1}{3}$ of a number is what per cent of 3 times $\frac{1}{5}$ of it?

37. $22\frac{1}{2}$ is what per cent of $7\frac{1}{2}$? What per cent of 5 pounds avoirdupois is $7\frac{1}{2}$ ounces? $\frac{3}{11}$ is 225% of what number?

COMMERCIAL DISCOUNTS

433. It is customary for manufacturers, wholesale dealers, and jobbers, to have a fixed price list for their products, and customers allowed certain discounts from their list price, determined by the current market value. Thus, a stove may be manufactured at $28 with a discount of 25% to dealers. The $28 is the *list price* and 25%, or $7 is the *discount.* The list price less the discount $7 = $21, the *net price,* or *cost.* This reduction from the price of an article, or from an invoice of merchandise, is called a **commercial** or **trade discount.**

To obviate the necessity of issuing a new catalogue whenever the market values change, business houses generally catalogue their lists higher than the actual selling price of the goods, and then quote prices subject to a trade discount. The discount is changed either by increasing or diminishing the single rate of discount already allowed, according as the cost of production is diminished or increased. If the discount is to be increased, the change is generally made by quoting a further discount. Thus, in a catalogue of hardware goods machine bolts are quoted at $8.40 per hundred. In the price list accompanying the catalogue a discount of 50% is allowed on all orders. In a price list issued later on account of a change in the cost of production, a discount of 50% and 10% is allowed. A dealer buying 500 bolts according to the first discount list would pay 5×$8.40 less 50% of 5×$8.40, or $21, while according to the second discount list he would pay $21 less 10% of $21, or $18.90.

434. Commercial discounts include *trade discounts, cash discounts,* and *time discounts.*

435. **Cash discounts** are reductions made from the selling price of an article to secure immediate payment. **Trade discounts** are reductions made from the list or marked price of articles belonging to *trade* or *commerce.* **Time discounts** are reductions from the amount of a bill of merchandise for payment within a definite time.

436. The **terms** are the conditions upon which a bill is sold and are usually printed, stamped, or written upon the "bill heads."

Thus, "Terms: 60 days, or 2% discount for cash." "Terms: net 60 days, or 2% in 10 days."

437. Space in which terms are to be written is frequently limited, and bookkeepers often use symbols and abbreviations to indicate the latter.

Thus, $^{n}/_{30}$ would mean, *net* 30 days; $^{3}/_{10}$, $^{n}/_{30}$, or 3% 10 da., *net* 30 da. would mean the bill is due in 30 days without discount, but an allowance of 3% is made for payment within 10 days.

Discount Series

438. If two or more discounts are quoted, one denotes a discount off the list price, the next a discount off the remainder and so on. This is known as a **discount series.**

The result of the computation in a discount series is the same regardless of the order in which they are taken. Thus, a series of 20%, 15%, and 10% is the same as one of 20%, 10%, and 15%, or one of 10%, 15%, and 20%.

439. The **list price,** sometimes called the *gross price,* is a fixed or catalogue price. The net price is what remains after the discount or discounts have been deducted.

Legal interest may be collected on all bills overdue, and when paid before maturity, a discount is usually allowed.

440. Computations in commercial discounts are neither more nor less than an application of percentage principles. The *list price* corresponds to the *base*, the *per cent of discount* to the *rate*, the *discount* to the *percentage*, and the *net price* of the bill to the *difference.*

ORAL EXERCISE

1. The list price of men's rubber boots is $3 per pair with a discount of 20%. What is the selling price?
2. Find the net amount of goods listed at $400, less 25% discount.

3. Find the net cost of a bill of goods amounting to $27, the discount being 11$\frac{1}{9}$%.

4. The list price of 1-inch wire brads is 30¢ per pound. How much will 100 pounds cost if they are sold at 60% off? How much will 100 pounds cost if they are sold at 50% and 10% off?

5. A stove listed at $40 is sold at 25% and 10% off. What is the net cost to the purchaser?

6. I bought a horse for $80 and sold it for $60. What was my loss per cent? I bought a horse for $60 and sold it for $80. What was my gain per cent?

CASE I

441. Given the list price and rates of discount to find the net amount of the bill.

Example. How much must be paid to satisfy a claim of $75, on which a discount of 25% and 10% is allowed?

Solution

$75 = gross amount of the claim.
25%, or $\frac{1}{4}$ of $75 = $18.75, the first discount.
$75 − $18.75 = $56.25, the remainder after the first discount.
10%, or $\frac{1}{10}$ of $56.25 = $5.63, the second discount.
$56.25 − $5.63 = $50.62, the net amount of the claim.

442. Rule. *Deduct the first discount from the list price, and each subsequent discount from each successive remainder.*

WRITTEN EXERCISE

Find the net amount of the following bills:

1. $440 less 25% and 10%.
2. $360 less 33$\frac{1}{3}$% and 10%.
3. $350 less 30% and 15%.
4. $6.80 less 40% and 5%.
5. $40.50 less 10%, 5%, and 2%.
6. $48 less 16$\frac{2}{3}$%, 10%, and 5%.

7. Find the cost of a bicycle listed at $75, 30% and 10% off.

8. The list price of an article is $150. If trade discounts of 25% and 16$\frac{2}{3}$% are allowed, what is the net price?

9. Which is the better offer, and how much, a series of 30% and 20%, or a single discount of 50%?

10. What is the difference on a bill of $650, between a discount of 30% and a discount of 25% and 6% off?

11. The list price of a certain piano is $1200, and trade discounts of 50%, 33⅓%, and 25% are allowed to dealers. If a dealer sells one of these pianos for $500, what is his per cent of profit?

12. Bought books at a discount of 20% on the retail price and sold them at the retail price; what per cent did I gain?

13. An article listed at $25 and bought subject to discounts of 20%, 10%, and 5%, was sold for $22.23. What was the per cent of gain on the investment?

14. A tradesman marks his hats $5 each, but takes off 5% for cash. If his profit is 25%, what did he pay per dozen for his hats?

Finding a Single Rate of Discount Equivalent to a Discount Series

Example. Trade discounts of 20%, 10%, and 8⅓% are equivalent to what single discount?

20% = first discount.
1.00 − .20 = .80
.80 × .10 = .08, 2d discount.
.80 − .08 = .72
.72 × .08⅓ = .06, 3d discount.
Then 20% + 8% + 6% = 34%.

Solution. Let 100% = the list price. Then 80% = the price after the first discount. 10% of 80% = 8%, the second discount which, subtracted from 80% = 72%. 8⅓% of 72% = 6%. Hence, the several per cents, 20% + 8% + 6% = 34%, the direct discount.

A Short Method of Finding a Single Discount Equivalent to a Series of Two Discounts

Example 1. Successive trade discounts of 25% and 20% are equal to what single discount?

25% + 20% = 45%.
45% − (25% × 20%) = 40%.

Solution. In a discount series of 25% and 20% the apparent discount is 45%, but as the second discount is not computed on 100%, but on 75%, 45% exceeds the true single discount by 20% of 25%, or 5%. Hence, 45% − 5% = 40%, the true discount.

Note. In a series of more than two discounts, the first two may be combined as illustrated in the example above, then that result and the third may be combined in the same manner. Thus:

Example 2. Successive trade discounts of 25%, 20%, and 10% are equal to what single discount?

$25\% + 20\% - 5\% = 40\%.$
$40\% + 10\% - 4\% = 46\%.$

Solution. Determine mentally to what single discount 25% and 20% are equivalent. $25\% + 20\% = 45\%$; $45\% - (25\% \times 20\%) = 40\%$. Then $40\% + 10\% = 50\%$, and $50\% - (40\% \times 10\%) = 46\%$. Therefore, the series of discounts 25%, 20%, and 10% is equivalent to a single discount of 46%.

ORAL EXERCISE

By inspection find a single rate of discount equivalent to the following:

1. 25% and 10%.
2. 30% and 20%.
3. 20% and 40%.
4. 25% and $33\frac{1}{3}\%$.
5. 20% and $12\frac{1}{2}\%$.
6. 50% and 10%.
7. 50% and 20%.
8. 50% and 50%.
9. 10% and 40%.
10. 60% and 20%.
11. 55% and 20%.
12. $8\frac{1}{3}\%$ and 36%.
13. $16\frac{2}{3}\%$ and 24%.
14. 20% and 15%.
15. 4% and 25%.
16. 70% and $66\frac{2}{3}\%$.
17. $11\frac{1}{9}\%$ and 18%.
18. $33\frac{1}{3}\%$ and 15%.
19. 35% and 10%.
20. $14\frac{2}{7}\%$ and 35%.
21. 25%, 20%, and 20%.
22. 20%, 15%, and 10%.
23. 25%, $33\frac{1}{3}\%$, and 10%.
24. 20%, 20%, and 20%.
25. 30%, 20%, and 10%.
26. 40%, 20%, and 10%.
27. 50%, 20%, and 10%.
28. 60%, 30%, and 10%.
29. 60%, 50%, and 20%.
30. 20%, 5%, and 10%.

CASE II

443. Given the net amount of a bill and the rate of discount to find the list price.

Example. After 10% and 5% had been deducted from the list price, a bill of goods was sold for $547.20. What was the list price?

$10\% + 5\% = 15\%.$
$15\% - (10\% \times 5\%) = 14\frac{1}{2}\%.$
$100\% - 14\frac{1}{2}\% = 85\frac{1}{2}\%.$
$\$547.20 \div 85\frac{1}{2}\% = \$640.$

Solution. Let 100% = the list price of the bill. Then deducting the discounts 10% and 5% from 100% as shown in the illustration, gives a net difference of $85\frac{1}{2}\%$ which must be equal to the selling price in money, or $547.20. Therefore, $\$547.20 \div 85\frac{1}{2}\% = \$640$, the list price of the bill of goods (430).

WRITTEN EXERCISE

1. What is the cash value of a bill of goods amounting to $2200 at 15% discount, and 3% off for cash?

2. The trade discounts of a piano sold subject to successive discounts of 33⅓%, 20%, and 5% from the list price was $304. Find the list price.

3. Silk marked $2 is bought at 25% and 20% off, and sold at an advance of 20% and 10% on the marked price. What is the gain per cent?

4. Sold 20 doz. feather dusters, giving the purchaser a discount of 10%, 10%, and 10%. His discounts amounted to $325.20. What was the price per dozen?

5. The list price is $7, the net price $4.50; one discount is 25%, what is the second discount?

6. A dealer in ready-made clothing marked an overcoat 33⅓% above cost, and in selling it dropped 10% on his marked price. What was his gain per cent?

7. Gloves are sold at $.90 a pair, which is 16⅔% less than the price they are marked, but which is 12½% more than they cost. What per cent above cost were they marked?

8. A jeweler marks goods 50% above cost, what discount from marked price can he give a customer and still make 33⅓% profit.

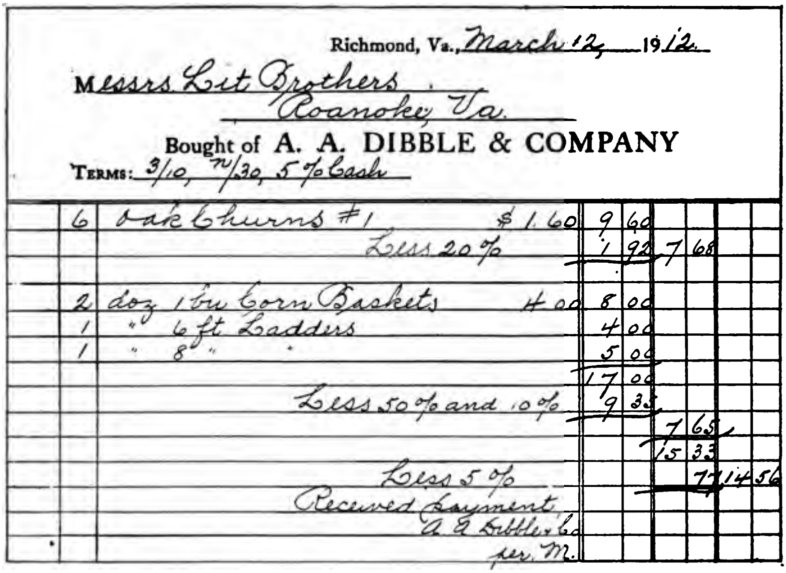
9. A school having $150 to spend for library books is offered a discount of 25% by one firm and discounts of 10% and 15% by another firm. Which offer is the better and how much on the list price?

CASE III

444. The terms and discount given to find the net amount of a bill to render.

Example. Render a bill for the following: March 2, 1912, Lit Brothers, Roanoke, Va., bought of A. A. Dibble & Co., Richmond, Va., 6 oak churns #1 at $1.60; 2 doz. 1 bu. corn baskets at $4; 1 doz. 6-ft. ladders at $4; 1 doz. 8-ft. ladders at $5. Terms: $^{8}/_{10}$ $^{n}/_{30}$, 5% cash. The bill is paid on the date of the sale. The churns are subject to a discount of 20%, the other articles 50% and 10%.

Solution



Richmond, Va., March 12, 1912

Messrs Lit Brothers.
Roanoke, Va.

Bought of A. A. DIBBLE & COMPANY

Terms: 3/10, n/30, 5% Cash

6	Oak Churns #1 $1.60	9 60		
	Less 20%	1 92	7 68	
2	doz 1 bu Corn Baskets 4 00	8 00		
1	" 6 ft Ladders	4 00		
1	" 8 "	5 00		
		17 00		
	Less 50% and 10%	9 35		
			7 65	
			15 33	
	Less 5%		77	14 56
	Received Payment			
	A A Dibble & Co			
	per M.			

WRITTEN EXERCISE

1. Sold a bill of goods amounting to $850 on 90 days time, at 8% discount, and deducted 5% for cash. What was the net price?

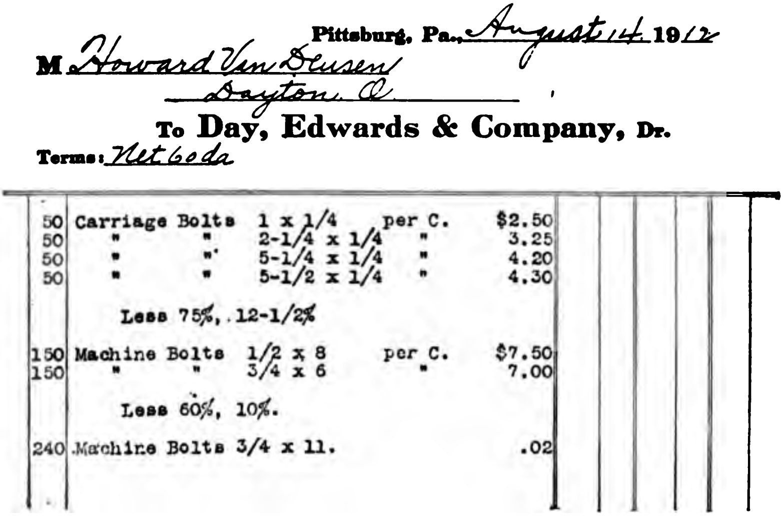
2. J. N. Burton, Atlanta, Ga., bought of Wilson Bros., St. Louis, Mo., June 1, 1912, on account 60 da., 3% 10 da., 1% 30 da.: 2 doz. pairs men's shoes at $2 less 20% and 5%; 12 pairs children's oxfords at $1.50 a pair; 1 doz. pairs men's kangaroo calf at $2.20 a pair; ½ doz. pairs Misses' sandals at $1.00 a pair; 1 gross leather laces at 75¢ a gross. Find the net amount of the bill if the above items are subject to a discount of 25% and 5%, except as indicated in the first.

3. An invoice of hardware amounting to $1600 was sold Oct. 15, on 60 days' credit, and subject to a trade discount of 30% and 10%, with an additional discount of 3% if paid in 20 days, or 5% if paid in 10 days. What sum will pay the bill Nov. 1st?

4. James Brownell, Toledo, O., bought of Fairman & Lin-

coln, Columbus, O., March 1, 1912. Terms: $^5/_{10}$, $^n/_{30}$. 4 pianos at $500 each, 5 pianos at $550 each, 4 pianos at $350 each, less a discount of 30% and 20% from each list price; 5 organs at $125 each, less 30% and 5% from the list price. Find the net amount of the bill Mar. 12.

5. Find the net amount of the following bill:



Pittsburg, Pa., August 14, 1912

M Howard Van Deusen

Dayton, O.

To Day, Edwards & Company, Dr.

Terms: Net 60 da.

50	Carriage Bolts	1 x 1/4	per C.	$2.50
50	" "	2-1/4 x 1/4	"	3.25
50	" "	5-1/4 x 1/4	"	4.20
50	" "	5-1/2 x 1/4	"	4.30
	Less 75%, 12-1/2%			
150	Machine Bolts	1/2 x 8	per C.	$7.50
150	" "	3/4 x 6	"	7.00
	Less 60%, 10%.			
240	Machine Bolts	3/4 x 11.		.02

PROFIT AND LOSS

445. Profit *and* **loss** are terms which denote the gain or loss arising in business transactions.

Gains and losses are usually estimated at a certain rate per cent on the cost, or the money or capital invested, therefore the operations involve the *same principles* as those of percentage. Discounts are reckoned on the marked or asking price.

446. A **profit** is realized when the *net selling price* exceeds the *whole cost*, and a **loss** when the *net selling price* is less than the *whole cost.*

The first cost of goods is called the *prime cost.* The prime cost increased by freight, drayage, etc., is called the *gross* or *full cost.* The *gross selling price* is the amount arising from the sale of goods. The gross selling price, less all charges such as trade discounts, commission, expense of delivery, etc., is called the ***net selling price.***

447. Corresponding terms to those in percentage are:

1. The **base** is the *cost*, or capital invested.
2. The **rate** is the *per cent* of profit or loss.
3. The **percentage** is *profit* or *loss.*
4. The **amount** is the cost *plus* the profit, or the *selling price.*
5. The **difference** is the cost minus the loss, or the *selling price.*

ORAL EXERCISE

1. Cost, $300; selling price, $400; rate of gain = ?
2. Cost, $400; rate of loss, 2%; selling price = ?
3. Selling price, $200; rate of gain, 33⅓%; cost = ?
4. Loss, $50; rate of loss, 10%; selling price = ?
5. Selling price, $150; gain, $50; gain per cent = ?
6. Cost, $600; rate of gain, 8%; gain = ?
7. Selling price, $30; rate of loss, 20%; loss = ?
8. Gain, $5; rate of gain, 12½%; cost = ?
9. Selling price, $55; loss, $5; loss per cent = ?
10. Cost, $500; loss, $100; loss per cent = ?
11. By selling an article for $21 I lost 12½%. At what price should I have sold it to gain 12½%?
12. By selling an article for $4 less than cost I lose 16⅔%, what would have been my per cent of gain had I sold it for $3 more than cost?

WRITTEN EXERCISE

1. By selling an article for $13.20 a merchant loses $2.40; for how much should he have sold it to gain 37½%?
2. I sold flour for $8.40 at a gain of 20%. Find the loss per cent if it had been sold for $6.86.
3. A building which costs $10,500 rents for $87.50 per month. The owner pays yearly taxes amounting to $162.50; insurance $23.75; repairs $136.80, and janitor's services $51.95. What annual rate of interest on his investment does the owner receive?
4. A machinist sold two seed-drills for equal sums of money.

He gained 25% on one and lost 25% on the other. His total loss was $9.60. Find the cost of each drill.

Suggestion. Let 100% = selling price of each. Then 100% ÷ 125% = cost of one, and 100% ÷ 75% = cost of the other.

5. Bought 500 barrels of flour and sold at an advance of 20%, thereby gaining $600. Find the cost price per barrel.

6. A merchant sold a quantity of goods to a customer at a gain of $33\frac{1}{3}\%$, and with the money bought another lot which he sold at 25% loss. What is the per cent of gain or loss on the whole?

7. A merchant sold cloth at 5% less than the marked price and yet made a profit of 25%. At what per cent advance on cost were the goods marked?

8. An importer receives an invoice of kid gloves billed at $680, pays a duty of 50% *ad valorem*, and sells them at an advance of $33\frac{1}{3}\%$ on their gross cost to him. How does the price paid by the purchaser compare with the exporter's price?

9. At a forced sale, a bankrupt sold a suit of clothes for $8, which was 20% less than its real value. If the suit had been sold for $12, what per cent above its real value would it have brought?

10. If $\frac{2}{3}$ of an article is sold for $\frac{4}{5}$ of its cost, what is the gain per cent?

11. If $\frac{7}{8}$ of an article is sold for $\frac{5}{6}$ of its cost, what is the loss per cent?

12. A grocer lost 20% by selling 120 gallons of oil for $1.62 less than cost. Find the cost of a gallon of oil.

13. A merchant marked goods at an advance of 40%, but in selling them he used a false balance, by means of which he gave 14 oz. to the pound. His total gain being $240, find the cost of the goods.

14. $\frac{7}{10}$ of the selling price of certain goods is 2% less than cost. Find the gain per cent at which the goods are sold.

15. A bought a house and sold it at a profit of 20%. This amount he invested in real estate which he afterwards sold at a profit of $33\frac{1}{3}\%$. With this amount he purchased a store which

he sold at a loss of 10%. If he had $8640 remaining, what amount had he at first?

16. A man buys 150 lb. of sugar and after selling 100 lb. finds that he has been selling at a loss of 5%. At what per cent advance on the cost must he sell the remaining 50 lb. that he may gain 10% on the entire transaction?

17. I bought a horse for $150 and sold it at an advance of 20%; I sold a buggy for $66, losing $33\frac{1}{3}$%; I sold a set of single harness for $15, gaining 25%. What did I gain or lose on the whole?

18. A manufacturer sold an article to a wholesaler at a gain of 25%, the wholesaler sold the same article to a retailer at a gain of 40%, and the retailer to the consumer at a gain of 20%. If the average gain was $88, what was the cost to manufacture the article?

19. A merchant sold a lot of goods at a gain of $33\frac{1}{3}$%, and with the proceeds purchased another lot which he sold for $120, at a loss of 20%. What was the gain or loss?

20. A merchant imported silk goods at a cost of $1425. Three months later he found that the sale of these goods aggregated $1350 and that the value of the stock unsold was $325. What was the per cent of gain on the sales?

WRITTEN REVIEW

1. A farm of 160 acres, costing $50 per acre, was sold at a profit of $1600. What was the per cent of gain?

2. $\frac{3}{4}$ is $\frac{2}{3}$% of what number?

3. 14 bushels is what per cent of 7 quarts?

4. Express $.00\frac{16\frac{2}{3}\times 3}{50\times\frac{2}{3}\times 6}$ as per cent.

5. A buggy listed at $125, 30% and 8% off, is sold on 90-days' time, 5% discount for cash. Find the cash price of the buggy.

6. A merchant closed out a stock of cloaks for $311.04 at a loss of 28%. Require the loss by the transaction.

7. A man buys goods at a discount of 20% and 10% from

the list price, and sells making 25% profit. How much does he discount the list price to his customers?

8. A man running an excursion boat gives to parties the following rates of discount: for every full 100, 33⅓% discount; for every full 50, 25% discount; for every full 10, 20% discount. If his round trip regular fare from Chicago to Milwaukee is $3.15, what will be the average cost per ticket for 375 excursionists?

9. A and B commenced business with equal sums of money. A gained a sum equal to ⅕ of his capital, and B lost $220. B then had ½ as much as A. How much had each at first?

10. Reports in the Bureau of Labor Statistics, from three manufacturing establishments, show that in one the average weekly wages paid to 262 operatives was $12.85; in another, to 355 operatives, $13.84; and in the third, to 128 operatives, $15.11. Find the average weekly wages for all three establishments, and the nearest per cent which the first is below the general average.

11. 4.0443 is $\frac{39}{100}$% of what number; 1.83 is ⅜% of what number?

12. If the list price of an article is 50% advance on cost, what other per cent of discount than 10% must be allowed to net 15% gain on the sale?

13. The stuff from a lead mine contained originally 15.9% of lead. After it had been washed, by which process the amount of lead was not diminished, the stuff contains 87.45% of lead; how much rock was washed away out of 216 tons 3 cwt. of the original stuff?

14. A has $5280, and B has $6400. What per cent is A's money less than B's? What per cent is B's more than A's?

15. If a grocer's pound-weight weighs only 15½ oz., how much is his fraudulent gain per cent on 4 barrels of sugar of 210 lb. each, true weight, at 6¢ a pound?

16. A huckster sold a quantity of potatoes and onions, gaining 37½% on the onions and 25% on the potatoes, 33⅓% of his profits being realized on the potatoes. At what price was each sold if his total gain was $450?

17. In a certain school 48% of the pupils are boys, and there are 39 girls. Find the number of boys.

18. What sum of money put on interest at 5% will amount to $299 in 3 years?

19. A sold an article to B and gained 25%, B sold it to C and gained 20%. C paid $30 for repairs and sold it to D for $140, gaining $16\frac{2}{3}$% on his total cost. How much more did D pay for the article than A?

20. 80 is 50% of 20% of 75% of $\frac{3}{4}$ of what number?

21. An agent in St. Paul, Minn., bought for me 240 bbl. of flour at $4.50 a barrel on a commission of 2%. The flour was shipped to Chicago, where another agent sold it for me at $5.20 a barrel on a commission of $2\frac{1}{2}$%. If the freight charges were $65, find my profit.

22. Assume that in problem 21, the flour was sold to J. D. Fleming, terms $^2/_{10}$, $^n/_{30}$. At the maturity of the bill Fleming refused payment and you placed the account in the hands of a lawyer who succeeded in collecting 80% of the amount due. If the fee for collecting was 5%, what was your net gain or loss?

23. What must be the marked price of goods that cost $740 that I may allow discounts of 30%, 10%, and 5% and still make a profit of 20%?

24. A merchant buys goods at a discount of 10% and 5% from the list price, and sells them at a discount of 5% and $2\frac{1}{2}$%. What per cent of the cost is profit?

25. A druggist buys acid at 40¢ a gallon and adds water so that when he sells it at 30¢ a gallon he gains 40%. What is the per cent of water in each gallon?

26. What is $\frac{7}{8}$% of $\frac{3}{4}$; $\frac{2}{3}$% of $16\frac{2}{3}$?

27. I sold $\frac{2}{5}$ of my property at a gain of $33\frac{1}{3}$%, and the remainder for $\frac{2}{3}$ of the cost of the whole, receiving in payment a note due in 3 months, without interest, which I had discounted at a bank at 6%. What was my gain per cent if my property was worth $24,000?

28. A merchant knows neither the weight nor prime cost of a caddy of tea. He recollects that if he had sold the whole quantity at 70¢ a pound he would have gained $7.00, and if he

had sold it at 50¢ a pound he would have lost $3.00. What was the weight and prime cost of the tea?

29. A lady put $864 in a savings bank, which was 25% of all her money, and afterwards deposited 45% of the remainder of her money. What per cent of all her money had she then in bank?

30. In billing a quantity of goods, a blundering clerk deducted 35% instead of 25% and 10%. If the mistake passed unnoticed, what was the buyer's gain per cent on the transaction, if the discount deducted was $420?

31. A merchant sold goods and gained 20%. If he had bought them for $60 less he would have gained 25%. What did they cost?

32. A man bought 100 bu. corn and 75 bu. wheat at the combined cost of $139.50. The corn cost 20% less per bushel than the wheat. What was the price per bushel of each?

33. A wholesale merchant sold a bill of goods on 60-day paper, giving 20%, 10%, and 5% off. He then discounted the note at a bank at 6%, receiving $677.16. What was the amount of the bill of goods?

34. A merchant having bought a lot of goods, sells $\frac{1}{3}$ at a loss of 4%. By what per cent must he raise that selling price in order that, by selling the remainder at the increased rate, he may gain 4% on the whole transaction?

35. Sold two horses for $240, losing on the first 20%, and gaining on the other 20%. What was my loss or gain, provided I received for the second three times as much as for the first?

36. If by selling tea at 47½¢ per pound, I lose 5%, at what price must I sell it to gain 15%?

37. Coffee costing 35¢ per pound is mixed with chicory worth 10¢ a pound in the proportion of 5 pounds of coffee to 2 pounds of chicory, and the mixture is sold for 34¢ a pound. Find the gain per cent.

38. A person's salary is $\frac{4}{5}$ of his total income, the other $\frac{1}{5}$ being interest on money which he has lent at 7%. It costs him 60% of his total income to live and the remainder, $700, is placed in the savings bank. If the rate of interest falls from 7%

to 5% and his salary and living expenses remain the same, how much less each year can he put in the bank?

39. A grocer bought a barrel of molasses containing 36 gallons at 45¢ a gallon, and retailed it at 50¢ a gallon, using by mistake a set of dry measures. What was his loss per cent?

40. What per cent is gained by buying coal at $5.25 for a long ton and retailing it at $6.50 a short ton?

41. Allowing 5% for shrinkage and waste, and 10% for bad debts, at what rate per cent above cost must goods be sold that a net profit of 25% may be realized?

42. A engaged in business for three years, adding each year the profits of the preceding year. His profits for each year were 25%, 30%, and 10% respectively. His capital at the end of the third year was $2145, how much had he at first?

43. A and B traded until they gained 6% on their stock; then $\frac{2}{5}$ of A's gain was $18. If A's stock was to B's as $\frac{2}{5}$ to $\frac{1}{3}$, what was the original stock of each?

44. A man buys 150 bbl. apples of 2 bu. 1 pk. each at $1.50 a barrel, and paid $14.40 for carriage. If loss by decay amounts to 20%, what is the least price per bushel at which he can sell them to gain 12½% on his outlay?

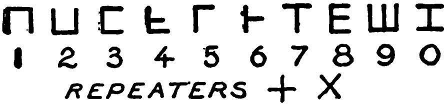
MARKING GOODS

448. It is customary for merchants to use a private mark to indicate the cost and selling price of goods. They usually select some word or phrase containing ten different letters and use it as a *key*. These letters are used to represent the nine digits and 0. In this way the cost and selling price will be understood only by those who know the key.

Any word may be selected, containing ten different letters, or ten different characters chosen; as,

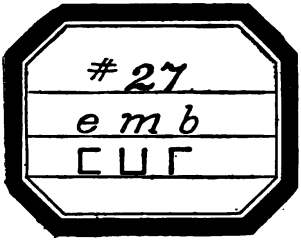
r	e	p	u	b	l	i	c	a	n
1	2	3	4	5	6	7	8	9	0

Repeaters: *m*, *n*.



Two different keys are often selected, one to represent cost and the other, selling price. The cost is generally written above and the selling price below a line on a tag, poster, or covering of the article. One or more extra letters, called repeaters, are used to avoid the repetition of a figure and to prevent giving any clew to the private mark used.

Using the above letters as cost, and the characters for the selling price, goods costing $2.25 and selling for $3.25 might be marked as shown in the margin. Fractions may be designated by additional letters or characters.



449. Jobbers, wholesalers, and retailers buy a great many articles by the dozen. Therefore, in marking them it is highy important that the student should be able to divide by 12 with great rapidity.

NOTE. Students should not be allowed to divide by any other method than *short division* in dividing by 12, as it is just as easy as dividing by any other number of one digit.

To divide by 12 with rapidity the decimal values of the twelfths should be *memorized.*

Table of Twelfths

TWELFTHS	FRACTIONAL VALUE	DECIMAL VALUE	TWELFTHS	FRACTIONAL VALUE	DECIMAL VALUE
1/12		$.08⅓	7/12		$.58⅓
2/12	1/6	.16⅔	8/12	2/3	.66⅔
3/12	1/4	.25	9/12	3/4	.75
4/12	1/3	.33⅓	10/12	5/6	.83⅓
5/12		.41⅔	11/12		.91⅔
6/12	1/2	.50	12/12	1	1.00

ORAL EXERCISE

State the cost per article when the cost per dozen is:

1. $24.00
2. $36.00
3. $27.00
4. $42.00
5. $64.00
6. $3.60
7. $7.20
8. $9.00
9. $3.40
10. $6.60
11. $12.24
12. $14.00
13. $10.00
14. $38.00
15. $11.00
16. $6.75
17. $3.75
18. $2.50
19. $4.00
20. $1.00

ORAL EXERCISE

1. Explain why, if the cost of a dozen articles is divided by 10, the result will give the retail price of one article with a profit of 20% added.

2. A merchant buys shirts for $18 per dozen, for what price must he sell them to make 20% profit?

3. Explain why, to make a profit of 33⅓%, the cost of a dozen articles may be divided by 10 and ⅑ of the result added.

4. What must be the asking price of a suit of clothes costing $24 in order to insure a gain of 25% and allow the purchaser a discount of 20%?

5. A merchant buys gloves for $12.50 per dozen, at what price must he sell them to make 50%?

6. Formulate a short method for finding the retail price of one article when the cost per dozen is given and the dealer wishes to make a profit of 33⅓%; 37½%; 40%; 50%; 66⅔%.

7. Hats costing $20 a dozen must be sold at what price to gain 33⅓%? 37½%? 25%? 40%? 66⅔?

WRITTEN EXERCISE

Using the following keys, write the cost and selling price of articles costing:

Cost Price

w	i	n	t	e	r	d	a	y	s
1	2	3	4	5	6	7	8	9	0

Selling Price

k	e	g	o	f	n	a	i	l	s
1	2	3	4	5	6	7	8	9	0

Repeaters: *x*, *z*

1. Cost $2.40, gain 25%.
2. Cost $5.25, gain 20%.
3. Cost $375, gain 33⅓%.
4. Cost $ .50, gain 40%.
5. Cost $ .75, gain 60%.

Repeaters: *m*, *c*

6. Cost $.90, gain 33⅓%.
7. Cost 12½¢, gain 40%.
8. Cost 16¢, gain 75%.
9. Cost $27, loss 16⅔%.
10. Cost $25, gain 50%.

11. A merchant sold a bill of goods that cost $125; the asking price was 40% in advance of the cost, from which a wholesale discount of 15% was allowed. What was the gain per cent?

12. An article marked 33⅓% profit is sold for $1.20. If the selling price is marked *kgs*, what is the rate of discount?

13. A retailer sells lemons at 35¢ per dozen and thereby gains 110% on their cost. Find the cost of each lemon.

14. How much less is a discount of ¼ and 15% than a discount of 40% from a bill of $346.40?

15. Goods marked at an advance of 20%, but really sold for 5% more than the marked price, bring $9450. What was the gain?

16. A hat marked $4.50 can be sold at 20% discount and a gain of 20% realized. What is the cost mark?

17. You list neck-ties bought for 27¢, at an advance of 66⅔% on the cost, but finding small sale for the article you determine to sell so as to gain but 33⅓%. What trade discount should you allow?

18. An invoice of tea costing $*ts* is marked so as to sell at 40% profit. Does the merchant gain or lose if the tea is sold at 30% discount from the marked price?

COMMISSION AND BROKERAGE

450. Manufacturers, produce dealers, farmers, and others frequently find it more convenient to employ some one to dispose of their goods, instead of selling direct to consumers. The person authorized to act in behalf of another is called an **agent, a commission merchant,** or a **broker,** while the person for whom the agent acts, is called the **principal.**

451. Produce is usually shipped to a commission merchant, and sold by him in his own name. The **net proceeds** is the amount belonging to the principal, and is the sum remaining after the commission and all other expenses are deducted from the total sales.

A commission merchant has actual possession and control of the goods of his principal; when he does not take possession, but simply contracts to buy or sell in the name of his principal, he is called a *broker.*

452. **Commission** or **brokerage** is the sum charged by the agent for transacting business for his principal, and is usually computed at a certain per cent of the amount realized on sales, or invested for the customer. In such cases the general principles of percentage are applied.

The commission for buying and selling some kinds of merchandise is usually computed at a certain price per unit of weight or measurement; as, grain per bushel, hay per ton, cotton per bale, etc.

453. An agent is frequently allowed an additional commission, called **guaranty,** for assuming the risk of any loss to his principal for sales made on credit, or guaranteeing the quality of goods bought or sold.

An agent is lawfully responsible for amounts due upon credit sales which are unauthorized by his principal.

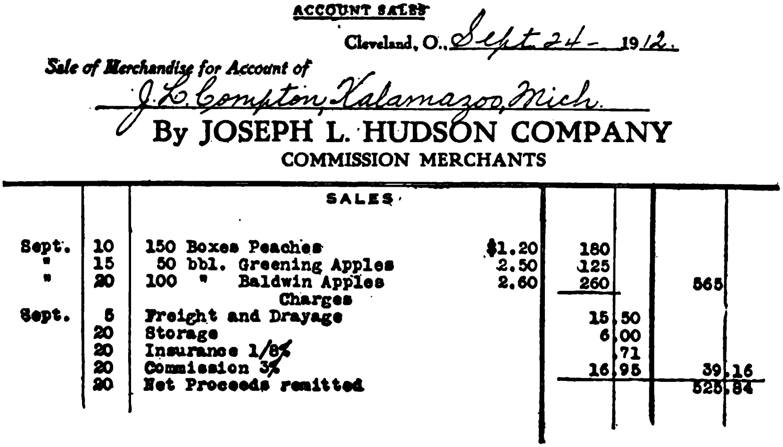
Shipments and Consignments

454. A quantity of goods sent to a commission merchant to be sold is called a **shipment;** while a quantity of goods received by the merchant to be sold on commission is called a **consignment.**

A *shipment* by the principal is a *consignment* to the agent.

455. The person who consigns or ships the goods is called the **consignor;** the person to whom the goods are sent is called the **consignee.**

456. An **account sales** is a detailed statement rendered by a commission merchant to the consignor, and contains a list of the goods sold, the charges in detail, and the net proceeds remitted or credited.



ACCOUNT SALES

Cleveland, O., Sept. 24 - 1912.

Sale of Merchandise for Account of

J. L. Compton, Kalamazoo, Mich.

By JOSEPH L. HUDSON COMPANY

COMMISSION MERCHANTS

		SALES					
Sept.	10	150 Boxes Peaches	$1.20	180			
"	15	50 bbl. Greening Apples	2.50	125			
"	20	100 " Baldwin Apples	2.60	260		565	
		Charges					
Sept.	5	Freight and Drayage		15	50		
	20	Storage		6	00		
	20	Insurance 1/8%			71		
	20	Commission 3%		16	95	39	16
	20	Net Proceeds remitted				525	84

Selling on Commission

457. Since *commission* is usually computed at a certain rate per cent, the principles of *commission and brokerage* involve the same elements that the fundamental principles in percentage do; as,

The amount of sale, purchase, or collection is the *base;* the rate of commission is the *rate;* the commission is the *percentage;* the entire cost is the *amount;* and the net proceeds is the *difference.*

Example. A commission merchant sold a consignment of flour for $1250, and charged 3% commission. What was the net proceeds?

Solution. $1250 × 3% = $37.50 commission.
$1250 − $37.50 = $1212.50 net proceeds.

458. Principle. *The commission is reckoned at a certain rate per cent upon the value of the sales.*

ORAL EXERCISE

By inspection find the commission in the following:

	Selling Price	*Rate of Commission*		*Selling Price*	*Rate of Commission*
1.	$150.	2%	5.	$ 670.50	$33\frac{1}{3}$%
2.	$550.	10%	6.	$1400.00	8%
3.	$840.	4%	7.	$2500.00	6%
4.	$666.	5%	8.	$3200.00	$12\frac{1}{2}$%

9. Amount of sales, $300; net proceeds, $295; what is the rate of commission?

10. Net proceeds, $196; rate of commission, 2%; what is the amount of sales?

11. Commission, $10; rate of commission, 5%; what is the net proceeds?

12. Net proceeds, $75; commission, $5; what is the rate of commission?

WRITTEN EXERCISE

1. Find the rate of commission that an agent receives for selling a house for $4750, if his commission is $95.

2. My commission at 5% on a sale of hay at $13.50 per ton was 12.48\frac{3}{4}$. How many tons did I sell?

3. Paid an agent $394 for selling a piece of property at 2½% commission. What did the property bring?

4. A commission merchant sold 15,460 bu. of potatoes at 80¢ per bushel; 840 bbl. flour at $6.25 per barrel. What was the amount of his commission at 2¼%?

5. An agent sold 26 copies of a book at $3.50 per volume and was allowed 35% commission. How much should he remit to his principal?

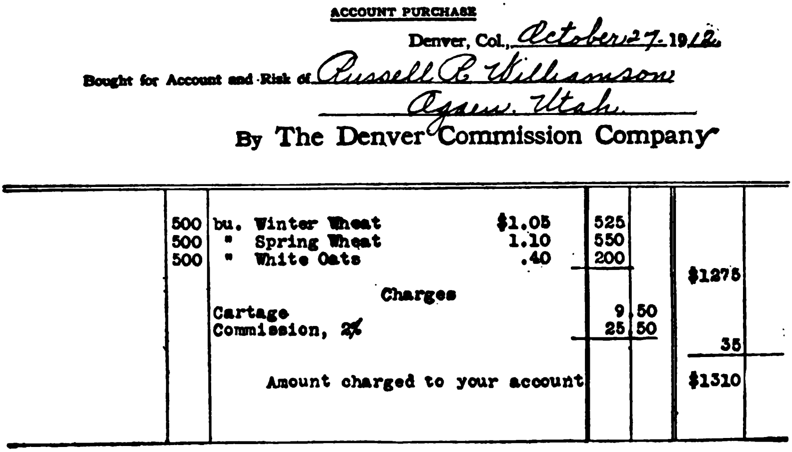
6. A commission merchant sold 600 baskets of grapes at 12¾¢ per basket, and after deducting $18.70 for transportation charges and commission at 6%, remitted the balance to the consignor. Find the amount of the remittance.

7. A produce broker remits to a consignor as the net proceeds of a sale of flour, $3615.75, after deducting his commission of 5% and $116.33 for freight and storage. Find the amount of the sale.

8. A broker sells gold and remits to the owner $8857. His brokerage at ⅛% is $68, at what price did the gold sell?

Buying on Commission

459. An **account purchase** is a detailed statement rendered by a purchasing agent to his principal, showing the cost of certain goods bought, and the charges or expenses attending the purchase.



ACCOUNT PURCHASE

Denver, Col., October 27 1912

Bought for Account and Risk of Russell R. Williamson
Ogden, Utah.

By The Denver Commission Company

	500	bu. Winter Wheat $1.05	525			
	500	" Spring Wheat 1.10	550			
	500	" White Oats .40	200			
					$1275	
		Charges				
		Cartage	9	50		
		Commission, 2%	25	50		
					35	
		Amount charged to your account			$1310	

Example. An agent is paid 1½% for purchasing goods. What amount does he purchase for a remittance of $1258.60, and what was his commission?

Solution. 100% = actual investment.
1½% = charges for buying.
100% + 1½% = 101½% = cost of investment.
$1258.60 ÷ 101½% = $1240, actual amount of purchase.
$1258.60 − $1240 = $18.60, commission.

460. Principle. *The commission is reckoned at a certain rate per cent upon the value of the purchases.*

1. Incidental expenses attending purchases, such as freight, insurance, storage, advertising, etc., are not percentage elements, since they are not based upon values represented by either proceeds or cost.

2. All expenses, other than those expressed by rates, must be added to the net proceeds or subtracted from the gross cost to obtain a result that is a percentage element.

ORAL EXERCISE

1. Commission, $5; rate of commission, ½%; what is the amount of sales?

2. Amount invested, $1000; rate of commission, 2½%; what is the total cost?

3. Amount invested, $600; rate of commission, 3%; what is the commission?

4. Amount remitted to agent, $309; rate of commission, 3%; what is the commission?

5. Total cost, $2100; commission, $100; what is the rate of commission?

6. Commission, $12; rate of commission, 4%; what is the amount of sales?

7. Amount of sales, $500; commission, $10; what is the rate?

WRITTEN EXERCISE

1. I send an agent $1890 to invest in western land at $15 an acre after deducting his commission of 5%. How many acres can he purchase?

2. I paid an agent $91.87½ at 3% for purchasing merchandise. What was the amount of his bill?

3. An agent receives $2647.08 to cover the cost of goods and commission at 8%. What is his commission?

4. A commission merchant sold goods to the amount of $4800, charging 3¼% commission. After paying $25 charges, he invested the balance in raw material and charged 2½% for the investment. How much was invested in raw material?

5. A merchant remitted to an agent $1412.45 with instructions to buy apples at $2.16⅔ a barrel after deducting his commission of 2½%. How many barrels did he buy?

6. An agent sold flour on a commission of 2%. He then bought sugar with the net proceeds, after reserving his commission of 1½%. If his entire commission was $75, and the sugar cost 5¢ a pound, how many pounds did he buy?

7. I remit to my agent in Baltimore $1374 to purchase peaches. After deducting his commission of 3% and $35 for other expenses, how many baskets at 65¢ each will the money purchase?

8. Rule a sheet of paper, copy the following account purchase, and make all necessary extensions:

NEW YORK, April 27, 1912.

Purchased by WM. D. GOODWIN,
For Account of BARRY & OSMUN,
Moline, Ill.

Telephone, 1322 Main.

	6375	yds. calico 4⅞¢				
	3790	" gingham 6⅝¢				
	3780	" gingham 7¼¢				
		Charges				
		Cartage	13	50		
		Commission and guaranty 2½%				
		Amount charged to your acc't				

9. A commission merchant sold 600 baskets of grapes at 16⅔¢ per basket, and after deducting $18.70 for transportation, $5 for storage, and his commission, he remits his principal $68.30 as the net proceeds of the sale. What was his rate of commission?

10. A country merchant forwarded 100 bbl. of apples to be

sold at $1.25 a barrel, the agent to receive a commission of 3% for selling. After paying $3.51 for cartage, and deducting his commission of 1½% for investing, he invested the proceeds in sugar at $7.73⅓ a barrel. How many barrels did he buy?

WRITTEN REVIEW

1. A Chicago merchant sent $2900 to an agent in New Orleans with instructions to purchase cotton at the rate of ⅔%. Find the commission.

2. My agent bought for me sugar at ¾% commission, receiving $300 for his trouble. He sold the sugar at a profit of 25% on the cost. His commission for selling was 1%. How much did I remit to him? How much did he return to me?

3. An agent remitted to his principal $1664.21 as the proceeds of a sale of furniture. Find the sum for which the furniture was sold, if the agent retained a commission of 5%.

4. An agent's commission on a sale of fruit was $31.15, the freight and storage paid by him was $5.20, and he remits $586.65. Find the rate of commission charged by the agent.

5. A commission merchant bought a lot, 50 feet frontage, with money realized from selling wheat at 2½%; the net proceeds of the wheat after deducting the commission being $23,887.50. Find the price per foot paid for the lot.

6. An agent sold a consignment of goods, and took $81 as his commission; he used the remainder in buying goods, deducting $79 for commission. If his rate for selling is the same as the rate for buying, what is that rate?

7. I sent $1800 to my Chicago agent for the purchase of flour at $6.50 per barrel; his charges were, for commission, 3%; drayage, 5¢ per barrel; and freight, $167. How many barrels did he buy, and how much unexpended money was left in his hands to my credit?

8. Prepare an account sales, under the current date, for the following, sold by you, for the account of G. S. Johnson Co., Trenton, N. J.: 100 bbl. Tea Roll flour at $6.50; 50 bbl. Snow Flake flour at $6.25; 75 bbl. XXX flour at $5.50; 80 bbl. Peerless flour at $6.80. Charges: freight, $42.25; cartage, $15.60; cooper-

age, \$7.50; commission, 3%; guaranty 1½%. Proceeds credited.

9. An agent sold a consignment of apples on a commission of 3%. After deducting his commission and reserving a sufficient sum to pay the freight at 20¢ per cwt., he bought flour at \$2.80 per cwt. on a commission of 2¼%. The total commission was \$63. Find the amount of flour bought.

INTEREST

ORAL EXERCISE

1. What is the interest on $1 for a year at 6%? at 4%? at 8%? at 3%? at 5%?

2. When the sum paid for the use of money is 6% annually, what must be paid for the use of $250 for 1 year? for 6 mo.? for 3 mo.? for 4 mo.?

3. A puts $400 in a savings bank and leaves it for 2 years, what can he draw out at the end of the two years besides the amount deposited, if the bank allows an annual rate of 3%?

4. If I borrow $250, and agree to pay 4% a year for the use of the money, how much will be due the lender in 5 years?

461. The premium paid for the use of money is called **interest.** The sum for the use of which interest is paid is termed the **principal,** and the per cent of interest is called the **rate.**

462. The **time** is the period for which the interest is computed. The sum of the principal and interest is called the **amount.**

463. Since interest is computed at a certain rate per cent, the cases of percentage apply, and the additional element of *time* is added. The elements are as follows: the *principal* corresponds to the *base* in percentage; the *interest,* to the percentage; the *rate of interest,* to the *rate;* and the *sum of the principal and interest,* to the amount.

464. In **commercial usage,** 360 days are considered a year, and 30 days a month. 365 days are considered a year in computing exact interest.

Legal Interest

465. **Legal interest** is interest computed at the rate established by law. Illegal interest, or a greater per cent than is sanctioned by law is called **usury.**

The penalties for charging usury in the several states differ, some of which are as follows: forfeiture of the excess interest; forfeiture of all the interest; forfeiture of principal and interest; forfeiture of principal and interest and also fine, or fine and imprisonment.

Contracts on which more than the legal rate can be charged and yet not be usurious are as follows:

1. Paper discounted at a bank—to the extent in which calculated.
2. On ship loans.
3. Demand loans of at least $5000 for which collateral is given.

466. The legal and maximum rates of interest allowed according to the laws of the different states and territories are shown in the following:

Table of Interest Rates

STATES AND TERRITORIES	LEGAL RATE	CONTRACT RATE
Alabama	8%	8 %
Alaska	8 "	12 "
Arizona	6 "	Any "
Arkansas	6 "	10 "
California	7 "	Any "
Colorado	8 "	Any "
Connecticut	6 "	6 "
Delaware	6 "	6 "
Dist. of Columbia	6 "	6 "
Florida	8 "	10 "
Georgia	7 "	8 "
Hawaii	8 "	12 "
Idaho	7 "	12 "
Illinois	5 "	7 "
Indiana	6 "	8 "
Iowa	6 "	8 "
Kansas	6 "	10 "
Kentucky	6 "	6 "
Louisiana	5 "	8 "
Maine	6 "	Any "
Maryland	6 "	6 "
Massachusetts	6 "	Any "
Michigan	5 "	7 "
Minnesota	6 "	10 "
Mississippi	6 "	10 "
Missouri	6 "	8 "
Montana	8 "	Any "
Nebraska	7 "	10 "
Nevada	7%	Any %
New Hampshire	6 "	6 "
New Jersey	6 "	6 "
New Mexico	6 "	12 "
New York	6 "	6 "
North Carolina	6 "	6 "
North Dakota	7 "	12 "
Ohio	6 "	8 "
Oklahoma	7 "	12 "
Oregon	6 "	10 "
Pennsylvania	6 "	6 "
Philippines	6 "	Any "
Porto Rico	6 "	12 "
Rhode Island	6 "	Any "
South Carolina	7 "	8 "
South Dakota	7 "	12 "
Tennessee	6 "	6 "
Texas	6 "	10 "
Utah	8 "	Any "
Vermont	6 "	6 "
Virginia	6 "	6 "
Washington	7 "	12 "
West Virginia	6 "	6 "
Wisconsin	6 "	10 "
Wyoming	8 "	12 "
Canada	5 "	Any "
Cuba	6 "	Any "

Simple Interest

467. Interest is either *simple*, *periodic*, or *compound*. **Simple interest** is interest on the principal only for the given time and rate.

468. There are numerous methods of finding simple interest, but only three are here given: 1. The *6-day method*. 2. *The 60-day method*. 3. *The 6% method*.

The *6-day method* and the *60-day method* are particularly adapted to finding the interest when the time is expressed in days; the *6% method* when the time is expressed in years and months, or years, months, and days.

The 6-Day Method

469. This method is based upon the fact that by pointing off *three* places in the principal gives the interest on any sum of money at 6% for 6 days. Hence, every 6 days equals \$.001 for each dollar, or as many mills as there are times 6 days.

Example 1. Find the interest on \$240 for 27 days at 6%.

$$\begin{array}{r} \$.240 \\ \underline{27} \\ 168 \\ 48 \\ 6\overline{)6.48} \\ \$1.08 \end{array}$$

Solution. Pointing off three places to the left gives \$.24 which is 6 times the interest for 1 day. Multiplying this result by 27 gives \$6.48 which is 6 times the interest for 27 days. Therefore, dividing this result by 6 gives \$1.08, the required interest.

$$\frac{\overset{\$.04}{\$.\cancel{2}\cancel{4}\cancel{0}}\times 27}{\cancel{6}} = \$1.08$$

The work may be greatly shortened by arranging the numbers as shown in the margin and reducing by the methods of cancellation.

470. When it is seen that the time in months and days is a multiple of 6, the result may be obtained by pointing off three places in the principal and multiplying by $\frac{1}{6}$ of the number of days. The result will be the interest at 6%. Thus:

Example 2. Find the interest on \$644 for 42 days at 6%.

$$\$.644\times 7 = \$4.508$$

Solution. Pointing off three places to the left gives \$.644 which is 6 times the interest for 1 da. Since 42 days are 7 times 6 days, 7 times \$.644 must be the interest for 42 days. 7 times \$.644 equals \$4.51, the required interest.

471. Rule. *Point off three places to the left in the principal, multiply by the number of days and divide by 6, the result is the required interest at 6%. Or,*

When it is seen that the time in months and days is a multiple of 6, point off three places and multiply by $\frac{1}{6}$ of the number of days.

NOTE. To find interest at any other rate than 6%, proceed as shown above, divide the interest found by 6 and multiply by the required rate.

ORAL EXERCISE

Find the interest on each of the following at 6%:

1. $400 for 6 da.
2. $550 for 6 da.
3. $333 for 6 da.
4. $500 for 12 da.
5. $250 for 18 da.
6. $650 for 12 da.
7. $1200 for 6 da.
8. $1500 for 21 da.
9. $1200 for 24 da.
10. $3500 for 12 da.
11. $1000 for 48 da.
12. $1200 for 30 da.
13. $4000 for 18 da.
14. $50.50 for 6 da.
15. $1200 for 1 da.
16. $500 for 72 da.
17. $1500 for 3 da.
18. $1000 for 33 da.

WRITTEN EXERCISE

Find the interest on the following:

1. $450 for 23 da. at 6%.
2. $356 for 40 da. at 6%.
3. $242 for 25 da. at 6%.
4. $666 for 42 da. at 5%.
5. $780 for 43 da. at 4%.
6. $250 for 63 da. at 6%.
7. $342 for 50 da. at 4%.
8. $956 for 35 da. at 3%.
9. $1235.50 for 34 da. at 7%.
10. $4325.60 for 45 da. at 6%.
11. $1234.75 for 52 da. at 8%.
12. $1004.33 for 72 da. at 4%.
13. $2002.45 for 90 da. at 6%.
14. $15.555 for 90 da. at 3%.
15. $2845.10 for 78 da. at 5%.
16. $9009.90 for 23 da. at 4%.

1. In figuring interest it is customary to disregard the mills, in final results, if they are less than 5, and to call them 1¢ more if they are 5 or more.

2. In computing the number of days between two given dates, reckon *from* the day upon which the sum was loaned, and *include* the day upon which it was paid.

WRITTEN EXERCISE

Find interest of the following at 6%, exact days:

1. $542 from Jan. 1 to Feb. 25; from Sept. 6 to Dec. 15.
2. $325 from June 8 to Sept. 18; from Aug. 6 to Nov. 24.
3. $650 from Mar. 23 to Sept. 11; from July 25 to Sept. 6.

4. $888 from May 21 to July 24; from July 25 to Sept. 6.
5. $98.50 from Aug. 6 to Nov. 2; from Mar. 5 to Apr. 20.
6. $85.60 from Sept. 9 to Dec. 1; from Aug. 1 to Nov. 1.
7. $859.25 from May 16 to Sept. 3; from June 15 to Aug. 7.
8. $990.60 from May 12 to Sept. 1; from July 16 to Oct. 1.
9. $368.50 from Aug. 24 to Dec. 31; from Mar. 1 to Oct. 20.
10. $345.75 from Sept. 23 to Nov. 23; from Mar. 13 to May 15.

472. From a foregoing exercise it must be clear that when the interest of any sum of money at 6% has been found, the interest of the same sum at 7% may be found by adding $\frac{1}{6}$ of the interest to the result; 8%, by adding $\frac{1}{3}$; 9%, by adding $\frac{1}{2}$; 5%, by subtracting $\frac{1}{6}$; 4%, by subtracting $\frac{1}{3}$, etc.

It is evident that pointing off three integral places in the principal, multiplying by the number of days and dividing by 6 gives the interest on any sum of money at 6% (Sec. 469); dividing by 24, gives 1½%; dividing by 18, gives 2%; dividing by 12, gives 3%; dividing by 9, gives 4%; dividing by 8, gives 4½%; dividing by 4, gives 9%; dividing by 3, gives 12%.

473. The *6-Day Method* is but the "Cancellation Method" which may be illustrated as follows:

Example. What is the interest on $540 for 23 days at 6%?

$$\frac{\$540\times.\cancel{0}\cancel{6}\times23}{\underset{6000}{\cancel{3}\cancel{6}\cancel{0}}}=\frac{\$.540\times23}{6\cancel{0}\cancel{0}\cancel{0}}=\frac{\$.54\times23}{6}=\$2.07.$$

NOTE. The above shows the different steps in the cancellation process, and may be carried still farther for illustrative purposes but this is deemed sufficient. The form is not recommended for practical purposes.

WRITTEN EXERCISE

Using the exact number of days find the interest on:

1. $450 from Jan. 4, 1912, to Mar. 23, 1912, at 6%; at 7%; at 4%; at 5%; at 3%.
2. $2450 from Apr. 3, 1911, to June 15, 1911, at 4%; at 2½%; at 5%; at 9%; at 12%.
3. $3350 from Sept. 4, 1913, to Dec. 1, 1913, at 6%; at 3%; at 9%; at 18%.

4. $540.75 from Nov. 23, 1912, to Jan. 31, 1913, at 8%; at 5%; at 12%; at 10%.

5. $446.60 from July 3, 1913, to Aug. 22, 1913, at 3%; at 4½%; at 5%; at 6%.

6. $909.90 from Feb. 3, 1911, to May 24, 1911, at 5%; at 4%; at 18%; at 15%.

The 60-Day Method

474. The *60-Day Method* or the *Banker's Method* is based upon the fact that by pointing off two places in the principal gives the interest on any sum of money at 6% for 60 days. Hence, every 60 days equals $.01 for each dollar, or as many cents as there are times 60 days.

This method not only teaches how to compute interest rapidly, but it also teaches the comparison or relation of numbers. It is particularly adapted to short periods of time, and is not recommended except when the time is expressed in days.

ORAL EXERCISE

1. What is the interest on $500 for 60 days at 6%? at 3%? at 12%? at 2%?

2. How can you find $\frac{1}{10}$ of a number? What is the interest on $100 for 60 days at 6%?

3. What is the interest on $1200 for 60 days at 6%? for 30 days at 6%? for 20 days at 6%?

4. What fractional part of 60 days is 30 days? 20 days? 15 days? 10 days? 90 days? 80 days?

ORAL EXERCISE

1. How long will it take $1.00 to draw $1.00 interest, or double itself, at 10%? at 5%? at 6%?

2. How can you find the time that it takes $1.00 to double itself at a given rate?

3. How can you find the number of years it takes any sum to double itself at 6%? How many years does it take? How many months? How many days, allowing 30 days to a month? Ans. 6000 da.

4. What part of 6000 days is 60 days? Ans. $\frac{1}{100}$. Then what part of itself does any sum draw at 6% in 60 days?

Example 1. What is the interest on $675.25 at 6% for 93 days?

$6.7525 = int. for 60 da.
3.3762 = " " 30 "
.3376 = " " 3 "
$10.4663

Solution. Removing the decimal point two places to the left gives the interest for 60 da. Now, as 93 da. is 60 da. increased by ½ and $\frac{1}{10}$ of ½ of itself, therefore, $6.7525 increased by $3.3762, ½ of itself, and $.3376, $\frac{1}{10}$ of ½ of itself, gives $10.47, the required interest.

Example 2. What is the interest on $320 for 14 da. at 6%?

$.320 = int. for 6 da.
.320 = " " 6 "
.106 = " " 2 "
$.746

Solution. Removing the decimal point three places to the left gives the interest for 6 da. Since 14 da. is 6 da. plus 6 da. and ⅓ of 6 da., $.32 doubled, increased by ⅓ of itself, or $.75 is the required interest.

475. Rule. *To find the interest by the 60-Day Method remove the decimal point two places to the left in the principal, and increase or decrease the interest for the basis of time by using aliquot parts.*

NOTE. It will be seen that in using the "6-Day Method" the decimal point must be removed *three* places to the left in the principal (Sec. 470).

WRITTEN EXERCISE

1. $1560 for 33 da. at 6%.
2. $3322 for 66 da. at 4%.
3. $85.50 for 90 da. at 5%.
4. $456.60 for 45 da. at 7%.
5. $875.25 for 10 da. at 6%.
6. $458.75 for 18 da. at 6%.
7. $324 for 37 da. at 2%.
8. $896 for 24 da. at 3%.
9. $724 for 43 da. at 3⅗%.
10. $1944 for 29 da. at 4%.
11. $1384 for 86 da. at 4½%.
12. $4968 for 75 da. at 5%.
13. $2472 for 34 da. at 6%.
14. $8488 for 18 da. at 7½%.
15. $378 for 53 da. at 8%.
16. $927 for 91 da. at 9%.
17. $524 for 186 da. at 10%.
18. $824.55 for 23 da. at 6%.
19. $793.27 for 57 da. at 9%.
20. $1624 for 180 da. at 2%.
21. $3284 for 90 da. at 4%.
22. $476 for 300 da. at 12%.
23. $2786 for 18 da. at 2%.
24. $832 for 36 da. at 10%.

Some Short Methods

476. Since interest is the product of three factors: ***principal***, ***rate***, and ***time*** many short methods may be employed in solving problems, based upon the foregoing principles.

As rate is a constant factor in solving problems in interest, it may be ignored for convenience, and the principal in dollars and the time in days be interchanged when necessary.

Example 1. Find the interest on $600 for 63 days at 6%.

$\$63\times\frac{1}{10}=\$6.30$

Solution. Since the interest for 600 days is $\frac{1}{10}$ of the principal, $\frac{1}{10}$ of \$63, or \$6.30 is the required interest on \$600 for 63 days.

Example 2. Find the interest on $300 for 57 days at 6%.

$\$57\times\frac{1}{20}=\$2.85$

Solution. Since the interest for 300 days is $\frac{1}{20}$ of the principal, $\frac{1}{20}$ of \$57, or \$2.85 is the required interest on \$300 for 57 days.

Example 3. Find the interest on $900 for 176 days at 8%.

$\$176\times\frac{2}{10}=\$35.20$

Solution. 8% is $\frac{1}{3}$ more than 6%. Increase the principal by $\frac{1}{3}$ of itself, and the result is \$1200. Interchange principal and time, the problem reads "Find the interest on \$176 for 1200 days at 6%." Since the interest for 600 days is $\frac{1}{10}$ of the principal, 1200 da. is $\frac{2}{10}$ of the principal, and $\frac{2}{10}$ of \$176, or \$35.20 is the required interest on \$900 for 176 da. at 8%.

Example 4. Find the interest on $375.60 for 90 days at 4%.

$\$375.60\times\frac{1}{10}=\$3.756$

Solution. 4% interest is $\frac{1}{3}$ less than 6%. Hence, if we decrease the time by $\frac{1}{3}$ of itself we have 60 da. Then, clearly, 60 da. at 6% is the same as 90 da. at 4%. Therefore, to find the interest on \$375.60 for 60 da. at 6%, remove the decimal point *two* places to the left, and we have \$3.76, the required interest.

NOTE. The above methods will apply advantageously in the solving of many problems, and the ingenious student may be able to devise other short methods equally as good.

WRITTEN EXERCISE

Find the interest on the following, and when possible use a short method:

1. $1000 for 24 da. at 1½%.
2. $600 for 18 da. at 2%.
3. $3600 for 36 da. at 3%.
4. $550 for 72 da. at 4%.
5. $4400 for 72 da. at 4½%.
6. $375.20 for 48 da. at 9%.
7. $250 for 33 da. at 12%.
8. $340 for 15 da. at 6%.
9. $728 for 65 da. at 7%.
10. $636 for 62 da. at 4%.
11. $1024 for 59 da. at 5%.
12. $1000 for 57 da. at 6%.
13. $1275 for 50 da. at 5%.
14. $300 for 47 da. at 6%.
15. $400 for 125 da. at 3%.
16. $850 for 110 da. at 10%.
17. $99.90 for 100 da. at 5%.
18. $3125 for 38 da. at 3½%.
19. $12,345 for 6 da. at 4%.
20. $290 for 72 da. at 12%.

Find the interest of the following by compound subtraction:

21. If $2150 is loaned July 15, 1912, at 6%, what is due Oct. 2, 1912?

22. A note for $685 with interest after 3 months, at 5%, was given Nov. 23, 1911, and paid Sept. 24, 1912. What was the amount due?

23. A merchant buys a bill of goods amounting to $1650 on the following terms: $^n/_{90}$, or less 5% cash. He accepts the latter terms, and borrows the money at 6% to pay the bill. How much does he gain?

24. $456.75 from May 10, 1913, to Dec. 1, 1913, at 7%; at 4%.

25. $7654.50 from July 27, 1912, to March 24, 1913, at 3%; at 9%; at 4%.

26. Sept 24, $560 is loaned at 5%, and Aug. 20, $775 is loaned at 4½. What is the amount due Nov. 30?

27. If a note for $375 on interest at 6%, dated July 14, 1911, be paid Sept. 24, 1911, what amount will be due?

28. Required the amount of $408.60 from May 20 to Oct. 16, 1912, at 5%.

The Six Per Cent Method

477. This method is particularly adapted to finding interest when the time is expressed in years and months, or years, months, and days because of the ease with which the interest of $1 can be computed. Thus:

The interest of $1 for 1 yr. =$.06
The interest of $1 for 1 mo. =$.005
The interest of $1 for 1 da. =$.000⅙

Example. Find the interest on $450 for 3 yr. 7 mo. 15 da. at 6%.

$.18 =int. on $1 for 3 yr.
.035 =int. on $1 for 7 mo.
.0025=int. on $1 for 15 da.
$.2175=int. on $1 for 3 yr. 7 mo. 15 da.
$.2175×450=$97.88, int. on $450 for 3 yr. 7 mo. 15 da. at 6%.

Solution. Since the interest on $1 for 1 year at 6% is $.06 for 3 years it is $.18; for 1 mo. it is $.005, for 7 mo. it is $.035; and for 1 da. it is $.000⅙, for 15 da. it is $.0025. Then $.18+$.035+$.0025=$.2175, the interest on $1 for the given time at 6%, and on $450 the interest is 450 times $.2175, or $97.88. Change to any other rate as in sec. 472.

478. Rule. *Find the interest on $1 for the given time at 6% and multiply the result by the principal.*

1. In using this method, to multiply by ⅔, write ⅓ twice; to multiply by ⅚, take ½ and ⅓.

2. It is sometimes shorter to find the interest on $1 for the given time at any given rate, and multiply by the principal. Thus,

479. To find the interest on $650 for 4 yr. 3 mo. at 4%, take 650 times $.17 (4¢×4¼), or $110.50; on $324 for 2 yr. 6 mo. at 10%, take 324 times $.25 (10¢×2½), or $81; on $420 for 3 yr. 9 mo. at 8%, take 420 times $.30 (8¢×3¾), or $126, etc.

WRITTEN EXERCISE

Find the interest on:

1. $768.50 for 2 yr. 4 mo. at 6%.
2. $1546.85 for 3 yr. 7 mo. at 6%.
3. $2990.90 for 2 yr. 6 mo. 24 da. at 8%.
4. $25,400 for 5 yr. 9 mo. 28 da. at 6%.
5. $660 from Feb. 24, 1911, to May 16, 1911, at 6%.
6. $5540 from Aug. 30, 1912, to Dec. 19, 1912, at 7%.
7. $333.30 from Mar. 16, 1913, to Aug. 14, 1914, at 5%.
8. $10,546.50 for 1 yr. 8 mo. 25 da. at 5%.
9. What is the amount of $3246 for 2 yr. 3 mo. 24 da. at 6%?

10. A man sold his house and lot for $9000; the terms were, $2500 cash on delivery, $3000 in 9 mo., $2500 in 1 yr. 6 mo., and the balance in 2 yr. 3 mo., with 6% interest. What was the whole amount paid?

11. A merchant bought a quantity of goods for $6400, and agreed to pay for them in four equal semi-annual installments, with interest at 5%. What was the total amount of money paid, the first payment being made at the time of the purchase?

12. A merchant bought a bill of goods invoiced at $1800, on 90 days' credit, but subject to a discount of 3% if paid in 10 days; not having ready money, he failed to take advantage of the discount and pays the bill at maturity. How much would he have gained by borrowing the money at 6%?

13. A note of $1250, dated Nov. 20, 1912, and bearing interest at 5% was paid Aug. 15, 1913; what was the amount paid?

14. A merchant marks his goods with two prices, one for cash and the other for 3 months' credit. If the cash price is $40, what ought the credit price to be, money being worth 6%?

Exact Interest

480. **Exact interest** is simple interest when the time is computed for the *exact number* of days on the basis of 365 days to the year, or the *exact* year.

In ordinary business transactions this method is seldom used but is strictly legal. It is used in government calculations, and its use is growing among business men.

481. The exact method of finding interest applies only when the time is less than a year expressed in days, as the interest for whole years is the same as in ordinary interest.

482. It is clear that exact interest is less than ordinary interest as the difference between the common year of 365 days and the commercial year of 360 days is 5 days, or $\frac{1}{73}$ of the common year.

483. **Rule.** *Find the ordinary interest, computed on the 360-day basis, and from the interest thus obtained subtract* $\frac{1}{73}$ *of itself.*

NOTE. Some prefer the method of multiplying the interest for one year by the number of days and dividing the product by 365. In using this method the work may be shortened, in many cases, by cancellation.

WRITTEN EXERCISE

Find the exact interest:

1. $345.35 for 72 da. at 6%.
2. $865.50 for 93 da. at 5%.
3. $654.25 for 47 da. at 7%.
4. $2354 for 124 da. at 10%.
5. $1254 for 142 da. at 4½%.
6. $437.80 for 63 da. at 4%.
7. $500 for 100 da. at 4½%.
8. $685 for 210 da. at 6%.
9. $543.20 for 84 da. at 5%.
10. $756.80 for 92 da. at 7%.
11. $15,865 for 123 da. at 5%.
12. $15,000 for 162 da. at 4%.
13. $4500 from Mar. 24, 1912, to Sept. 1, 1912, at 6%; at 9%.
14. $675.50 from Jan. 6, 1913, to Oct. 4, 1913, at 5%; at 3%.
15. $3458 from July 1, 1911, to Feb. 6, 1912, at 6%; at 3%.
16. What is the interest on four U. S. bonds of $5000 each, at 3% from June 1 to Dec. 1 of the same year?
17. What is the interest on a $10,000 U. S. bond at 2½% from April 1, 1911, to Oct. 1, 1914?

Problems in Interest

484. In every problem in interest there are four distinct elements involved, *principal*, *rate*, *time*, and *interest*. When any three of these are given, the other can be found.

The practical business problem is to find the interest when the principal rate, and time are given. However, the principles involved in the following cases are frequently met with in business and consequently no other part of this book should be more thoroughly mastered.

CASE I

485. The principal, time, and interest given, to find the rate.

Example. At what rate will $450 in 1 yr. 4 mo. 18 da. produce $24.90 interest?

Solution

The interest on $450 for 1 yr. 4 mo. 18 da. at 1% is $6.225.
$24.90 ÷ $6.225 = 4. ∴ the rate is 4%.

486. Rule. *Divide the given interest by the interest on the given principal for the given time at 1%.*

NOTE. When the amount is given, find the interest by subtracting the principal from the amount.

ORAL EXERCISE

Find the rate of interest:

	Principal	*Time*	*Interest*		*Principal*	*Time*	*Interest*
1.	$100	1 yr.	$ 7.	4.	$ 500	3 yr.	$75.
2.	$400	2 yr.	$48.	5.	$ 800	½ yr.	$40.
3.	$200	3 yr.	$36.	6.	$1000	¼ yr.	$10.

WRITTEN EXERCISE

At what rate will:

1. $654 in 8 mo. 24 da. produce $28.78 interest?
2. $1000 in 9 mo. 9 da. produce $54.25 interest?
3. $963 in 1 yr. 4 mo. 12 da. produce $65.81 interest?
4. $1260 in 96 da. produce $13.44 interest?
5. $375 in 1 yr. 5 mo. produce $31.875 interest?
6. $565.50 in 1 yr. 4 mo. 18 da. amount to $588.97?
7. $2640 in 11 mo. 11 da. amount to $2860.06?
8. $525 in 1 yr. 11 mo. 18 da. produce $309.75 interest?
9. I invest $25,000 in a business that pays me $250 a month. What annual rate of interest do I receive?
10. If the use of $3750 for 3 yr. 8 mo. 25 da. is worth $336.25, what is the use of $100 for 1 year worth?

CASE II

487. The principal, rate, and interest given to find the time.

Example. In what time will $500 at 6% produce $52.50 interest?

Solution

The interest on $500 for 1 yr. at 6% is $30.
$52.50 ÷ $30 = 1¾. ∴ the time is 1 yr. 9 mo.

488. Rule. *Divide the given interest by the interest on the given principal at the given rate for 1 year.*

The integral part of the quotient will be years. Reduce the decimal, if any, to months and days (320).

ORAL EXERCISE

Find the time in each of the following:

	Principal	*Rate*	*Interest*		*Principal*	*Rate*	*Interest*
1.	$500	6%	$90.	4.	$800	7%	$112.
2.	$1200	5%	$90.	5.	$600	3%	$42.
3.	$1500	4%	$150.	6.	$750	6%	$75.

WRITTEN EXERCISE

In what time will:

1. $2500 at 6% produce $225 interest?
2. $1500 at 7% produce $54.25 interest?
3. $8750 produce $1260 at 2% a month?
4. $735 at 5% amount to $742.66? at 6%? at 4%?
5. $1301.64 at 5% amount to $1522.92?

6. How long will it take any sum of money to double itself at 2½%? at 3%? at 5%? at 6%? at 10%? at 12½%?

7. If the interest on $900 for 6 mo. 20 da. is $20, what is the interest on $2760 at the same rate per annum for 1 yr. 3 mo. 18 da.?

8. March 11, 1911, there was due on a note for $500, and interest at 6%, $509.60. On what date was the note given?

9. A certain sum of money was put at interest at 8%, Sept. 4, 1900. When was the interest equal to the principal?

10. Sept. 16, 1911, there was due $441.96 on a note of $438 at 5%, exact interest. On what date was the note given?

CASE III

489. The rate, time, and interest given, to find the principal.

Example. What principal will produce $97.50 interest in 1 yr. 3 mo. 18 da. at 6%?

Solution

The interest on $1 for 1 yr. 3 mo. 18 da. at 6% = $.078.
$97.50 ÷ $.078 = 1250. ∴ $1250 is the principal.

490. Rule. *Divide the given interest by the interest on $1 for the given time at the given rate.*

ORAL EXERCISE

Find the principal in the following:

	Interest	*Rate*	*Time*		*Interest*	*Rate*	*Time*
1.	$72	6%	6 yr.	4.	$64	4%	$2\frac{2}{3}$ yr.
2.	$72	4%	$1\frac{1}{2}$ yr.	5.	$45	6%	6 mo.
3.	$50	5%	$1\frac{1}{4}$ yr.	6.	$40	4%	72 da.

WRITTEN EXERCISE

What principal will produce:

1. $346.50 interest in 3 yr. 6 mo. at 6%?
2. $617.50 interest in 1 yr. 8 mo. 12 da. at 6%?
3. $375.60 interest in 2 yr. 4 mo. 6 da. at 4%?
4. $173.97 interest in 4 yr. 4 mo. at 3%? at 5%?
5. $183.75 interest in 147 da. at 9%? at 7%?

6. The interest on a note for 2 yr. 3 mo. 18 da. at 8% was $155.02. What was the face of the note?

7. The sum of A's and B's money on interest for 4 yr. 6 mo. at 6%, gives $5400 interest. How much money has each, if 4 times B's equals A's?

CASE IV

491. The amount, time, and rate given, to find the principal.

Example. What principal will amount to $668 in 2 yr. 10 mo. at 4%?

Solution

The amount of $1 for 2 yr. 10 mo. at 4% = $$1.11\frac{1}{3}$.
$668 ÷ $$1.11\frac{1}{3}$ = 600. ∴ $600 is the principal.

492. Rule, *Divide the given amount by the amount of $1 for the given time, at the given rate.*

ORAL EXERCISE

Find the principal in the following:

	Amount	*Time*	*Rate*		*Amount*	*Time*	*Rate*
1.	$612	2 yr.	6%	4.	$412	9 mo.	4%
2.	$880	2½ yr.	4%	5.	$1020	120 da.	6%
3.	$1175	3½ yr.	5%	6.	$517.50	6 mo.	7%

WRITTEN EXERCISE

What principal will amount to:

1. $1028 in 4 mo. 24 da. at 7%?
2. $1888.2 in 2 yr. 6 mo. 12 da. at 6%?
3. $495.07 in 3 yr. at 8%? At 5%? At 4%?
4. $761.44 in 3 yr. 4 mo. 24 da. at 5%? At 9%?
5. $918.73 in 217 da. at 5½%? At 10%? At 3%?
6. A sum of money at 4½%, simple interest, amounted in 1 yr. 7 mo. 18 da. to $375.725. Find the principal.
7. What sum will amount to $354.09 in 7 mo. at 3% per annum?
8. Find the interest on that sum of money which in 1 yr. 5 mo. 2 da. at 6% amounted to $358.16.
9. Two notes for equal sums, at interest 8 mo. 12 da. at 5½% and 6% per annum respectively, together amount to $5201.25. Find the principal made payable by each note.
10. A man with $25,000 invested in his business makes 12½% profit annually. He sells out and invests the $25,000 at 6% and works on a salary of $1800 per annum. Does he make or lose by the change, and how much?

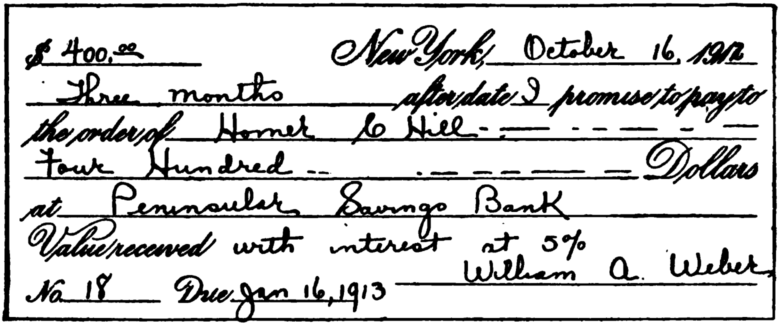
COMMERCIAL PAPER

Promissory Notes

493. A **promissory note,** or a *note,* is a written promise made by one or more persons to pay to another or his order or bearer a certain sum of money at a specified time.

All notes to be valid must contain a *promise to pay.* Hence, the term, "a promissory note." The mere acknowledgment of a debt is not a note.

Form of a Promissory Note



$400.00 New York, October 16, 1912

Three months after date I promise to pay to

the order of Homer C Hill

Four Hundred Dollars

at Peninsular Savings Bank

Value received with interest at 5%

No. 18 Due Jan 16, 1913 William A. Weber

494. The party who makes the note and whose promise is contained therein is called the *maker*, and the party to whom the promise is made is called the *payee*.

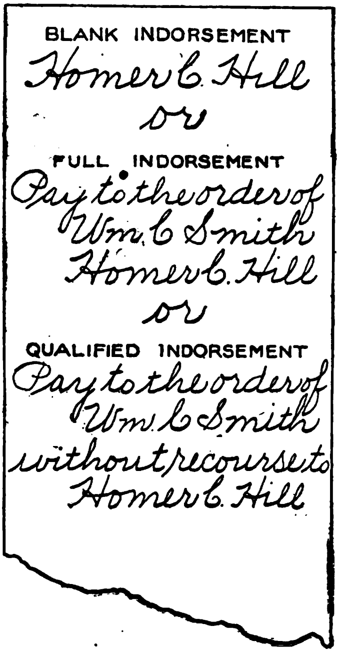
There is no exact form of note required by law; in ordinary business practice it is printed in blank and filled in. It may or may not draw interest, but will not unless provided for in the note. If the note is without interest the words "with interest" are omitted.

495. In the above note William A. Weber is the ***maker;*** Homer C. Hill, the *payee;* and $400, the *face*. The note is ***negotiable*** and may be transferred by Weber to any other person by indorsement.

496. A promissory note or bill of exchange must be made payable to "order" or "bearer" to be negotiable, though these words are not necessary to the validity of the instrument.

1. Notes drawn payable to "bearer" may be transferred by delivery without the indorsement of the holder, but it is not wise to take such paper without his indorsement.
2. If the words "the order of" were omitted from the foregoing note, it would be called a ***non-negotiable*** note and would not be negotiable either by indorsement or delivery.
3. Non-interest bearing notes become interest-bearing after maturity at the legal rate of the place where payable.

497. If in the foregoing note, Homer C. Hill, the payee, wishes to dispose of the paper or obtain the money before the note matures, he indorses it by writing his name upon the back. He is then called the *indorser;* the person to whom he transfers it, the *holder* or *indorsee.* This indorsement may be made in different ways, several of which are here shown:



BLANK INDORSEMENT

Homer C. Hill

or

FULL INDORSEMENT

Pay to the order of
Wm. C. Smith
Homer C. Hill

or

QUALIFIED INDORSEMENT

Pay to the order of
Wm. C. Smith
without recourse to
Homer C. Hill

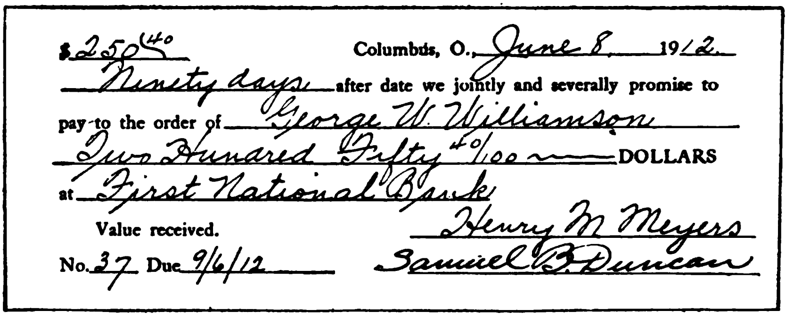
Assuming that Homer C. Hill, the payee, sold the note to Wm. C. Smith, and effected the transfer by writing his name on the back of the note as shown in the first illustration, it is called a *blank indorsement.* This makes the instrument payable to bearer. Or,

Assume that he effected the transfer by a *full indorsement,* an indorsement which specifies that it is made payable to the order of Wm. C. Smith, as shown in the second illustration. If Smith wishes to transfer, he must again indorse to his transferee. Or,

Assuming that he wishes to avoid any personal responsibility, he would indorse as shown in the third illustration. This is called a *qualified indorsement.* By such an indorsement Hill stipulates that he will not be liable if the maker does not pay, but he is held to impliedly warrant that the signatures of the maker and all prior indorsers are genuine. The intent of such indorsement is to pass title only to the instrument.

498. Other forms of indorsement are: "Pay to Wm. C. Smith only," called *restrictive;* "Pay Wm. C. Smith at maturity if his son Henry is attending college," called a *conditional indorsement.* Many other forms of such indorsement as the latter may be given.

499. Other forms of notes are: "On demand I promise to pay," etc., called a *demand note;* a *joint note,* and a *joint* and *several note. Demand notes* draw interest after payment has been demanded. The following illustrates a *joint* and *several note:*



$250 40/100 Columbus, O., June 8, 1912
Ninety days after date we jointly and severally promise to
pay to the order of George W. Williamson
Two Hundred Fifty 40/100 DOLLARS
at First National Bank
Value received. Henry M Meyers
No. 37 Due 9/6/12 Samuel B. Duncan

If the words "and severally" were omitted in the above note, it would be a joint note. The holder may sue and collect of either of the parties whose names are signed to the paper, or of both.

If the note should be drawn in the form of the several note, but be signed by Henry M. Meyers and also by one or more sureties, it would be construed in law to be a joint and several note.

In a few of the states the distinction between joint, and joint and several notes has been abolished, and all notes signed by two or more parties have been declared to be joint and several.

500. A note is *dishonored* when the maker refuses payment or is unable to pay at maturity; a draft or check, if the drawee does not accept or pay it upon presentation.

Drafts, bills of exchange, and promissory notes must be protested if dishonored. Notice of such dishonor must be given, not later than the next day, to the drawer of a bill of exchange and to each endorser if a bill or note, and any drawer or indorser to whom such notice is not given is discharged.

501. Protest is a formal declaration in writing and under seal, of an officer called a Notary Public, certifying to the demand and dishonor to pay a note or bill.

502. Drafts, bills of exchange, and promissory notes formerly had *days of grace*, that is, three days were added to the time stated in which the instrument should become due.

1. As days of grace have been abolished in nearly all the states, all problems in this text omit them.

2. It is generally the law that if the date of maturity of a note or bill

falls on Sunday or a legal holiday, the instrument is payable on the next succeeding business day.

3. In the states in which days of grace are yet allowed, if the last day of grace is a holiday or Sunday, the instrument is payable on the preceding day.

503. The **maturity** of negotiable paper is the date upon which it is due. In all computations of this kind, reckon from the day upon which the paper was drawn or accepted.

When the time is expressed in days, the exact number of days, regardless of the number of months included, is counted; as, a note dated Feb. 24, 1911, and due 60 days after date, would fall due April 25, 1911.

But when the time is reckoned by the month, as it is when the instrument is made payable one or more months after date, the note falls due on the corresponding date of the month in which it is due. Thus, a note dated Jan. 31, 1911, due one month after date, would mature Feb. 28, 1911, where no grace is allowed, and if dated Feb. 28, it would be due March 28.

WRITTEN EXERCISE

Carefully write interest-bearing notes as follows:

1. A time note for $625, due in 60 days, in which you are the maker, John H. Bloom the payee; present date, and interest at 5%.

2. A demand note for $380, in which you are the maker, the First National Bank the payee; present date, and payable at the above named bank, with interest at 7%.

3. Find the amount due at maturity in note No. 1; and in note No. 2, if demand and payment were made 47 days after date.

4. A 60-day note was paid at maturity, March 15, 1911. What was the date of the note?

5. A retail merchant settled with a wholesale firm 28 days after balance on account was due by giving an interest-bearing note for 90 days. If the date of settlement was May 10, 1912, when would the note mature?

Periodic Interest

504. **Periodic,** or *annual interest* is simple interest on the

principal increased by the simple interest on such interest as remains unpaid at the stated period as promised in the note.

Periodic interest signifies interest payable at certain periods of time, but the term as generally applied is made to mean an extra payment of interest as a penalty for not paying when due.

Periodic interest is technically legal, and enforceable in many states. To secure it in any state, it should be specified by contract. In states where it is not sanctioned, money loaners, frequently require the debtors to give coupon notes for the interest, such notes to draw interest if not paid when due.

Example. Find the amount of $1600 for 3 yr. 8 mo., with interest payable annually at 6%.

Solution

Face of note on interest for the given time	=$1600.00
Interest on $1600 for 3 yr. 8 mo. at 6%	= 352.00
3 items of annual interest ($96 each) are unpaid:	
Int. for the first year remains unpaid for 2⅔ yr.	
Int. for the second year remains unpaid for 1⅔ yr.	
Int. for the third year remains unpaid for ⅔ yr.	
Int. on the annual interest = int. on $96 for 5 yr. =	28.80
Total amount due at the end of 3 yr. 8 mo.	$1980.80

505. **Rule.** *To the given principal and its interest to the date of settlement, add the interest of each deferred interest payment from the date it is due to the date of settlement. The result will be the amount due at periodic, or annual interest.*

NOTE. The *period* of computation for periodic interest may be yearly, semiannually, or quarterly as agreed upon.

WRITTEN EXERCISE

1. Find the total interest, payable annually, on $640 for 2 yr. 6 mo. at 6%.
2. Annually, on $1250 for 4 yr. 4 mo. at 6%.
3. Annually, on $4400 for 3 yr. 3 mo. 12 da. at 6%.
4. Semiannually, on $2200 for 2 yr. 3 mo. at 5%.
5. Semiannually, on $465.50 for 2 yr. 3 mo. 18 da. at 6%.
6. Quarterly, on $500 for 1 yr. 8 mo. 24 da. at 4%.
7. Sept. 1, 1912, a man borrows $600 at 6% interest, payable

annually. If nothing is paid until Jan. 16, 1915. how much is due?

8. A coupon note for $1800 is issued May 15, 1911, at 5% interest, payable annually. The coupon notes draw 6% interest if not paid when due. Find the amount that will settle the note Oct. 3, 1914, if the first coupon note was paid when due.

NOTE. Frequently coupon notes draw a higher rate of interest than the note itself, if not paid when due.

9. A note of $1500 was given Apr. 20, 1912, with interest at 6%, payable quarterly. The first three interest payments were paid when due, but no subsequent payments were made until May 10, 1914, at which time note and interest were paid in full. What amount was required?

Compound Interest

506. Compound interest is interest not only on the principal, but its unpaid interest combined at regular intervals.

507. In compound interest the interest is added to the principal at the end of each interest period. Then the amount becomes the new principal for the next interest period.

The period of computation may be yearly, semiannually, or quarterly as agreed upon. The U. S. Supreme Court has decided that compound interest is illegal, and many states do not sanction the collection of it, but if agreed upon by the parties to the contract, the taking of it does not constitute usury.

Most savings banks allow a small rate of compound interest on balances running on deposit for a full interest period. It is also used by life insurance companies, building and loan associations, etc., who wish to compute the income from reinvestment of interest when due. For such work compound interest tables are used.

Example. If interest is compounded annually, what will be the amount of $500 for 3 yr. 4 mo. at 4%?

Solution

$500.00 = first principal.
20.00 = int. on $500 for 1 yr. at 4%.

$520.00 = amount of new principal for 2d year.
20.80 = int. on $520 for 2d year at 4%.

$540.80 = amount of new principal for 3d year.

21.632 = int. on $540.80 for 3d year at 4%.

$562.432 = amount of new principal for ⅓ of an int. period.
7.499 = int. on $562.432 for 4 mo. at 4%.

$569.931 = amount due at end of 3 yr. 4 mo. at 4%.

508. Rule. *Find the amount of the given principal for the first period of time; this amount will be the principal for the second period. Continue in like manner for the full time, and the final amount less the first principal will be the compound interest.*

WRITTEN EXERCISE

1. Find the compound interest of $800 for 2 yr. 3 mc., interest compounded annually at 6%.
2. Find the compound interest on $1200 for 3 yr. 5 mo., interest compounded annually at 5%.
3. What will $750 amount to in 2 yr. 6 mo. 6 da. at 4% compound interest?
4. Find the compound interest of $840 for 2 yr. 3 mo. 27 da. at 4%, payable quarterly.
5. What will be the amount at compound interest of $1250 in 3 yr. 7 mo. 18 da. at 5%, interest payable semiannually?
6. What is the difference betweem the simple and compound interest of $600 for 3 yr. 3 mo. 3 da. at 4%?
7. A man deposited $5000 in a savings bank July 1, 1910, with interest to be added thereto every 6 mo. at the rate of 3%. No withdrawals having been made, what was the balance due Jan. 1, 1912?

509. The computation of compound interest may be abridged by using the following table.

It is important, however, that the student should thoroughly master the method of finding compound interest without the use of the table.

To use the table, multiply the given principal by the number in the table corresponding to the given number of years and the given rate. If the interest is not annual, reduce the time to periods, and the rate proportionally. Thus, 2 yr. 6 mo. by quarterly payments, at 8% is the same as 10 yr. at 2%; and 2 yr. 6 mo. semiannual payments, at 6%, the same as 5 yr. at 3%.

WRITTEN EXERCISE

1. A man deposited $1000 in a savings bank when his son

was 12 years old. How much will this amount to when he is 21 years old, interest being compounded semiannually at 4%?

2. Mr. A. whose life is insured for $2500, pays an annual premium of $75. How much would this amount to at 4% compound interest in 20 years?

3. How much should a man invest at compound interest, 6%, for his son who is now 5 years old so that when he is 21 years of age, he may have $10,000?

Compound Interest Table

Yrs.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	5 per ct.	6 per ct.	Yrs.
1	1.020000	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000	1
2	1.040400	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600	2
3	1.061208	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016	3
4	1.082432	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477	4
5	1.104081	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226	5
6	1.126162	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519	6
7	1.148686	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630	7
8	1.171659	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848	8
9	1.195093	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479	9
10	1.218994	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848	10
11	1.243374	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299	11
12	1.268242	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197	12
13	1.293607	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928	13
14	1.319479	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904	14
15	1.345868	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558	15
16	1.372786	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352	16
17	1.400241	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773	17
18	1.428246	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339	18
19	1.456811	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600	19
20	1.485947	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136	20

Sinking Fund

510. A **sinking fund** is a sum of money set aside at regular periods in such a way that its gradual accumulations will enable it to meet and extinguish a debt at maturity.

It is practically impossible in a corporation or public loan to arrange for the payment of large masses of debt at one time, neither is it wise, hence the *sinking fund* is used with great effect in modern public loans of a limited currency.

The successful application of a sinking fund depends on the regular investment of a given sum to be applied to the reduction of debt and the accumulation of all the interest upon such sums for the same purpose.

ORAL EXERCISE

1. How long, at simple interest, will it take $1 to double itself at 4%? at 5%? at 3%? at 10%?

2. About how long will it take any sum of money to double itself at 4%, compound interest? at 5%? at 6%?

3. If you put $1 at compound interest Jan. 1, and another at the beginning of each year for 20 years, how much would you have at the end of the 20th year, interest being compounded annually at 5%? (See table below).

511. Table showing the sum to which $1, paid at the beginning of each year will increase at compound interest, in any number of years not exceeding 20.

Yrs.	2%.	3%.	3½%.	4%.	5%.	6%.	Yrs.
1	1.0200	1.0300	1.0350	1.0400	1.0500	1.0600	1
2	2.0604	2.0909	2.1062	2.1216	2.1525	2.1836	2
3	3.1216	3.1836	3.2149	3.2465	3.3101	3.3746	3
4	4.2040	4.3091	4.3625	4.4163	4.5256	4.6371	4
5	5.3081	5.4684	5.5502	5.6330	5.8019	5.9753	5
6	6.4342	6,6625	6.7791	6.8983	7.1420	7.3938	6
7	7.5830	7.8923	8.0517	8.2142	8.5491	8.8975	7
8	8.7546	9.1591	9.3685	9.5828	10.0266	10.4913	8
9	9.9497	10.4630	10.7314	11.0061	11.5779	12.1808	9
10	11.1687	11.8078	12.1420	12.4864	13.2068	13.9716	10
11	12.4121	13.1920	13.6020	14.0258	14.9171	15.8699	11
12	13.6803	14.6178	15.1130	15.6268	16.7130	17.8821	12
13	14.9739	16.0863	16.6770	17.2919	18.5986	20.0151	13
14	16.2934	17.5989	18.2957	19.0236	20.5786	22.2760	14
15	17.6392	19.1569	19.9710	20.8245	22.6575	24.6705	15
16	19.0121	20.7616	21.7050	22.6975	24.8404	27.2129	16
17	20.4123	22.4144	23.4997	24.6454	27.1324	29.9057	17
18	21.8406	24.1169	25.3573	26.6712	29.5390	32.7600	18
19	23.2974	25.8704	27.2797	28.7781	32.0660	35.7856	19
20	24.7833	27.6765	29.2695	30.9692	34.7193	38.9927	20

WRITTEN EXERCISE

1. A father put $125 in a savings bank on the day his son was 10 years old, and the same sum on each subsequent birthday; what will the son be worth when 21 years old, compound interest 4%?

2. A certain township borrowed $25,000 to build a union schoolhouse and agreed to pay 5% compound interest. What sum must be invested annually in securities, paying 5% compound interest, as a sinking fund to pay the debt in 10 years?

3. A railroad company bought $500,000 worth of rolling stock payable in 10 years, with 4% compound interest, and bonded its road to secure the indebtedness. What sum of money must be set apart annually, at 5% simple interest, to pay the debt when it becomes due?

NOTE. The money in a *sinking fund* is sometimes used to buy in or pay off some of the debt itself, from time to time under conditions provided; or may be held until the maturity of the debt and then applied to its payment, and, in the meantime, invested in other securities, so that it may increase in amount.

4. A man can rent his house for $480, payable at the beginning of the year, or $125 payable quarterly in advance. Which is the more profitable, money worth 8%?

Present Worth and True Discount

512. The **present worth** of a debt due at some future time without interest is its value now.

513. The **true discount** is the difference between the face of the debt and the present worth.

Theoretically, *present worth* is a sum of money that if placed at interest today for the given time, would amount to the face of the debt. It is clear then that an obligation of $106, due in one year, has the same value as $100, ready money, since the $100, if loaned for a year, considering money worth 6%, will have the same value at the end of the year as the future obligation.

In actual business, true discount is little used, though the principle involved is an important one in the solving of many problems. Banks and merchants generally use *bank discount*, that is, interest on the face of the debt.

514. Present worth and true discount are simply interest elements, and the computations involved come under the cases of interest problems in which the time and rate are given, and the sum due at maturity is the amount.

Example. What is the present worth and true discount of a debt of $465.30 due in 6 mo. 24 da., when money is worth 6%?

Solution.

$1.034 = amount of $1 for 6 mo. 24 da. at 6%.
$465.30 ÷ $1.034 = $450, present worth.
$465.30 − $450 = $15.30, true discount.

515. **Rule.** *Divide the amount of the debt at maturity by the amount of $1 for the given time at the given rate, the quotient will be the present worth. Subtract the present worth from the amount of the debt to find the true discount.*

WRITTEN EXERCISE

1. Find the present worth of a debt of $301.60 due in 8 mo. 21 da., money worth 6% per annum.

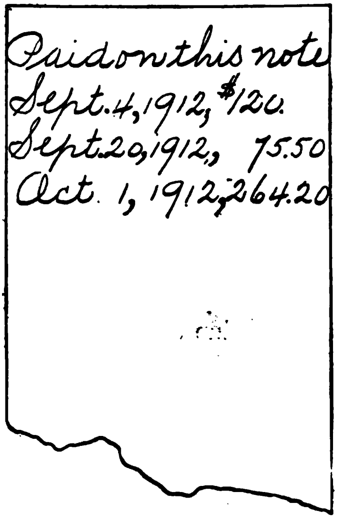
2. A merchant buys a bill of goods amounting to $2480. He can have 4 months' credit, or 5% off for cash; if money is worth 10% to him, what will be his gain at the end of the time by paying cash?

3. Find the sum whose true discount for 3 years is $12 more at 6% than at 5% per annum.

4. A man offers to sell his farm for $8000, or for $9000 payable in three equal installments, at the end of 1, 2, and 3 years, without interest. Considering money worth 6%, what would be gained to the buyer by paying cash?

5. Find the difference between the interest and true discount on $2460 for 9 mo. 24 da. at 5%.

6. A man bought a house and lot for $4800, and after keeping it for 9 mo. sold it for $5200 and took in payment a note due in 6 mo. without interest. If money was worth 6% per annum, what was his actual gain?



Paid on this note
Sept. 4, 1912, $120.
Sept. 20, 1912, 75.50
Oct. 1, 1912, 264.20

PARTIAL PAYMENTS

516. **Partial payments** are payments in part of the amount due on a note, bond, or other interest-bearing obligation.

It frequently happens that the maker of a note, not being able to pay the whole amount at one time, makes several partial payments. These are endorsed on the back of the note as shown in the marginal illustration, or some other words of similar intent. Such payments may be made either before or after maturity of the paper.

Special receipts are sometimes given for partial payments on notes and other interest-bearing obligations.

517. There are *two methods* in regular use for computing interest when partial payments have been made, viz: The *United States Method*, and the *Merchants' Method.*

United States Method

518. The **United States method** of partial payments is taken from the decision of the Supreme Court of the United States and prevails when appeal is made to the courts.

The above method was founded upon the decision of Chancellor James Kent of New York, in 1826. The principle is, that neither interest nor payment shall draw interest. It has been adopted in nearly all the States.

The Supreme Court of the United States has decided that compound interest in unlawful, yet this method upholds it, because it makes interest due as often as payment is made, instead of yearly, semiannually, or quarterly, as in compound interest. However, this method is not ordinarily used except when the time between the date of the note and its payment is more than one year.

519. By *this method* the amount of the note is found to the time when the payment, or the sum of the payments, equals or exceeds the interest due. From this amount the payment, or sum of the payments, is subtracted. This operation is repeated to the time of the next payment and so on.

Example. May 15, 1910, a speculator bought a piece of real estate for $6500, paying $3500 in cash, and giving a note for the remainder at 6% interest, with the privilege of paying all or part of it any time within 2 yr. 6 mo. The following payments were made and indorsed on the note by the payee: March 4, 1911, $550; Oct. 24, 1911, $40; Jan. 18, 1912, $1500; Sept. 6, 1912, $800. How much was due upon the note Nov. 12, 1912?

Solution

Face of note		$3000.00
Interest from May 15, 1910, to Mar. 4, 1911 (9 mo. 19 da.) .		144.50
Amount due Mar. 4, 1911		3144.50
Payment Mar. 4, 1911		550.00
New principal to draw interest from Mar. 4, 1911 . . .		2594.50
Interest from Mar. 4, 1911, to Oct. 24, 1911 (7 mo. 20 da.)	$99.46	
Interest exceeds the payment and a new principal is not formed.		
Interest from Oct. 24, 1911, to Jan. 18, 1912 (2 mo. 24 da.)	$36.32	
Total interest due Jan. 18, 1912		135.78*
Amount due Jan. 18, 1912		2730.28
Sum of the payments since Mar. 4, 1911 ($40+$1500) . .		1540.00
New principal to draw interest from Jan. 18, 1912 . .		1190.28
Interest from Jan. 18, 1912, to Sept. 6, 1912 (7 mo. 18 da.) .		45.23
Amount due Sept. 6, 1912		1235.51
Payment Sept. 6, 1912		800.00
New principal to draw interest from Sept. 6, 1912 . . .		435.51
Interest from Sept. 6, 1912, to Nov. 12, 1912 (2 mo. 6 da.) . .		4.79
Amount due Nov. 12, 1912, the final date of settlement . .		$440.30

520. Rule. *Find the amount of the principal from the date of the note to such time as the payment, or payments, shall exceed the interest then due. From this amount subtract the payment or the sum of the payments made, the remainder will be a new principal.*

Proceed in the same manner with the remaining payments to the time of settlement.

Condensed Form for Written Work

DATES			INT. PERIODS			%'s	PRINCIPALS	INTERESTS	AMOUNTS	PAYMENTS
yr.	mo.	da.	yr.	mo.	da.					
1910	5	15								
1911	3	4		9	19	$.048⅙	$3000.00	$144.50	$3144.50	$ 550.00
1911	10	24		7	20	.038⅓	2594.50	99.46	2730.28	40.00
1912	1	18		2	24	.014	2594.50	36.32	1235.51	1500.00
1912	9	6		7	18	.038	1190.28	45.23	440.30	800.00
1912	11	12		2	6	.011	435.51	4.79	Ans.	

The above outline, taken from the foregoing illustrative problem, shows a condensed form for simplifying written work.

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* If it can be determined mentally that the interest is larger than the payment, find the interest at once to the next date, and subtract the sum of the payments from the amount obtained as though no payment had been made.

Use *compound subtraction* in solving the following problems by the United States method.

WRITTEN EXERCISE

1. How much is due August 15, 1912, on an interest-bearing promissory note for $250 at 6%, dated Buffalo, N. Y., June 1, 1910, on which $30 was paid December 24, 1911, and $10 January 5, 1912?

2. Draw a note for $600, given July 1, 1910, for 3 years at 6% and compute the interest due Oct. 1, 1914; the following indorsements having been made: Sept. 1, 1910, $100; July 1, 1911, $125; Dec. 31, 1913, $200.

3. The following payments were made on a demand note of $1000 drawn March 1, 1910, with interest at 6%; June 1, 1910, $300; Sept. 1, 1910, $10; Jan. 1, 1911, $100; June 1, 1911, $400. How much would remain due on June 1, 1912?

4. On the note below indorsements were made as follows: Nov. 25, 1909, $500; July 18, 1910, $50; Sept. 1, 1910, $600; Dec. 28, 1910, $75. What was the balance due Feb. 10, 1911?

$1750.00 New York, Nov. 22, 1907.

Two years after date I promise to pay to the order of Spencer & Williams, Seventeen Hundred Fifty Dollars, at the Chemical National Bank, with interest at 7%.

Value received. Russell B. Williamson.

5. On a mortgage note for $4000, dated April 4, 1909, and bearing interest at 6%, the following payments were made: Jan. 2, 1910, $1000; May 24, 1910, $25; Sept. 30, 1910, $50; Feb. 20, 1911, $1800. What was due Dec. 16, 1911?

6. On the note below indorsements were made as follows: Jan. 4, 1910, $200; Jan. 4, 1911, $100; June 5, 1911, $500.

$1800.00 Wheeling, W. Va., Aug. 2, 1909.

Two years after date I promise to pay S. P. Dunham & Co., or order, Eighteen Hundred Dollars, at their office, with interest at 8% until paid.

Value received. George W. Dean.

What amount was due Jan. 3, 1912?

Merchants' Method

521. Business men and banks usually settle notes and accounts running for one year or less, upon which partial payments have been made, by the *Merchants' Method.*

The merchants' method is based on custom rather than on legal authority, and is a more equitable one than the United States method. The method provides that:

1. *The amount of the note shall be found from the date of issue to the time of settlement.*
2. *The amount of each payment shall be found from its date to the time of settlement.*
3. *The sum of the amounts of the payments is then subtracted from the amount of the principal, and the result is the amount due.*

Example. On a note for $700, dated Oct. 15, 1911, due in one year, with interest at 6%, the following payments have been made: Mar. 9, 1912, $300; June 1, 1912, $250. Find the amount due at maturity, merchants' method.

Solution

Face of note, or principal, Oct. 15, 1911		$700.00
Interest on the note to Oct. 15, 1912 (1 yr.) at 6%		42.00
Amount due at maturity if no payments had been made		$742.00
First payment, Mar. 9, 1912	300.00	
Interest on the same to Oct. 15, 1912 (220 da.)	11.00	
Second payment, June 1, 1912	250.00	
Interest on the same to Oct. 15, 1912 (136 da.)	5.67	566.67
Balance due, Oct. 15, 1912		$175.33

NOTE. As the interest periods in this method are usually for less than a year, compute the interest for the *exact number of days* in using the method, unless otherwise stated. Merchants' method is sometimes used when the note runs for more than a year, although it is to the disadvantage of the payee.

WRITTEN EXERCISE

1. What was due Aug. 1, 1911, on a note for $800 bearing 6% interest, dated Oct. 1, 1910, and having the following in-

dorsements: Dec. 6, 1910, $250; Jan. 24, 1911, $50; Mar. 20, 1911, $300; May 1, 1911, $60?

2. On a note for $1500, dated May 12, 1911, payable on demand, with interest at 5%, payments were made as follows: June 10, 1911, $220; Sept. 1, 1911, $350; Dec. 18, 1911, $500. What was due at settlement, Feb. 20, 1912?

3. On July 6, 1910, R. D. Ruskin sold the stock and good-will of his grocery business to C. D. Chase for $8500. Chase paid $6000 cash, and gave a note due in 9 months, bearing 8% interest for the balance. Before the maturity of the note he made the following indorsements: Aug. 24, 1910, $500; Oct. 1, 1910, $1200; Jan. 1, 1911, $400. How much was due at maturity?

4. On the note below indorsements were made as follows: Aug. 4, 1911, $125; Sept. 30, 1911, $75; Nov. 24, 1911, $350. How much was due at maturity?

$700.00 Scranton, Pa., June 18, 1911.

One year after date I promise to pay Wm. L. Smith, or order, Seven Hundred Dollars with interest at 6%.

Value received. Leonard B. Ivens.

5. On Sept. 1 you borrowed $10,000 at the First National Bank on your demand note secured by a deposit of 100 shares of Union Pacific Railroad Stock at $150. June 1, the following year, the bank demanded a return of the loan; in the meantime you had made indorsements as follows: Oct. 20, $3000; Dec. 16, $2500; Mar. 18, $1500. On demand you paid the remainder of the note and interest, and withdrew the collaterals. What was the amount of the last payment, money being loaned at 4%?

6. On an open account of $150, due April 1, a merchant charges 6% interest, per annum. How much would settle the account Dec. 1, if the following payments had been made: June 1, $40; Sept. 1, $50; Nov. 1, $20? (Compound subtraction.)

522. New Hampshire and Connecticut have rules of their own for solving problems in partial payments. In these states it is left for the teacher to present the rule.

WRITTEN REVIEW

Find the following:

1. $\frac{3}{4}\%$ of $1250.
2. $6\frac{2}{3}\%$ of $420.
3. 108% of 25 men.
4. $888\frac{8}{9}\%$ of 99999.
5. .11% of 10000.
6. .3% of 5.
7. .1% of .1
8. .2% of 1750.
9. 4.4% of 875.
10. $3\frac{1}{3}\%$ of $18\frac{3}{4}$.

11. Find the exact interest on $2150 from March 12, to April 5, at 6% per annum.

12. Find the date of a note of $760 at 8%, simple interest, which, when it matured December 1, 1911, amounted to $919.60.

13. If the interest is $12.57, the time 8 mo. 2 da., and the rate per annum $5\frac{1}{2}\%$, what is the principal?

14. Bought a farm for $10,000, payable $\frac{1}{2}$ cash, the remainder in one year, with interest at 6%. I sell immediately for $12,000, payable in 3 months, with interest at 4%. What is my present gain, money being worth 5% per annum?

15. If the interest on $100 for 3 yr. 4 mo. at 6% is $20, for how long must $350 remain at interest at $4\frac{1}{2}\%$ to produce $42?

16. A contractor sends in a tender of $20,000 for a certain work, a second sends in a tender of $19,000 but stipulates to be paid $2000 every 3 months. Find the difference between the tenders, supposing the work in both cases to be finished in 2 years, and money to be worth $7\frac{1}{2}\%$, simple interest.

17. A merchant buys a stock of goods for $5640 and gets 3% and 5% off from the list price, and borrows the money at 6% for 60 days to pay for the same. What per cent does he make if he sells the goods at the list price?

18. What is .9% of 450? 1250% of $2400? $\frac{3}{4}\%$ of $\frac{7}{8}$?

19. What sum must a person save annually, commencing at 21 years of age, so that he may be worth $25,000 when he is 40 years old, if he gets 6% compound interest on his money?

20. At what rate per cent will $1100 amount to $1254 in 3 yr. 1 mo. 10 da.?

21. Which is more profitable, to buy flour at $7.50 a barrel on 6 mo. credit, or at $7 cash, money being worth 6%?

22. A merchant desiring to pay a bill of $1650, 3 months before it was due, was allowed a discount equivalent to the interest on the face of the bill for the unexpired time at 6% per annum. How much greater was this discount than the true discount?

23. J. H. Roberts borrowed $1200 at 6%, and agreed to pay interest annually. Having paid no interest, he wishes to settle at the end of 3 yr. 3 mo. 20 da. What is the amount due?

24. A merchant buys of a New York firm 85 barrels of sugar at $18.50 per barrel, giving in payment a 90-day note, dated June 20. If the New York firm has the note discounted July 2, following, what will be the proceeds?

25. What sum of money at interest for 1 yr. 3 mo. 5 da. at 6% per annum, will amount to $258.20?

26. A man agrees to pay for a farm $1600 a year, for 5 years. What sum paid now would be equivalent to this price, money being worth 4% per annum, payable yearly?

27. The amount of a certain principal in a certain time, at 5% is $517.20, and the amount for the same time at 8% is $539.52. Find the time.

28. A merchant bought $1750 worth of goods at a discount of 25% and 20% from list prices, and sold them at the regular retail price on 6 months' credit. Money being worth 6%, what per cent profit did he make?

29. April 15 a Philadelphia speculator bought 5000 bu. of wheat at 90¢; May 12 he sold 2000 at 96¢; May 30, 1000 at 88¢; and July 24, the remainder at $1. Allowing interest at 6%, and storage charges 1¢ per bushel, what was his gain?

30. A merchant keeps his deposit with the First National Bank, and has left with the bank certain securities valued at $1500 for overdrafts, the bank charging 6% on all overdrafts that were not settled within 3 days. June 6, there was an overdraft of $80 that was settled June 15; June 25, $235.40, that was settled July 14; July 20, $450.20, that was settled July 22; July 28, $547.50, that was settled Aug. 15. How much interest did the merchant have to pay?

31. Two highway districts buy a road machine for $285, and pay the freight from the manufactory, one district paying

3/7 and the other 4/7 of the entire cost. The cost to the first district being $127.50, how much was charged for freight?

32. Find the rate of income on 4% interest-bearing bonds bought at $115.

33. Bought goods to the amount of $1500, 1/5 payable in cash, 1/3 in two months, 1/3 in four months, and the balance in 6 months. What sum paid down would discharge the whole debt, money being worth 6%?

34. I sent my agent $6724 to be invested in wheat at $1.02½ per bushel, allowing him a commission of 2½% on the purchase. I paid for storage 2¢ per bushel per month, and 1% per month for the use of the money. After three months my agent sold the wheat at $1.20 per bushel, charging 2% commission for making the sale, and took in payment a note for the amount for one month which I had discounted at a bank at 6%. Did I gain or lose, and how much?

EXAMINATION TEST

Selected from State Papers

1. If 25% of what I receive for an article is profit, what is my per cent of gain on the cost of the article?

2. A man is offered a house and lot for $3000 on 1 yr's time or 5% off for cash. He chose the former, did he gain or lose and how much, money being worth 6%?

3. If an article had cost 12½% more, my profit would have been 5% less. What was my per cent gain?

4. Bought 50 gross of buttons for 25%, 10%, and 5% off, and disposed of the lot for $35.91 at a profit of 12%. What was the list price of the buttons per gross?

5. If equal sums be put at interest for 1 yr. 8 mo. 12 da. at 5½% and 7% per annum, the difference in interest received on the two principals will be $7.65. Find the sum invested in each case.

6. A speculator sold a piece of real estate at a profit of 30%. He then put with the sum received $7,000 and bought another piece, which he sold at a gain of 37½%. If his total gain was $10,500, what was the cost of the first piece?

EQUATIONS AND CASH BALANCES

Equation of Accounts

523. An **account** is a record of business transactions.

524. **Equation of accounts** is the process of finding the mean time when the balance of an account may be paid without loss to the debtor or creditor.

When several items in a personal account are not paid when due, it often happens that the holder of the account suffers a loss; and when items are paid before maturity, the holder usually realizes a gain. To equitably adjust these losses and gains, accounts are equated.

When an account has but one side it usually means *Equation of Payments*. *Equation of Accounts* includes both sides.

ORAL EXERCISE

1. The interest of $25 for 1 year equals the interest of $1 for how many years at the same rate?

2. The interest of $40 for 2 years equals the interest of $20 for how many years at the same rate?

3. If I borrow $50 for 1 year, how long should I lend $100 to balance the interest?

Analysis. Since the interest on $50 for 1 year equals the interest of $1 for 50 times 1 year, or 50 years, the interest of $100 equals the interest of $1 for $\frac{1}{100}$ of 50 years, or $\frac{1}{2}$ year = 6 mo.

525. **The term of credit** is the time which must elapse before a debt becomes due.

526. **The average term of credit** is the time which must elapse before several sums due at different times may be equitably discharged in one sum.

527. **The equated date** is the date at which the several sums due may be paid together without loss to either party.

528. The **focal date** is an assumed date of settlement, or the date when several accounts are compared for the purpose of finding the actual due date.

Any date may be used as a focal date or any rate of interest may be used and the result will be the same. We recommend the use of the latest date, and 6% as the rate of interest.

Retail accounts are not often equated, only wholesale, commission, and foreign accounts.

All accounts, or notes, even though they contain no interest-bearing clause, bear interest after maturity.

ORAL EXERCISE

1. How long will it take $40 to be equivalent to the use of $80 for 2 months?

2. If I use $10 of A's money for 6 months, how much of my money should A use for 2 months in return for the favor?

3. The interest on $50 for 3 months plus the interest on $50 for 4 months is equal to the interest on $100 for how many months?

4. If I pay one half of an account 20 days before the whole account is due, how long after the whole account is due may I have in which to pay the balance?

5. C owes D $600 and pays $300 30 days before the account is due. How long after the account is due may D have in which to pay the balance?

6. If I am entitled to 60 days' credit on $200, how long a term of credit should I have on the remainder if I pay $100 at once?

Simple Accounts

529. To find the equated date when the items are all debits or all credits and no term of credit is given.

Example. On what date may the total of the following account be paid without interest?

S. B. Yard & Co.

1911 May	10	To Mdse.		240							
	30	To Mdse.		300							
June	16	To Mdse.		200							

DATE	AMOUNT	DAYS	INTEREST
May 10	$240	37	$1.48
30	300	17	.85
June 16	200	0	
	$740		$2.33

The amount of the account = $740.
Int. on $740 for 1 da. = 12⅓¢
$2.33 ÷ $.12⅓ = 19, or 19 da.
June 16–19 da. = May 28, 1911, the due date.

Solution. Take the latest date June 16, as the focal date. The *first* charge, $240, would then draw interest from May 10 to June 16, or 37 da. The interest on $240 for 37 da. = $1.48. The *second* charge, $300, would draw interest from May 30 to June 16, or 17 da. The interest on $300 for 17 da. = $.85. The *third*, being dated on the assumed date of settlement, does not draw interest. It is clear then that if settlement was made June 16, the loss to the holder of the account would be the sum of the interests, or $2.33, and that the loss per day would be the interest on $740 for 1 da., or 12⅓¢. Therefore, $2.33 ÷ $.12⅓ = 19, or 19 days before June 16 the holder would lose nothing. June 16–19 da. = May 28, 1911, the due date.

Proof. To prove that $740 may be paid May 28 without interest, it is necessary to show that the interest on the items *dated before May 28*, the equated date, is offset by the discount on the items *dated after May 28*.

Sometimes there is a variance of 1¢ or more between the interest and discount in proving the work. This is due to the fact that the equated date cannot always be correct to the fraction of a day, as we take no account of fractional days if it be less than ½ in finding the equated date.

530. From the foregoing illustrative problem the following rules may be deduced:

1. Select the latest date as the focal date.

2. From the focal date, as a standard, compute the time in exact days, and interest of each of the other items.

3. Divide the sum of the interest by the interest of the sum of items for one day. The quotient will be the number of days' average time from the focal date.

4. Count back the average time from the focal date, and the result will be the equitable date of payment.

If the fraction in the quotient is ½ day or more, 1 day is added; if less than ½ day, it is rejected. If the latest due date is used as the focal date its item has no interest, but it must be included in the sum of debts.

There are good reasons why the latest date should be selected in equating accounts as it removes the objection that is often raised that the account is not likely to be settled before it was made.

WRITTEN EXERCISE

In each of the following accounts find the equated date. Prove the work and assume that all the dates are in 1911.

1. *F. S. Stokes, Dr.*

Jan. 2,	To Mdse.....	$ 80.
Jan. 27,	To Mdse.....	120.
Feb. 4,	To Mdse.....	200.
Feb. 20,	To Mdse.....	100.

2. *F. B. Moore, Dr.*

June 12,	To Mdse.....	$240.
June 24,	To Mdse.....	360.
July 8,	To Mdse.....	400.
July 30,	To Mdse.....	500.
Aug. 15,	To Mdse.....	600.

3. *W. P. Gladen, Dr.*

Mar. 1,	To Mdse.....	$ 50.
Apr. 1,	To Mdse.....	75.
May 20,	To Mdse.....	150.
May 25,	To Mdse.....	120.

4. *John E. Gill, Dr.*

Sept. 18,	To Mdse.....	$540.
Oct. 10,	To Mdse.....	200.
Nov. 12,	To Mdse.....	580.
Nov. 30,	To Mdse.....	800.
Dec. 31,	To Mdse.....	900.

5. Bought a bill of goods Sept. 1, amounting to $800, of which ¼ is to be paid in cash, ½ in 3 months, and the balance in 4 mo. When may the whole be equitably paid in one sum?

531. To find the equated date when the items are all on one side and terms of credit are given.

Example. On what date may the total of the following account be paid without interest?

George E. Hall

1911 Mar.	24	To Mdse., 10 da.	600				
April	16	To Mdse., 20 da.	500				
	30	To Mdse., 15 da.	800				
May	24	To Mdse., 30 da.	400				

Solution

Date	*Term of Credit*	*Maturity*	*Item*	*Interest Period*	*Interest*
Mar. 24	10 da.	Apr. 3	$600	81 da.	$8.10
Apr. 16	20	May 6	500	48	4.00
30	15	15	800	39	5.20
May 24	30	June 23	400	0	
			$2300		$17.30

After adding the term of credit to each item as indicated in the account, assume June 23, the latest date, as the *focal date.*

It will be seen that the amount of the account equals $2300. Hence, it must be clear that if settlement was made June 23, the loss to the holder of the account would be the sum of the interests, or $17.30, and that the loss per day would be the interest on $2300 for 1 da., or $.38⅓. Therefore, $17.30 ÷ $.38⅓ = 45, or 45 da. before June 23 the holder would lose nothing.

June 23 − 45 da. = May 9, 1911, the due date.

Proof. To prove that $2300 may be paid May 9 without interest we must show that the interest on the items dated before May 9 is offset by the discount on the items dated after May 9.

Items dated before May 9.

Date	*Interest Period*	*Item*	*Interest*
Apr. 3	36 da.	$600	$3.60
May 6	3 "	500	.25
			$3.85

Items dated after May 9.

Date	*Interest Period*	*Item*	*Interest*
May 15	6 da.	$800	$ .80
June 23	45 "	400	3.00
			$3.80

$3.85 − $3.80 = $.05. As this is less than the interest on the amount of the account for ½ day, therefore the solution must be correct.

532. From the foregoing illustrative problem the following rule may be deduced:

To the date of each item add the term of credit and proceed as in 530.

WRITTEN EXERCISE

In each of the following accounts find the equated date and prove the work. Assume that all the dates are in 1911.

1. *S. P. Dunham & Co., Dr.*

Mar. 2, To Mdse., 10 da., $200
Apr. 1, To Mdse., 30 da., 300
Apr. 10, To Mdse., 20 da., 100
Apr. 25, To Mdse., 60 da., 150

2. *Brown, Wilson & Co., Dr.*

Sept. 20, To Mdse., 15 da., $500
Oct. 1, To Mdse., 10 da., 750
Oct. 18, To Mdse., 20 da., 600
Nov. 15, To Mdse., 6 da., 800

3. *J. P. Everitt & Co., Dr.*

May 1,	To Mdse., 10 da.,	$100
May 21,	To Mdse., 20 da.,	125
May 28,	To Mdse., 30 da.,	140
June 10,	To Mdse., 60 da.,	80
June 18,	To Mdse., 40 da.,	160

4. *W. J. Schultz & Co., Dr.*

July 1,	To Mdse., 2 mo.,	$150
July 6,	To Mdse., 1 mo.,	100
July 21,	To Mdse., 30 da.,	220
July 28,	To Mdse., 60 da.,	140
July 31,	To Mdse., 10 da.,	60

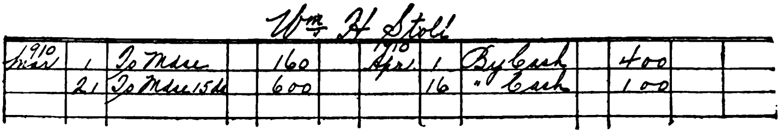
5. I bought goods as follows: Oct. 5, 1911, on 4 mo., $450; Nov. 8, on 2 mo., $600; Dec. 16, on 3 mo., $700; and Jan. 5, 1912, on 3 mo., $800. Find the equated time for one payment.

6. A merchant has the following charges against a customer: Sept. 8, 1910, $875 on 90 da.; Oct. 6, 1910, $700 on 30 da.; Nov. 10, 1910, $1000 on 60 da.; and Dec. 1, 1910, $500 on 4 mo. Find the equated time for one payment.

Compound Equations

533. To find the equated date when an account has both debits and credits.

Example. Find the equated date for paying the balance of the following account:



Wm H Stoll

1910 Mar	1	To Mdse		160		1910 Apr.	1	By Cash		400		
	21	To Mdse 15 da		600			16	" Cash		100		

Solution

Take April 16, 1910, as the focal date.

Debits

Date	*Term of Credit*	*Due*	*Items*	*Int. Period*	*Int.*
Mar. 1	00 da.	Mar. 1.	$160	46 da.	$1.23
Mar. 21	15 da.	Apr. 5.	600	11 da.	1.10
			$760		$2.33

Credits

Date	*Term of Credit*	*Due*	*Items*	*Int. Period*	*Int.*
Apr. 1	00 da.	Apr. 1	$400	15 da.	$1.00
Apr. 16	00 da.	Apr. 16	100	00 da.	.00
			$500		$1.00

\$760 − \$500 = \$260, the balance of the account. \$2.33 − \$1.00 = \$1.33, the interest due the creditor on Apr. 16.

The interest on \$260 for 1 da. = \$.04⅓.

\$1.33 ÷ \$.04⅓ = 31, the number of days.

If the account were settled Apr. 16 there would be ***interest for 31 days*** due the holder of it. Therefore the balance of the account is due 31 days before Apr. 16. Apr. 16 − 31 da. = Mar. 16, the equated date.

Proof. The interest on \$160 from Mar. 1 to Mar. 16 = \$.40. Discount on \$600 from Mar. 16 to Apr. 5 = \$2.00. \$2.00 − \$.40 = \$1.60.

The discount on \$400 from Mar, 16 to Apr. 1 = \$1.07. Discount on \$100 from Mar. 16 to Apr. 16 = \$.52. \$1.07 + \$.52 = \$1.59. Therefore, the difference between the interest and discount of the debit side is practically the same as the total discount of the credit side, hence the solution must be correct.

534. General Principles. When no term of credit is mentioned, the transaction is understood to be for cash, and the payment is due at once.

If any item of an account is not paid when due, it shows that the holder of the account suffers a loss, and if an item is paid before it is due, it shows that the holder of the account realizes a gain.

A loss is offset by dating back, a gain is offset by dating forward.

535. Rule. *Find the interest of each item for the time from the focal date to the due date of each item, and divide the difference between the sum of the debit interest and the sum of the credit interest by the interest of the balance of the items for 1 day; the quotient will be the number of days between the focal date and the date of settlement.*

When the balance of account and interest are both on the same side, add this to the focal date; if on opposite sides, count back; the result will be the date of settlement.

WRITTEN EXERCISE

Find the equated date of the following accounts:

1.

J. L. Stafford

1911 Jan	10	To Mdse		160	1911 Jan	20	By Cash		100		
″	15	″ Mdse		200	Feb	6	″ Cash		350		
	29	″ Mdse		300							

2. *John H. Perrine*

1913					1913				
Mar.	1	To Mdse.	200		Apr.	1	By Cash	400	
	23	To Mdse.	300			16	By Cash	100	
	30	To Mdse.	150		May	1	By Cash	200	
Apr.	10	To Mdse.	250						

3. *Fred. R. Peppler & Co.*

1910					1910				
Apr.	10	To Mdse.	180		Apr.	27	By Cash	140	
	24	To Mdse., 10 da.	220			30	By Cash	160	
May	1	To Mdse., 1 mo.	350		May	10	By Cash	250	
	16	To Mdse., 2 mo.	200						

4. *P. R. Brown & Son*

1912					1912				
June	4	To Mdse., 30 da.	600		June	16	By Cash	200	
	21	To Mdse., 40 da.	300			27	By Cash	300	
July	1	To Mdse., 10 da.	200		July	20	By Cash	440	
	21	To Mdse., net	240		Aug.	1	By Cash	160	
	29	To Mdse., 30 da.	200						

5. *Elmer E. Wyckoff*

1912					1912				
Dec.	16	To Mdse.	125	50	Dec.	26	By Cash	100	
1913					1913				
Jan.	1	To Mdse., 10 da.	76	80	Jan.	15	By 30-da. note with int.	100	
	16	To Mdse., 15 da.	220	55					
	27	To Mdse., 20 da.	150	60		20	By Cash	100	
						28	By 30-da. note without int.	200	

NOTE. Interest-bearing notes have no term of credit.

6. *M. Cramer's Sons*

1911					1911				
Oct.	16	To Mdse., net	400		Oct.	30	By Cash	350	
	28	To Mdse., 1 mo.	300		Nov.	28	By Cash	300	
Nov.	22	To Mdse., 30 da.	600		Dec.	20	By Cash	500	
Dec.	24	To Mdse., net	800			24	By 60-day note with int.	900	

NOTE. Any items which are paid when due may be omitted in equating the account.

Equation of Account Sales

536. An **account sales** is an itemized statement of goods sold by an agent for his principal, with his expenses and charges.

537. The method of averaging *account sales* is the same as that used in averaging an account. The *invoice,* or *sales* form the credit side of the account, and the *expenses* the debit side.

Commission and guarantee are sometimes considered due on the date of the last sale, but when there are long intervals between sales, commission and guarantee are generally considered due on the *average due date of the sales.* When goods are sold for cash an account sales is seldom averaged.

WRITTEN EXERCISE

1. Copy and complete the following account sales and find the equated date for paying the net proceeds. Consider the commission as due on the date of the last sale.

Pittsburg, Pa., July 16, 1912.

Sales for the Account of C. J. Heppe & Son,
Trenton, N. J.

By MACDONALD-CAMPBELL COMPANY,
Commission Merchants.

1912		Sales.				
June	10	500 bu. wheat..............$.85	...			
	25	250 " oats, 30 days........ .40	...			
July	1	600 " corn, 60 days....... .52	...			
	10	500 " wheat.............. .86	...		...	
		Charges.				
June	1	Freight....................	15			
	4	Cartage....................	35			
July	8	Cash advanced............	500			
		Com. and Guarantee 5%..	...	..		
		Net proceeds....			...	..

2. Arrange the following in the form of an account sales and find when the net proceeds are due by equation. Consider the commission as due on the average due date of the sales:

F. W. Messlor & Co., Minneapolis, Minn., sold for the account and risk of W. E. Thoma, Chicago, Ill., 1000 bbl. flour as follows: Sept. 1, 1911, 300 bbl. Tea Roll at $5.75, cash; Sept. 24, 1911, 500 bbl. Snowflake at $6, 30 da.; Oct. 16, 1911, 200 bbl. Roller Process at $6.10, cash. Charges were as follows: Aug. 25, 1911 freight and cartage, $225; Aug. 28, 1911, inspection, $15; Oct. 16, 1911, storage, $45; commission and guaranty, 3%.

Cash Balance

538. It is the usual custom of business houses to render a statement, at certain periods, to their customers who buy on account. These statements are usually made monthly, quarterly, semiannually, or annually.

539. The amount due upon an account at any given date is called the **cash balance** of an account.

In settling mercantile accounts, *if interest is not charged,* the cash balance is the difference between the sides of the account. *If interest is charged* on the account, the cash balance is the difference between the two sides of the account after interest has been added to overdue items and discount deducted from items not due at the date of settlement.

Interest charges are usually regulated by custom, or previous agreement. Wholesalers, as a rule, charge interest on all overdue accounts, but retailers do not. However, they frequently close their personal accounts at the end of the year and then charge interest on the balances brought down from the date of closing to the date of settlement.

540. This section treats only of accounts in which interest is charged. The exact time in days is found, and the days are regarded as 360ths of a year.

Example. What is the cash balance due on the following account Sept. 1, 1910, interest being charged on overdue items at the rate of 6%?

William A. Steele

1910					1910				
July	1	To Mdse., 20 da.	600		July	17	By Cash		500
	16	To Mdse., 30 da.	500			31	By Cash		300
	30	To Mdse., 25 da.	250		Aug.	16	By Cash		350

Solution

Debits

Date	*Term of Credit*	*Maturity*	*Items*	*Interest Period*	*Interest*
July 1	20 da.	July 21	$600	42 da.	$4.20
16	30 da.	Aug. 15	500	17 da.	1.42
30	25 da.	24	250	8 da.	.33
			$1350		$5.95

Credits

Date	*Items*	*Interest Period*	*Interest*
July 17	$500	46 da.	$3.83
31	300	32 da.	1.60
Aug. 16	350	16 da.	.93
	$1150		$6.36

The debit footing and interest = $1350 + $5.95 = $1355.95
The credit footing and interest = $1150 + $6.36 = $1156.36
Balance due Sept. 1, 1910 = $199.59

541. From the foregoing illustrative problem the following rules may be derived:

1. Find the date each item matures.

2. Compute the interest on each item from the date of its maturity to the date of settlement.

3. From the sum of the debit items plus the interest subtract the sum of the credit items plus the interest, and the result will be the cash balance.

Note. If the account has been averaged, the amount due at a given date may be found by calculating the interest on the balance of the account from the time it is due to the date of settlement.

If the date of settlement is earlier than the average date, subtract the interest from the balance of the account; if later than the average date, add the interest.

WRITTEN EXERCISE

1. If money is worth 6% per annum, what is the cash balance due on the following account, Oct. 15, 1910?

Moore, Gill & Co.

1910					1900				
Sept.	1	To Mdse.	300		Sept.	12	By Cash	200	
	20	To Mdse., 1 mo.	200			25	By Cash	200	
	30	To Mdse., 2 mo.	150		Oct.	1	By Cash	100	

NOTE. If certain items fall due after the date of settlement, the interest on such items should be transferred to the opposite side of the account.

2. What is the cash balance of the following account, Jan. 1, 1912, interest at 6%?

Wm. G. Sumner & Co.

1911					1911				
July	16	To Mdse., 2 mo.	500		July	20	By Cash	400	
Aug.	10	To Mdse., 3 mo.	700		Aug.	20	By Note, 30-da. with int.	1000	
Sept.	1	To Mdse., 1 mo.	800						
	24	To Mdse., 2 mo.	600						

3. What is the cash balance of the following account, May 1, 1912, if money is worth 5%, per annum?

The Witwer Baking Co.

1912					1912				
Jan.	16	To Mdse., 1 mo.	650		Jan.	24	By Cash	500	
	30	To Mdse., 2 mo.	760		Feb.	15	By 30-da. acceptance	1000	
Feb.	14	To Mdse., 3 mo.	840						
	29	To Mdse., net	500		Mar.	4	By Cash	1200	
Mar.	4	To Mdse., 2 mo.	900						

4. Equate the following account and find the cash balance due Dec. 4, 1912, if money be worth 8%:

Hinckley-Cornell Co.

1911					1911				
Oct.	4	To Mdse., net	300		Oct.	10	By Cash	250	
	20	To Mdse., 30 da.	400			30	By Cash	450	
Nov.	10	To Mdse., 30 da.	500		Nov.	30	By Cash	750	
	28	To Mdse., 30 da.	660		Dec.	1	By Cash	500	
Dec.	1	To Mdse., 30 da.	750			12	By Note, 30 da. no int.	1000	
	15	To Mdse.. 30 da.	800						

NOTE. See note under Sec. 541.

BANKS AND BANKING

Bank Discount

ORAL EXERCISE

1. Learn your bank's rules for discounting. Which is greater, bank discount or true discount?

2. If a note falls due on Sunday or a legal holiday when must it be paid? When is a 30-day note dated July 3 due?

3. When does a 1-mo. note, dated Jan. 28 become due? If dated Jan. 29? If dated Jan. 31?

4. In payment for merchandise sold you receive a non-interest bearing note for $500. Discount the note for 60 days at 6% and calculate the proceeds.

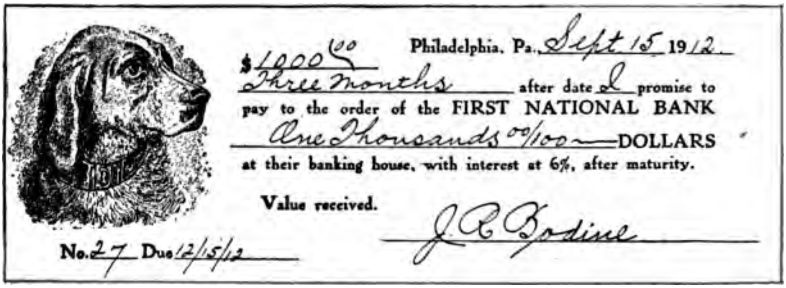
542. A **bank** is an institution chartered by law for the purpose of receiving deposits, loaning money on promissory notes, bonds, and mortgages, and furnishing a circulating medium.

543. The deduction made by a bank in buying commercial paper is called **bank discount.** *It is simple interest on the face of the paper,* paid in advance, for the number of days it has to run.

The chief functions of banks are to receive deposits, to lend money, and to buy, sell, and collect drafts or bills of exchange, but a considerable part of the business of most banks is the discounting of merchants' notes before they are due, or extending them credit on their individual note. The bank may, however, ask for *security.* In this case the borrower must have some responsible person indorse the note, or he must deposit *collateral security* in the form of mortgages, stocks, etc.

It is a mistake to say that banks deal only in money, for they deal in credit transactions as well. They, however, have a capital of their own which serves as a guarantee fund. Neither do all bank deposits represent money intrusted to banks by individuals, as most of them represent credit loaned to individuals by banks. Thus, if a bank accepts a promissory note for $2000, it may give in return a deposit credit for $2000 less the discount and will thereby add that sum to its deposits.

544. Assuming that J. R. Bodine desires to increase his credit at the First National Bank of Philadelphia, and for this purpose gives the bank his note. The following is a common form for such note. It does not differ materially from an ordinary note.



$1000.00 Philadelphia, Pa., Sept 15 1912

Three months after date I promise to pay to the order of the FIRST NATIONAL BANK

One Thousand 00/100 DOLLARS

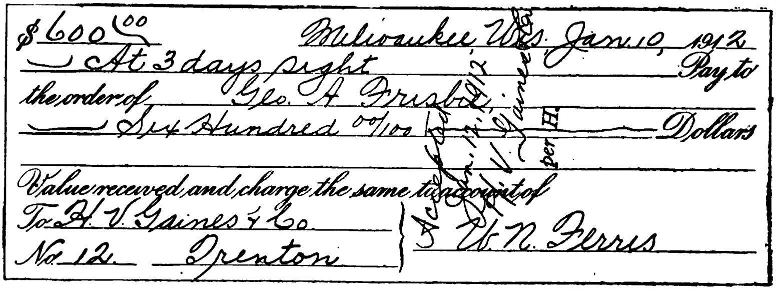
at their banking house, with interest at 6%, after maturity.

Value received. J. R. Bodine

No. 27 Due 12/15/12

Mr. J. R. Bodine would take the above note to the bank and, if required, either furnish an indorser or collateral security, such as bonds, mortgages, etc., which he would assign to the bank. If the bank authorities are satisfied, he would then receive credit for the proceeds of the note which would be $1000 less the interest on the same for 3 mo. at whatever rate it was their custom to discount. If the rate is 6%, the proceeds would be $1000 − $15, int. for 3 mo., or $985. The deduction or interest is called ***bank discount***.

545. The form of draft below is called a **commercial draft** and is frequently used by business houses as a method of collecting accounts, the merchant in such cases depositing it with his banker for collection. Such a draft may be defined as the written order of one party to another for the payment of a specified sum of money to the order of himself or to a third person.



$600.00 Milwaukee, Wis. Jan. 10, 1912

At 3 days sight Pay to

the order of Geo. A. Frisbie

Six Hundred 00/100 Dollars

Value received and charge the same to account of

To H. V. Gaines & Co.

No. 12 Trenton

W. N. Ferris

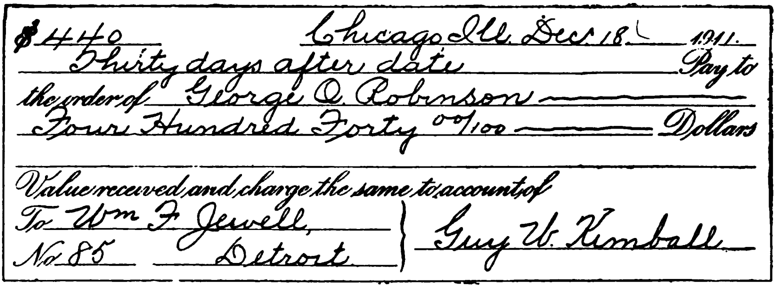
Accepted Jan. 12, 1912

H. V. Gaines & Co.

per H.

It is assumed in the above that H. V. Gaines & Co. owes W. N. Ferris $600 past due. W. N. Ferris draws the above *time draft* payable to the order of himself and deposits it with his banker for collection. The bank sends it to a Trenton bank to collect. The Trenton bank demands payment, or an acceptance of H. V. Gaines & Co. They accept it by writing across the face, preferably in red ink. After writing these words across the draft, H. V. Gaines & Co. have agreed to pay $600 and the draft becomes the same as a promissory note. This gives them 3 days in which to pay the draft after the date of acceptance. On the date of maturity the bank collects $600 of H. V. Gaines & Co. and returns the same to the Milwaukee bank where W. N. Ferris receives credit for it.

546. A draft drawn a certain time after date, or a certain time after sight, is called a **time draft.** A draft drawn payable "at sight," that is, payable on presentation, is called a **sight draft.**



$440 Chicago Ill. Dec 18 1911.

Thirty days after date Pay to

the order of George Q. Robinson

Four Hundred Forty 00/100 Dollars

Value received and charge the same to account of

To Wm F. Jewell,

No. 85 Detroit

Guy W. Kimball

547. A draft drawn for a certain time *after date* begins to mature from the date on which it was written, but a draft payable *after sight* begins to mature from the date of its acceptance.

548. The maturity of negotiable paper is the date upon which it is due. The whole sum specified to be paid at maturity is called the value, or amount, of the paper. The proceeds of a note or other negotiable paper is its value at maturity less the bank discount.

Commercial paper maturing on Sunday or a legal holiday is not legally due until the day following, except in a few states whose statutes make payment the day preceding.

When discounting notes, or other negotiable paper, banks count forward by days or months as stated in the paper and usually reckon 360 days in the year. Thus, a note dated May 15 at 60 days will mature May 15+60 days, or July 14. A note dated May 15 due in 2 mo. will mature July 15. (See 503.)

549. The parties to a draft are the **drawer, drawee,** and **payee.** The *drawer* is the person who draws or writes the draft. The *drawee* is the one on whom the draft is drawn. The *payee* is the person to whom it is made payable.

ORAL EXERCISE

Each of the following notes is discounted on the date of issue. Find the date of maturity and the discount.

	Date of Note	*Time*	*Face*	*Rate*
1.	Jan. 15, 1911	2 mo.	$600	6%.
2.	Mar. 12, 1911	60 da.	$1000	6%.
3.	July 8, 1911	4 mo.	$800	9%.
4.	June 6, 1911	10 da.	$900	6%.

5. Which is more advantageous to you to issue a 60-day or 2-months' note dated July 31?

6. A note for $500, dated July 1, 1911, due in 60 da., without interest, was discounted at a bank July 21. What was the term of discount?

550. Maturity Calendar.. Nearly all banks use some form of a *maturity calendar* containing a table for calculating exact days between two dates, but these are too extensive to show their use in a work of this kind. The apt student may easily and quickly learn their use. It is important that the student should know how to make these calculations without the use of such devices.

WRITTEN EXERCISE

Find the maturity and term of discount in the following:

1. What is the maturity and term of discount of a note dated Mar. 16, payable in 3 mo., and discounted May 10?

Solution. To find the maturity of a note due in 3 mo. from Mar. 16, count ahead 3 mo., obtaining *June 16 as the due date.* Then the exact days from May 10, the date of discount, to June 16 the due date, gives 37 days, the *term of discount.*

	Date of Note	*Time*	*Date Discounted*		*Date of Note*	*Time*	*Date Discounted*
2.	Sept. 24	3 mo.	Oct. 6.	8.	Dec. 2	2 mo.	Dec. 6.
3.	Jan. 16	60 da.	Feb. 4.	9.	Mar. 5	3 mo.	Mar. 5.
4.	Oct. 20	90 da.	Nov. 2.	10.	May 15	60 da.	May 15.
5.	July 5	60 da.	July 8.	11.	Aug. 4	4 mo.	Aug. 4.
6.	Feb. 10	2 mo.	Feb. 24.	12.	Apr. 9	60 da.	May 25.
7.	June 12	4 mo.	Aug. 6.	13.	Nov. 4	60 da.	Dec. 6.

Find the date of maturity of each of the following acceptances:

	Date	*Time after Date*		*Date*	*Time after Date*
1.	July 16	30 da.	4.	Sept. 6	2 mo.
2.	June 12	60 da.	5.	April 4	60 da.
3.	Mar. 23	2 mo.	6.	Aug. 16	20 da.

Find the date of maturity of each of the following acceptances:

	Date Accepted	*Time after Sight*		*Date Accepted*	*Time after Sight*
1.	June 14	2 mo.	5.	July 12	30 da.
2.	Aug. 10	60 da.	6.	Sept. 4	60 da.
3.	Mar. 23	1 mo.	7.	May 15	30 da.
4.	Jan. 14	2 mo.	8.	Oct. 29	40 da.

CASE I

551. To find the bank discount and the proceeds of a note or draft, the face, time, and rate per cent being given.

Example 1. Find the bank discount and proceeds of a note for $1200 due in 38 da., discounted at 6%.

Solution

$.006⅓ = rate for the term of discount.
$.006⅓ × 1200 = $7.60, the bank discount.
$1200 − $7.60 = $1192.40, the proceeds.

Example 2. Find the proceeds of a note for $900, dated Dec. 1, 1911, payable in 60 days with interest at 7% and discounted at a bank Dec. 15 at 6%.

Solution

$900 = face of the note.
$10.50 = interest on $900 for 60 da. at 7%.
$900 + $10.50 = $910.50, value of the note at maturity.
$6.98 = bank discount on $910.50 for 46 da. at 6%.
$910.50 − $6.98 = $903.52, the proceeds.

552. **Rule.** *Compute the interest on the amount of the note due at maturity for the time and rate, and the result is the bank discount.*

Subtract the discount from the amount of the note at maturity, and the remainder is the proceeds.

The discount of an interest-bearing note is computed on the amount of the note at its maturity. Remember that banks compute interest for the *actual number of days* a note has to run, whether a note is drawn for *months* or *days*.

Some banks charge a small fee for collection and exchange in addition to the interest in discounting notes which are payable in other cities or towns.

WRITTEN EXERCISE

1. Find the proceeds of a note for $1350, discounted at a bank 4 mo. 5 da. before it was due, at 6% per annum.

2. A note for $350, drawing interest at 7%, due in 90 days, is discounted at a bank at 5% for 60 days. What are the proceeds?

3. Find the proceeds of a 90-da. note, dated Jan. 31, 1911, for $2040 and discounted at a bank, Feb. 25, 1911, at 6%.

4. A note of $600 due in 90 da., drawing interest at 6%, is given Jan. 15, 1911. 35 days afterward it is discounted at a bank at 6%. What are the proceeds?

5. A note for $456.60, dated Aug. 12, 1911, due in 90 days, without interest, was discounted at the bank, Oct. 5, 1911, at 6%. Find the proceeds.

6. Find the proceeds of the following note if it is discounted the day it is issued at 6%:

$200. NEWBURGH, N. Y., Dec. 16, 1912.

Three months after date, I promise to pay to the order of Alvin Clark Two Hundred Dollars, at the First National Bank of Newburgh. Value received. EDWARD FISH.

7. Find the proceeds of the following note if it is discounted at a bank the day it is issued at 6%:

$440. JACKSON, MICH., Apr. 16, 1912.

Four months after date, I promise to pay to the order of Wesley J. Sears Four Hundred Forty Dollars, at the First National Bank of Jackson. Value received. CLARENCE E. HAYNES.

8. Find the proceeds of the following joint note if it is discounted at the bank, Dec. 1, 1912, at 6%:

$850. ST. PAUL, MINN., Oct. 18, 1912.

Six months after date, we promise to pay to the order of W. C. Stephens Eight Hundred Fifty Dollars, at the Second National Bank of St. Paul. Value received.

JACOB H. BROWN.
JOHN G. WARREN.

9. Assuming that the model note, page 264, was discounted the date of its issue at 6%, find the proceeds.

10. Assuming that the model note, page 266, is discounted at the bank, July 1, 1912 at 7%, find the proceeds.

11. Assuming that the model draft, payable to George O Robinson, page 297, is discounted at the bank, Dec. 23, 1911, find the proceeds, discount 6%.

12. A man having a 6% note for $2000, dated Mar. 1, 1911, payable in 90 days, gets it discounted at a bank Mar. 10, 1911, at 5%, and invests the proceeds in wheat at $.90 a bushel. How many bushels of wheat does he buy and what was the unexpended balance?

13. A merchant's bank account is overdrawn $1865.50, and he presents to the bank the following notes which are discounted Sept. 4, 1911, at 6% and placed to his credit. What is his balance?

Face	*Date*	*Time*	*Rate of Interest*
$875	Aug. 4, 1911	90 da.	7%
$900	July 5, 1911	3 mo.	no int.
$688.25	June 9, 1911	4 mo.	6%

14. The proceeds of a note for $137.50 discounted 40 days before maturity were $136.30. What was the rate of discount charged on the face of the note?

15. What is the difference between the bank discount and the true discount of $1319.50, due in 90 days, discounted at 6%?

16. Find the discount at 6% of a $300 note due in 2 years, bearing interest at 8%.

17. I wish to make 8% per annum on my money. What must I pay for a note of $360 due in 3 yr. 6 mo. hence, bearing 6% interest.

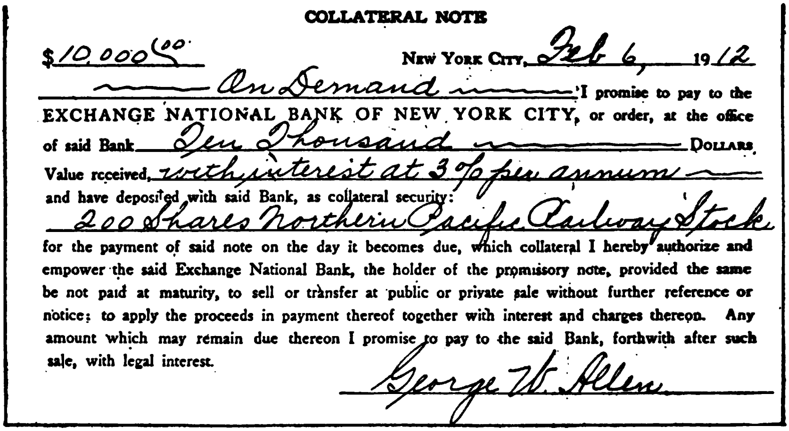
18. Find, by exact interest method, the proceeds of a note for $250, due Jan. 10, 1912, and discounted at a bank at 6% per annum, Dec. 3, 1911.

19. A merchant can discount a note at his bank at 6%, 365 days to the year, or through a broker at 5%, 360 days to the year, broker's commission ¼%. How much better is the latter method on a note of $5000, payable in 4 months, dated and discounted July 16?

Call Loans

553. In the large cities brokers and stock speculators frequently borrow large sums of money of banks, for speculative purposes, on their individual note, and deposit with the bank as security for such loans, bonds, stocks, negotiable warehouse receipts, or other collaterals. Such loans are payable on demand, or on giving one day's notice, and are termed "call" or demand loans. Interest on such loans is paid at the end of the time.

The illustration below shows a form of *collateral note* such as used by banks for *call* or *demand loans*.



COLLATERAL NOTE

$10,000 00/100 NEW YORK CITY, Feb 6, 1912

On Demand I promise to pay to the EXCHANGE NATIONAL BANK OF NEW YORK CITY, or order, at the office of said Bank Ten Thousand DOLLARS.

Value received, with interest at 3% per annum

and have deposited with said Bank, as collateral security:

200 Shares Northern Pacific Railway Stock

for the payment of said note on the day it becomes due, which collateral I hereby authorize and empower the said Exchange National Bank, the holder of the promissory note, provided the same be not paid at maturity, to sell or transfer at public or private sale without further reference or notice; to apply the proceeds in payment thereof together with interest and charges thereon. Any amount which may remain due thereon I promise to pay to the said Bank, forthwith after such sale, with legal interest.

George W. Allen

It is understood that if the above note is paid on demand the stock will be returned to Mr. Allen. If not paid, the stock will be sold and the proceeds of the sale will be applied in settlement of the note. If the proceeds from the sale of the stock be in excess of the loan, such excess will be returned to Mr. Allen, or placed to his credit.

The rate of interest on *call* or *demand loans* is generally lower than loans on time. Merchants prefer *time loans* as they do not wish to be embarrassed by having the loans called in at an unexpected time. Time loans are usually given for 30, 60, or 90 days, and if the borrower requires the money for a longer period, they may be renewed at the option of the bank.

CASE II

554. To find the face of a note when the proceeds, time, and rate are given.

Example. For what sum must I draw my note, payable in 60 days, that the proceeds when discounted at a bank at 6% will be $231.66?

Solution. Let $1 = face of the note.
Bank discount on $1 for 60 da. = $.01.
$1.00 − $.01 = $.99, proceeds of $1.
But the proceeds of the note = $231.66.
$231.66 ÷ $.99 = 234. Therefore the face of the note is 234 × $1 = $234.

555. **Rule.** *Divide the given proceeds by the proceeds of $1 for the given time and rate, and the result is the face of the note.*

WRITTEN EXERCISE

1. What is the face of a note at 30 days, the proceeds of which, when discounted at a bank, at 6%, are $1777.07?

2. The proceeds of a note due in 4 mo., and discounted in bank at 6%, were $543.90. What was the face of the note?

3. The proceeds of a 45-day note that had been discounted in bank at 7½% were $990.625. What was the bank discount?

4. The bank discount on a note 2 mo. 12 da. before it was due, at 6% per annum, was $2.34. Find the face of the note.

5. A merchant received a 6-months' note for goods sold at 50% profit. He immediately discounted the note at a bank at 6%, receiving $1866.28. What did he pay for the goods?

6. For what sum must I write a note, payable in 90 da., that when discounted in bank at 5% the proceeds shall be $1224.50.

7. On July 29 I presented at a bank a 90-da. note dated June 29 and bearing 6% interest; it was discounted at 8% and I received $1201.76. What was the face of the note?

8. A 4-months' note, dated June 18, was discounted in bank Aug. 31, at 6%. If the proceeds were $1254.38, for what sum was the note written?

9. May 10, G. W. Scheetz bought merchandise of me amounting to $2200, less 25% and 10%. Terms: cash. Not having the money, he gave me his 60-da. note, dated this day, for an amount equivalent to the cash value of the goods. What was the face of the note, money worth 6%?

10. Wishing to speculate in land, I obtained at a bank on my note payable in 3 months money enough to buy 50 acres of land at $75 per acre, and also borrowed of a friend sufficient to buy another lot of the same size and value, giving my note payable on demand, interest 6%. I sold both lots in time to take up my note at bank when it became due, and found that the price received for 36 acres would pay for it. What did I gain by my speculation, supposing that I paid my demand note at the same time as that in bank?

11. A merchant had his note discounted in a bank for 90 days at 6%, receiving $512.20. How much more would he have received for his note if discounted at true discount at the same rate and time?

12. A speculator bought 10 village lots, and gave a 4-months' note in payment. This note was immediately discounted in bank at 8%, and the bank discount was $192. What was the average price of the lots?

13. A note dated Apr. 24, 1912, payable in 3 months with interest at 7%, was discounted May 25, 1912, at 8%. If the proceeds were $2208.65, what must have been the face of the note?

14. I purchased through a broker 1000 lb. of tea at 27½¢. Commission 3%, and gave the broker a 36-da. note, which when

discounted at 6% just covered the amount due. If the note bore interest at 4%, what was its face?

Kinds of Banks

556. Banks are divided into the following classes: *National, State,* and *Private.*

557. **National banks.** In addition to doing a general banking business, National Banks have authority to issue circulating notes against bonds deposited with the United States Treasurer. The government does not own nor conduct the bank, but only authorizes its establishment and prescribes its mode of doing business.

National banks are organized under the National Banking Act of February, 1863, and the amendments that have been made thereto. *State Banks* are organized under the laws of the State in which they are located. The former are subject to rigorous supervision by Federal authorities, while all banks organized under state laws are subject to similar supervision by state authorities.

National banks that issue notes which circulate as money are often termed *banks of issue,* while those which do a discount business, but do not issue bank notes, are known as *banks of discount.* Previous to July 1, 1866, state banks issued notes which circulated as money, but on that date the government imposed a tax of 10% on the circulation of all state banks, which had the effect of destroying the privilege of issuing bank bills.

558. **Powers of national banks.** Aside from the powers which usually belong to corporate bodies a national bank may discount and negotiate notes, drafts, bills of exchange, etc.; receive deposits; buy and sell exchange, coin, and bullion; loan money on personal security; and issue circulating notes.

National banks are prohibited from holding real estate other than the buildings in which they do business, or from making loans on real estate or on security of their own shares of capital except to secure debts previously contracted. Real estate purchased or mortgaged to secure a pre-existing debt shall not be held for a longer period than five years.

By "personal security" is meant stocks and bonds, bills of lading, warehouse receipts, choses in action, or other personal chattels. In the absence of collateral, banks usually endeavor to get at least two names to all paper which they discount or upon which they make loans, but such is not required under the statute.

Each stockholder of a national bank is individually responsible for all debts to the extent of the par value of his stock in addition to the amount invested in such stock.

559. Circulation. Every national bank before it can be authorized to commence business must deliver to the Treasurer of the United States, registered interest-bearing bonds of the United States, to an amount not less than one-fourth of its capital, if the capital is $150,000 or less. If the capital is in excess of $150,000, $50,000 in bonds must be delivered. This entitles the bank to receive from the Comptroller of the Currency circulating notes in blank, registered and countersigned, equal in amount to the par value of the bonds so deposited. These notes when signed by the president, or vice-president, and cashier of the bank, circulate as money, and are known as "National Bank Bills."

A national bank may at any time increase its circulation as its capital may be paid up or increased, or it may decrease its circulation as provided in the law governing such banks.

Every national bank shall, at all times, keep and have on deposit in the Treasury of the United States, in lawful money of the United States, a sum equal to 5% of its circulation, to be held and used for the *redemption of such circulation;* which sum shall be counted as a part of its lawful reserve.

560. Reserve. Every national banking association in either of the following cities: Albany, Baltimore, Boston, Cincinnati, Chicago, Cleveland, Detroit, Louisville, Milwaukee, New Orleans, New York, Philadelphia, Pittsburg, St. Louis, San Francisco, and Washington, shall at all times have on hand, in lawful money of the United States, an amount equal to at least 25% of the aggregate amount of its deposits; and every other association 15%.

The law provides that the directors of any national bank may, semiannually, declare a dividend of so much of the net profits of the association as they shall judge expedient; but each association shall, before the declaration of a dividend, carry one-tenth part of its net profits of the preceding half year to its *surplus fund* until the same shall amount to 20% of its capital stock.

The failure of a national bank to make good its reserve of lawful money, the Comptroller may, with the concurrence of the Secretary of the Treasury, appoint a receiver to wind up the business of the association.

561. State banks. The functions of state banks are the same as that of national banks, only they do not issue circulating notes and are not restricted as to their loan privileges.

It is claimed that state banks possess some advantages over national banks; namely, they are not subjected to so rigid an examination; they may certify checks in excess of the depositor's balance in bank; and are not required to hold United States bonds.

562. Private banks are organized and operated by an individual or firm. Their only safety consists in the financial responsibility of the individual or individuals who conduct them. *Bankers and brokers* usually buy and sell stocks and bonds for their customers, and in addition to this carry on a general banking business.

563. The profits of a bank arise mainly from loaning money, discounting notes, dividends on stocks and bonds held, and collection and exchange.

WRITTEN EXERCISE

1. A national bank in Columbus, O., has deposits amounting to $875,642. How large must be its reserve?

2. A bank has a circulation of $50,000, how much is its redemption fund? What is the amount of bonds on deposit to secure its circulation?

3. A bank having failed was placed in the hands of a receiver, who declared a dividend of 75% in favor of the depositors. M's balance was $3456.25, N's $5426, and O's account was overdrawn $540. How much did M and N receive, and how much did O have to pay?

4. How long will it take a national bank with a capital of $250,000, to produce the surplus fund if the average semiannual net profits are $25,000?

5. The net earnings of a bank whose surplus is less than 20% of its capital are $8764.50. What amount must be carried to the surplus account if the capital stock is $200,000, and what are the undivided profits after declaring a dividend of 4%?

6. The combined deposits of the national banks of New

Jersey for a certain period amounted to $16,248,325. How large was the reserve required?

7. A national bank has a capital of $125,000, and a circulation of $80,000. What is the remaining amount of circulation it may call for by depositing the necessary amount of bonds?

8. A national bank becoming insolvent found its capital stock, consisting of $200,000, impaired to the extent of $14,000. What was the rate per cent of the assessment made upon the stockholders to make good the deficiency, and how much was Mr. C obliged to pay, who owned 65 shares?

9. The Detroit national banks at a certain period held $3,843-091 in specie, $1,678,734 in legal tenders, $4,762,765 due from reserve agents, and a redemption fund with the U. S Treasurer of $750,240. If the deposits were $42,295,678, what amount of reserve was required, and what was the surplus reserve?

Savings-Bank Account

564. A **savings bank** is an institution, chartered by the state, for the deposit and safe keeping of small sums of money. They are intended to encourage economy and thrift among the people.

The deposits in a savings bank are not subject to a check as in other banks, and many banks reserve the right of requiring the depositor to give from 30 to 60 days' notice when he wishes to withdraw his funds; but this right is seldom exercised.

As safety is considered paramount to profit, the law usually prescribes the character of the investments of the savings institutions. In some states they are allowed to invest only in United States Government bonds and bonds of their state. Outside of this, investments which are usually permitted, are loans on pledges of stocks and bonds, and mortgage loans.

565. The **interest term** is the period which must elapse before dividends of interest are declared. Interest is computed on an even number of dollars, and all fractions of a dollar are rejected.

Dividends in most banks are declared semiannually, beginning Jan. 1, and July 1 of each year, but in some banks they are declared quarterly; in others, deposits made on or before the first of any month draw interest from the first days of these months respectively. Interest is added at the end of every interest term on the smallest balance on deposit during the entire term, and if the interest is not withdrawn it draws interest the same as a deposit. Savings banks therefore pay *compound interest.*

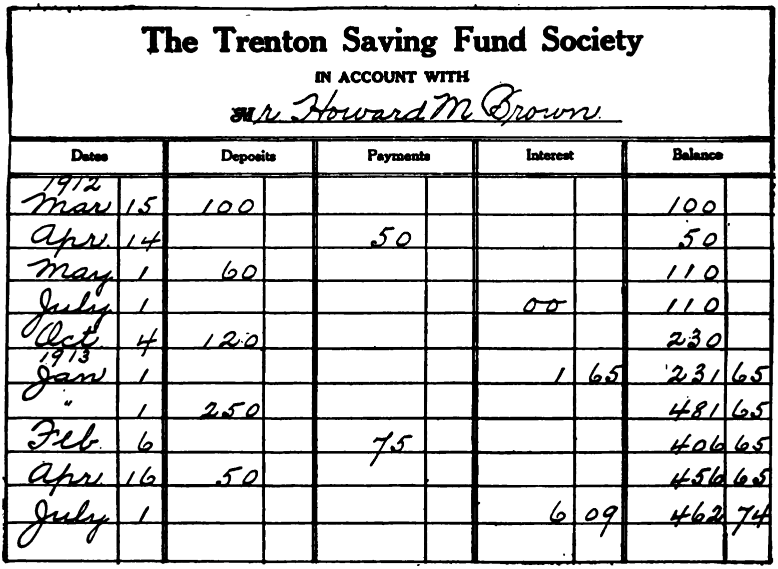
566. Depositors in savings banks are furnished a small book called a *pass book,* in which all deposits and amounts drawn out are recorded, together with the interest credited to the depositor at the expiration of the interest term.

The by-laws of many savings banks do not permit a larger balance, to the credit of each depositor, than $5,000.

In examples in this book no interest is computed upon the cents in the balance, and in the interest any fractional part of a cent is dropped.

Example 1. What will be the balance due on the following account on July 1, 1913, interest being allowed semiannually at 3%?

Deposited Mar. 15, 1912, $100; May 1, 1912, $60; Oct. 4, 1912, $120; Jan. 1, 1913, $250; Apr. 16, 1913, $50. Drew out Apr. 14, 1912, $50; Feb. 6, 1913, $75.



The Trenton Saving Fund Society

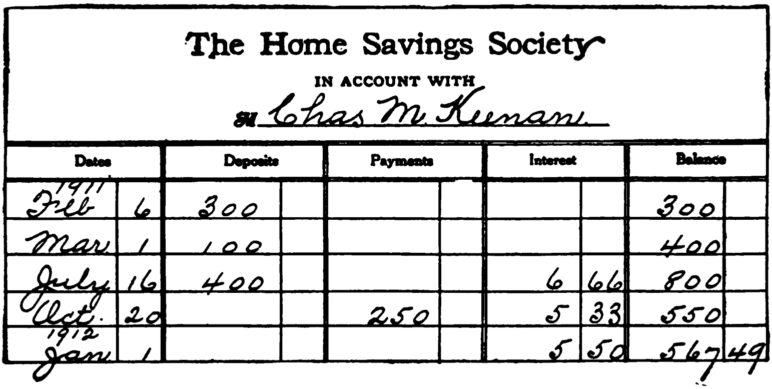
IN ACCOUNT WITH

Mr. Howard M. Brown

Dates		Deposits		Payments		Interest		Balance	
1912 Mar.	15	100						100	
Apr.	14			50				50	
May	1	60						110	
July	1					00		110	
Oct.	4	120						230	
1913 Jan.	1					1	65	231	65
"	1	250						481	65
Feb.	6			75				406	65
Apr.	16	50						456	65
July	1					6	09	462	74

Explanation. Interest begins July 1, and Jan. 1 on the smallest balance for any interest period. Hence, at the end of the first interest term, no interest was allowed because no part of the deposits was in the bank during the entire term. The smallest balance during the second term was $110. The interest upon $110 for 6 mo. = $1.65, which, added to the previous balance, gives the balance Jan. 1, 1913. The smallest balance from Jan. 1, 1913, to July 1, 1913, was $406.65. The interest upon $406 for 6 mo. = $6.09, which added to the account July 1 = $462.74, the balance due.

Example 2. In the Home Saving Society interest dividends are declared annually and the interest days are the first day of each month. Verify the balance due on the following account Jan. 1, 1912, at 4%.



The Home Savings Society

IN ACCOUNT WITH

M Chas M. Keenan

Dates		Deposits		Payments		Interest		Balance	
1911 Feb	6	300						300	
Mar	1	100						400	
July	16	400				6	66	800	
Oct.	20			250		5	33	550	
1912 Jan	1					5	50	567	49

Explanation. The first interest period begins March 1. The smallest balance on deposit, beginning Mar. 1 and continuing until Aug. 1, was $400. The interest on $400 for 5 mo. = $6.66. The smallest balance from Aug. 1 to Oct. 1, was $800. The interest on $800 for 2 mo. = $5.33. The smallest balance from Oct. 1 to Jan. 1, 1912, was $550. The interest on $550 for 3 mo. = $5.50. Then the last balance $550, and the several interest items = $567.49, the balance due Jan. 1, 1912.

NOTE. Deposits made from the first to the fourth of the month in some savings banks draw interest from the first of the month; however, these matters are governed by the by-laws of each institution.

WRITTEN EXERCISE

1. Find the balance due Jan. 1, 1912, on the following account, assuming that the interest days are the first day of each month: *Deposits*, July 16, 1911, $200; Oct. 1, $150; Oct. 24, $320. *Drew out*, Aug. 1, $80; Nov. 8, $50. Interest at 3%, annually.

2. B made the following deposits in a savings bank: Mar. 12, 1911, $220; May 2, $80; July 1, $120; Sept. 20, $175; Oct. 1, $90. Interest at 4%; interest days, Jan. 1, Apr. 1, July 1, and Oct. 1. What is the balance due B Jan. 1, 1912, if the interest term is 1 year?

3. In the Dime Savings Bank, Detroit, interest dividends are declared semiannually and the interest days are Jan. 1, Apr. 1, July 1, and Oct. 1, with interest at 4%. D deposits as follows: Jan. 1, 1911, $340; Mar. 12, $400; May 1, $250; Sept. 30, $124.50; Oct. 24, $60; Nov. 1, $100. Withdrawals: Feb. 20, $160; May 19, $75; Oct. 10, $200. How much is due D Jan. 1, 1912?

4. What was due July 1, 1913, on the following pass-book if the interest term is 6 months, and the interest days are Jan. 1, Apr. 1, July 1, and Oct. 1 and the rate is 4% per annum?

Dr. PEOPLE'S SAVINGS BANK in Acc't with Oswald N. Page Cr.

1912 Aug	15	To Cash	300		1912 Sept	15	By Check	180	
Oct.	1	" Cash	250		Dec	18	" Check	50	
Nov	10	" Cash	200		Mar	4	" Check	120	
1913 Jan.	1	" Interest	*	**					
	1	" Cash	440						
Mar	31	" Cash	220						
May	20	" Cash	325						
July	1	" Interest	**	**					

Bankers' Daily Balances

567. Most **banks** and **trust companies,** as well as savings banks, pay interest on the daily balances of their depositors.

Whether interest is allowed on the balances of a *commercial account* is usually regulated by the custom of the bank and the size of the depositor's daily balance. As a rule, banks do not allow interest on small balances subject to check. *Active accounts*, in which the balances change frequently, interest is often allowed on an even number of hundred dollars when the balance amounts to $500 or more.

568. Various forms of daily balances, as kept with depositors are in use, but the following will illustrate a general form. Another form is shown on page 27.

Example. Verify the balance due on the following account May 1, 1912, interest settlements being made monthly at 2%.

Charles H. Gifford

Explanation	Date		Debit		Credit		Credit Balances		Days	Credit Interests	
	1912 Mar	1					840	50	9		40
Currency		10			900		1740	50	5		47
Check		15	250				1490	50	5		39
N. Y. Draft		20			1150	50	2641		4		58
Check		24	782				1859		3		30
Check		27	210				1649		5		44
Interest	Apr.	1			2	58	1651	58			
Currency		1			560		2211	58	4		49
Curr&Chks		5			1260	90	3472	48	4		76
Check		9	942				2530	48	7		97
Check		16	150				2380	48	0		
Currency		16			470	52	2851		5		78
N. Y. Draft		21			740		3591		6	1	17
Check		27	390				3201		1		18
Check		28	1275				1926		3		32
Interest	May	1			4	67	1930	67			

Explanation. Beginning Mar. 1, Charles H. Gifford's balance was $840.50, which was on interest until the 10th, or 9 da. before the account was disturbed. The interest on $800 for 9 da. is $.40. On the 10th a deposit of $900 was made, making a credit balance of $1740.50. The interest on $1700 for 5 da., or until the 15th when the account is disturbed again, is $.47. Continuing in this manner to Apr. 1, the credit interests amount to $2.58 which is placed to the credit of the depositor. On this date the net credit is $1651.58. The same process is continued until May 1, when it is seen that Mr. Gifford's balance, or *net credit*, is $1930.67.

WRITTEN EXERCISE

1. The Trenton Trust and Banking Co. allows interest to

its depositors on daily balances at 3% per annum, payable semiannually. Find the cash balance of the following account with J. H. Adams, Oct. 1, 1912. July 1, 1912, deposited \$800; July 20, \$960; Aug. 1, \$700; Aug. 23, \$400; Sept. 4, \$750. Checked out, July 15, \$340; Aug. 12, \$632.50; Sept. 5, \$375. Interest allowed on an even number of hundred dollars when the balance amounts to \$300 or more.

NOTE. Some accountants shorten the process of finding the interest by means of the *product method*, as follows: *multiply each item by the interest days, add the products and find the interest on the sum at the given rate for 1 da., the result will be the required interest.*

2. The Philadelphia Trust Co. allows interest to its depositors on daily balances at 4% per annum on an even number of hundred dollars when the balance amounts to \$200 or more. Find the cash balance of the following account with Amos R. Wells, July 1, 1912, interest payable quarterly: Jan. 1, 1912, deposited \$150; Feb. 4, \$325; Mar. 30, \$600; May 2, \$1200; June 6, \$280. Checked out, Feb. 20, \$150; Mar. 10, \$240; Apr. 28, \$320; May 24, \$500.

3. Find the balance due on the following bank account Jan. 1, 1912, assuming that interest is allowed on daily balances at 3% per annum, and added to the account monthly:

Dr. COMMERCIAL & SAVINGS BANK, PITTSBURG, PA. Cr.

1911 Oct	5	To Cash	350		1911 Oct	20	By Check	50	
″	28	″ Cash	420		″	29	″ Check	140	
Nov	6	″ Cash	700		Nov	16	″ Check	200	
″	20	″ Cash	940		Dec	8	″ Check	180	
Dec	8	″ Cash	100		″	24	″ Check	220	

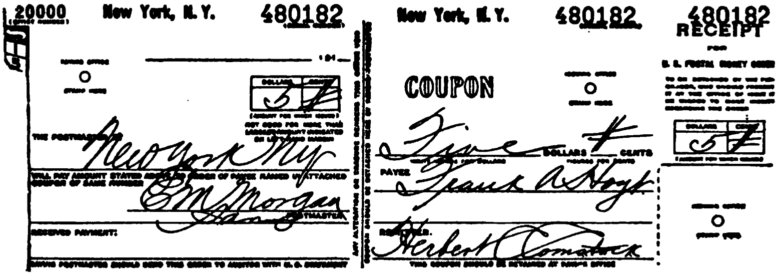
Exchange

569. Exchange treats of methods of canceling indebtedness between distant places without the actual transfer of money. This may be accomplished in any one of the following ways:

1. By a postal money order 2. By an express money order

3. By a telegraphic money order
4. By a bank check
5. By a bank draft
6. By a personal draft

570. Postal money order. This is a government order drawn by the postmaster, at one office, and payable at another to the person named in the order.



20000 New York, N. Y. 480182

New York Ny

WILL PAY AMOUNT STATED TO ORDER OF PAYEE NAMED IN ATTACHED COUPON OF SAME NUMBER

E. M. Morgan

POSTMASTER

RECEIVED PAYMENT:

PAYING POSTMASTER SHOULD SEND THIS ORDER TO AUDITOR WITH U. S. STATEMENT

New York, N. Y. 480182

COUPON

Five DOLLARS — CENTS

PAYEE Frank A Hoyt

REMITTER Herbert H Comstock

THIS COUPON SHOULD BE RETAINED AT PAYEE'S OFFICE

480182 RECEIPT FOR U. S. POSTAL MONEY ORDER

5

The maximum amount for which a single money order may be issued is $100. When a larger sum is to be sent additional orders must be obtained. Identification by the payee is necessary unless the sender waives it and in case payment is made to the wrong person the government cannot be sued.

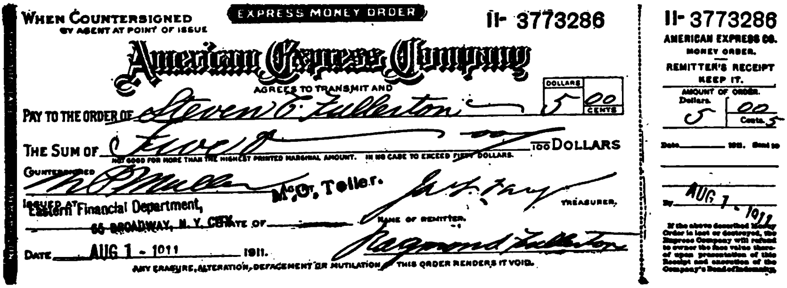
Application for money orders must be made in writing on a blank furnished by the post-office department, and such application must be preserved at the office of issue for four years from date of issue. More than one transfer of a money order, by the holder, is prohibited by law.

At present the rates charged for postal money orders are as follows:

From $ 0.01 to $ 2.50	3¢	From $30.01 to $ 40.00	15¢
From $ 2.51 to $ 5.00	5¢	From $40.01 to $ 50.00	18¢
From $ 5.01 to $10.00	8¢	From $50.01 to $ 60.00	20¢
From $10.01 to $20.00	10¢	From $60.01 to $ 75.00	25¢
From $20.01 to $30.00	12¢	From $75.01 to $100.00	30¢

571. Express money order. This is an order for the payment of money, issued by an express company and payable at any of its offices.

The rates for express money orders are the same as for postal money orders. They may be obtained for any number of dollars and bear any number of indorsements. The fact that they are transferable by indorsement the same as notes, checks, etc., makes them valuable as a means of exchange.



WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE

EXPRESS MONEY ORDER

11- 3773286

American Express Company

AGREES TO TRANSMIT AND

PAY TO THE ORDER OF Steven C. Fullerton — DOLLARS 5 CENTS 00

THE SUM OF Five — 00/100 DOLLARS

NOT GOOD FOR MORE THAN THE HIGHEST PRINTED MARGINAL AMOUNT. IN NO CASE TO EXCEED FIFTY DOLLARS.

COUNTERSIGNED — AGENT, M. O. Teller. — TREASURER.

ISSUED AT Eastern Financial Department, 65 BROADWAY, N. Y. CITY STATE OF — NAME OF REMITTER. Raymond Fullerton

DATE AUG 1 - 1911 1911.

ANY ERASURE, ALTERATION, DEFACEMENT OR MUTILATION OF THIS ORDER RENDERS IT VOID.

11- 3773286

AMERICAN EXPRESS CO.

MONEY ORDER.

REMITTER'S RECEIPT KEEP IT.

AMOUNT OF ORDER.

Dollars 5 Cents 00 5

Date — 1911. Sent to

By AUG 1 - 1911

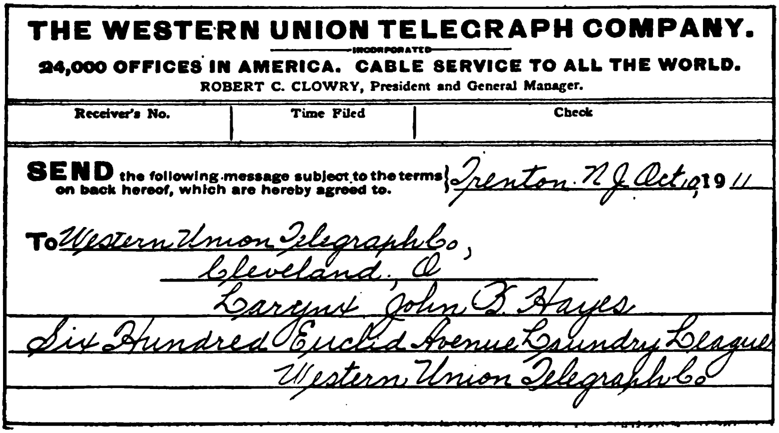
If the above described Money Order is lost or destroyed, the Express Company will refund to owner the face value thereof upon presentation of this Receipt and execution of the Company's Bond of Indemnity.

572. Telegraphic money order. This is a telegram of an express or telegraph company at any given office directing another agent of the company at some designated place to pay to the person named therein a certain sum of money.

The principle of the telegraphic money order is the same as the postal, or express, money order. No money is transferred from one place to another. The cost of exchange for a telegraphic money order is usually 1% of the amount to be transmitted plus twice the rate for a single message.

Telegraphic money orders are usually sent in cipher; that is, in a secret word or character which is understood only by the company and its agents. The following illustrates the principle of a secret code:

Code Word.	*Phrase or Sentence.*
Larynx	Pay to........of........your city
Launch	Fifty dollars
Laundry	Five hundred dollars
League	to such person as the telegraph company's agent believes to be the above-named payee.



THE WESTERN UNION TELEGRAPH COMPANY.

INCORPORATED

24,000 OFFICES IN AMERICA. CABLE SERVICE TO ALL THE WORLD.

ROBERT C. CLOWRY, President and General Manager.

Receiver's No.	Time Filed	Check

SEND the following message subject to the terms on back hereof, which are hereby agreed to. Trenton, N. J. Oct. 10 1911

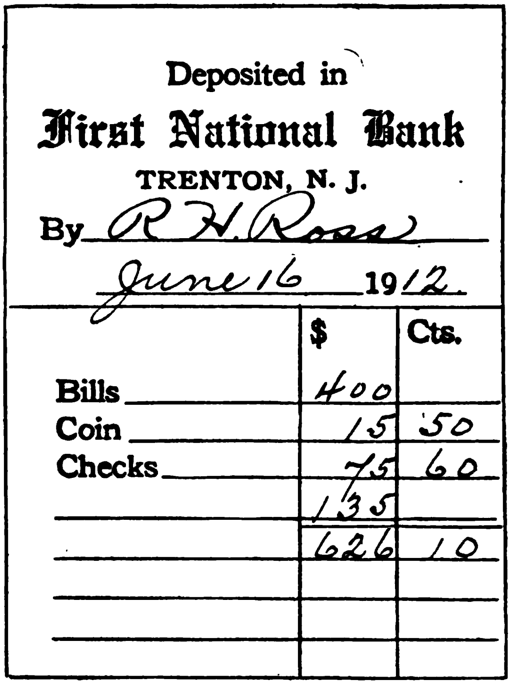
To Western Union Telegraph Co.,
Cleveland, O.
Larynx John B. Hayes
Six Hundred Euclid Avenue Laundry League
Western Union Telegraph Co.

The above telegram translated would read: "Pay to John B. Hayes of 600 Euclid Avenue, your city, five hundred dollars, to such person as the telegraph company's agent believes to be the above-named person." Of course, it is understood that the sender and receiver must each have a code.

573. A bank check. A check is an order on a bank or banker by a depositor for the payment of a certain sum of money to the person therein named, to his order or to bearer.

A check purports to be drawn on a deposit of funds. It is not money, it is only a means of getting money; neither is it a legal tender.

574. It is the custom of business men to keep their money on deposit with a bank, or trust company, and make payment by means of a *check*. Hence, a person desiring to open a *commercial account* with a bank may deposit therewith certain funds, as is shown by the *deposit ticket* in the accompanying illustration. He is then given a bank pass-book in which is credited the amount of his deposit. The bank book is the depositor's receipt, and, in this respect, it is the same as any other receipt. He is now privileged to issue checks upon this fund so long as they do not exceed the amount on deposit. The following illustrates the form of a check:



Deposited in

First National Bank

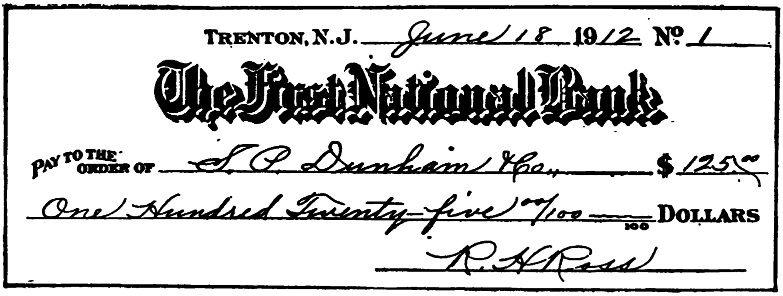
TRENTON, N. J.

By R. H. Ross

June 16 1912

	$	Cts.
Bills	400	
Coin	15	50
Checks	75	60
	135	
	626	10

The laws governing the transfer of negotiable paper and the endorsement of the same as outlined in a previous section, apply in the case of checks.



TRENTON, N.J. June 18 1912 No. 1

The First National Bank

PAY TO THE ORDER OF S. P. Dunham & Co. $125.00

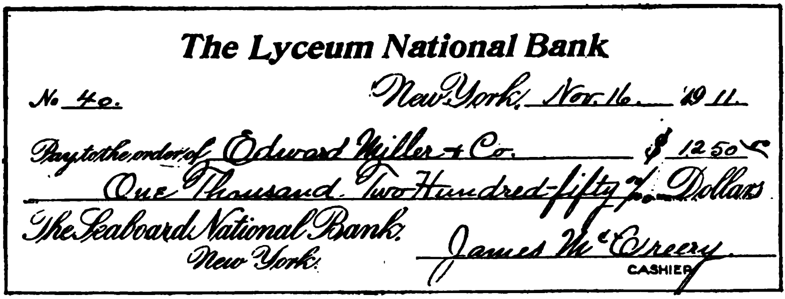
One Hundred Twenty-five 00/100 DOLLARS

R. H. Ross

A check must be presented for payment within a reasonable time after its issue, or the drawer will be discharged from liability thereon to the extent of the loss caused by the delay. A depositor whose check has been dishonored—that is, payment refused—may bring action against the bank for damages to his commercial credit, provided he had sufficient funds in the bank for its payment; but the holder has no right to such action.

Checks may be drawn payable to the order of a designated payee, or "bearer," or "cash." Any one may collect on checks payable to *bearer*, or *cash*.

575. A bank draft. This is an order by one bank directed to another, to pay a specified sum of money to a third party, or to his order.



The Lyceum National Bank

No. 40. New York, Nov. 16 1911

Pay to the order of Edward Miller & Co. $1250.00

One Thousand Two Hundred fifty 00/100 Dollars

The Seaboard National Bank,
New York.

James McCreery
CASHIER

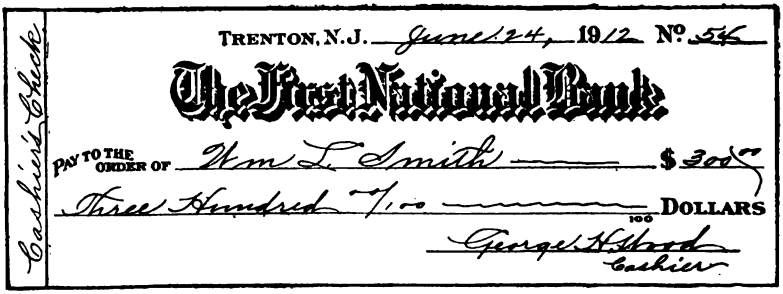
Nearly all banks, at least those located in the smaller cities, usually keep money deposited in banks in New York, Chicago, or other financial centers, against which they draw drafts to sell to their customers. Drafts on New York are very extensively used for making remittances in all parts of the country. These drafts are used to make payment to a person who does not care to take the risk of a private check or be called upon to pay the cost of cashing it.

The deposits which banks make with their *correspondents* in these different financial centers are composed principally of drafts and checks on firms and individuals in and near the city in which the correspondent bank is located, and money sent to the correspondent bank by express. Banks usually charge the purchaser of a draft a small fee in excess of its face value. This fee or premium is called *exchange.* Bankers frequently make a charge called *collection* for collecting out-of-town checks or drafts deposited with them, though there is no uniformity in the matter, as they are usually influenced by the size of a customer's bank account.

576. The form of draft called a **commercial draft** plays a prominent part in the collection of accounts and the payment of bills at distant places. These have been discussed on pages 296 to 298.

577. Other methods of depositing money than those already explained are by means of the *cashier's check,* and *certificate of deposit.*

578. A *cashier's check* differs from a depositor's check only in that it is signed by the cashier of the bank. It is issued in exchange for cash deposited or a depositor's check.



Cashier's Check

TRENTON, N.J. June 24, 1912 Nº 56

The First National Bank

PAY TO THE ORDER OF Wm L. Smith $300^{00}

Three Hundred $\frac{00}{100}$ DOLLARS

George H. Hood
Cashier

Cashier's checks are many times issued for the payment of the general expenses of a bank, notes discounted for cash, etc. In New York City they are sometimes issued instead of the New York draft. As New York City is the banking center of this country the wisdom of the course is apparent.

579. A **certificate of deposit** is practically the check of a bank upon itself, and is usually issued to persons who do not intend to become regular customers.

A certificate of deposit may be made payable to the depositor or to his order, or bearer, or a designated person. If the purchaser is a stranger he should be required to leave his signature with the bank, so that when the instrument is presented for payment the indorsement may be compared with the original signature.

A certificate of deposit is not subject to a check.

ORAL EXERCISE

1. What should you do if you desired to purchase a post-office money order, or express money order?

2. When C owes D $200, and D owes C $200, how may the accounts be settled without any transfer of money taking place?

3. What must you do to secure payment on a bill of exchange?

4. When A in Baltimore owes B in New York $50, and C in New York owes A $150, how can A pay his indebtedness to B without remitting the money?

5. Ascertain from your postmaster to what extent the postal authorities are liable if a registered letter or package is lost.

6. What will be the total cost of a telegraphic money order from New York to Baltimore for $50?

WRITTEN EXERCISE

1. A bank charged $\frac{1}{8}$% on a draft for $568. What was the cost of the draft?

2. I paid my banker $376.47 for a draft on Chicago. If the rate of exchange was $\frac{1}{8}$%, what was the face of the draft?

3. C. W. Green & Co., Marion, Ind., are indebted as follows: Rankin Bro., Indianapolis, Ind., $365.50; J. W. Morrison, Canton, O., $760; Chase, Brown & Co., Rockford, Ill., $965.40; Sears, Roebuck & Co., $22.24. They pay the amounts by remitting as follows: Rankin Bro., by New York draft, exchange costing $\frac{1}{8}$%; J. W. Morrison, and Chase, Brown & Co., express money orders; and Sears, Roebuck & Co., postal money order. Find the total cost of the remittances.

4. If the cost of exchange for a telegraphic money order is 1%, and the message 50¢, find the total cost of transmitting $125.

Domestic Exchange

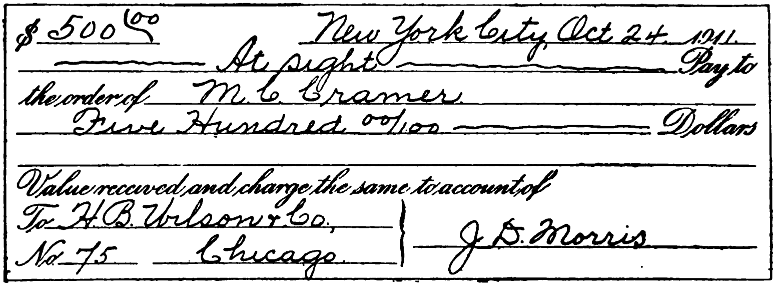
580. Domestic exchange treats of remittances that are payable in the country in which they are made.

According to the laws of many states, drafts drawn in one state and payable in another, in which they are subject to different laws, are termed *foreign* bills of exchange.

A large part of the exchange of the United States is effected through the banks of New York, Chicago, Philadelphia, Boston, St. Louis, Baltimore, and San Francisco. The financial centers of Europe are London, Paris, Antwerp, Geneva, Amsterdam, Hamburg, Frankfort, Bremen, Berlin, and Vienna.

581. A bill of exchange, or draft, is an order for the payment of money written by one person addressed to a person in another locality. (See 545, 546, and 575).

582. The transactions underlying **bills of exchange** are shipments of merchandise, of goods of all kinds, and any other transaction through which a person in one locality becomes a debtor or creditor of a person in another.



$\$500^{00}$ New York City Oct 24. 1911.
At sight Pay to
the order of M. C. Cramer.
Five Hundred 00/100 Dollars

Value received and charge the same to account of
To H. B. Wilson & Co.
No. 75 Chicago.
J. D. Morris

In the above bill, J. D. Morris is the *drawer*, H. B. Wilson & Co. the *drawee*. The third person entering into the transaction is the person to whom the bill of exchange is made payable, or the *payee*, who undertakes the collection of the debt either direct or through the mediation of one or more bankers to whom the bill of exchange is subsequently made payable by transfer of ownership effected through endorsement.

Rate of Exchange

583. When a rise or fall in exchange is spoken of, or it is said that exchange is high or low, reference is had to the variations in the price at which transactions in bills of exchange are made. Exchange is at a *premium* when it costs more than its face value; at a *discount* when it sells for less than its face value; and at *par* when it sells for its face value.

That which gives rise to the *fluctuations in the rates* of exchange is the difference in the indebtedness on the part of one locality to another. If A in Chicago owes B in New York $500, while A in Chicago is creditor to C in New York to the extent of $400 only, it becomes evident that in order to settle the account $100 must be shipped in currency, unless C in New York may be induced to purchase of B $500 worth of exchange on Chicago, although only $400 are needed for his immediate requirements. A discount must be offered C to compensate him for the loss of interest which he incurs by having $100 lie idle in Chicago, or, on which the interest return may be less than the income he would derive by actually employing the $100 in his business in New York. To induce C to make the sacrifice, the discount offered must ordinarily be about equal to the cost of forwarding currency to the amount of $100. If C insists on a larger discount than the cost of shipment, then it would be more advantageous to ship currency. In other words: the excess of supply of Chicago exchange in New York causes the holder to part with it at a discount, while scarcity of New York exchange in Chicago would contrariwise compel a premium, both discount and premium being, however, limited, under ordinary circumstances, to the cost of shipping actual currency.

The premium in the rate of exchange between New York and Chicago will not ordinarily exceed $\frac{1}{20}$%. On small sums, say $500 or less, exchange is usually at a uniform rate. This premium is an allowance to the bank for their trouble in issuing the draft and the cost of shipping money to the banking centers on which the drafts are drawn, when balances at these points become low.

Banks make no charge for cashing bank drafts drawn on the trade centers of the country, like New York and Chicago. The banks transmit them to their correspondent for credit to the places on which they are drawn.

In certain cases bankers in discounting notes and drafts payable in distant places charge interest for the time required for the return of the money when the note or draft is paid; and in case of drafts drawn a certain number of days after sight, they sometimes charge interest for the time required for the acceptance of the draft.

WRITTEN EXERCISE

1. What is the value in Cleveland of a draft on New York for $5000 at $\frac{1}{20}$% premium?

2. Find the cost of a draft on New York for $1234.50, exchange at $\frac{1}{15}$% premium.

3. Find the face of a draft that cost $720.60, at $\frac{1}{12}$% premium.

4. Find the face of a draft that cost $15,685.45, at $\frac{1}{16}\%$ premium.

5. How large a draft may be bought at $\frac{1}{20}\%$ discount, with $6000?

6. A bank draft for $12,500 was bought for $12,506.25. Was exchange at a premium or at a discount, and what rate?

7. What is the cost of a draft on New York for $1575, payable in 60 days, exchange being at $\frac{1}{8}\%$ premium and interest 6%?

Solution

Cost of $1 of draft at $\frac{1}{8}\%$ premium = $1.00125.
Bank discount of $1 at 6% for 60 da. = $.01.
$1.00125 − $.01 = $.99125 cost of $1 time draft.
$.99125 × 1575 = $1561.22, cost of the draft.

8. Exchange being at par, what must be paid for a 45-day draft on Boston, if the rate of interest is 6%, and the face of the draft is $650?

9. A traveler paid $1553.76 for a 30-day draft on Denver. Exchange being at $\frac{1}{10}\%$ premium, and interest at 6%, what was the face of the draft?

10. I hold a 60-day draft on Chicago for $2750. I sell the draft at $\frac{1}{4}\%$ premium and with discount off at 5% per annum. What do I receive?

NOTE. Bankers always discount the face and not the value of a draft.

11. An attorney, having made collections for a firm in Baltimore, to the amount of $2450, is instructed to remit by sight draft. If the attorney charges a commission of $1\frac{3}{8}\%$, and exchange is $\frac{1}{10}\%$ premium, how much will the firm in Baltimore receive?

12. Find the cost of a 90-day draft on New Orleans for $1760, exchange $\frac{3}{16}\%$ premium, interest 6%.

13. Suppose the draft on page 297 for $440 was accepted Dec. 20, and discounted at a Chicago bank Dec. 23. If the bank charged $\frac{1}{10}\%$ collection and 6% interest, how much was placed to the credit of the drawer, allowing 3 days for the return of the money?

14. A banker in Detroit discounts a draft for $4000, payable

in San Francisco 30 days after sight. What would be the proceeds, exchange being $\frac{1}{8}\%$ discount, interest 5%, and allowing 6 days for the acceptance and 6 days for the return of the money?

15. Sept. 20, W. D. Wiseman, Rockford, Ill., drew a sight draft on A. H. Lee & Co., Worcester, Mass., in favor of himself, for $672, and left it with the Rockford National Bank for collection. The Rockford bank sent the draft to a Worcester bank for collection, and 7 days later W. D. Wiseman was informed that the draft had been collected, and the amount, less $\frac{1}{8}\%$ collection, placed to his credit in the Rockford National Bank. How much was placed to his credit?

16. My agent sold for me 500 bbl. flour at $6.75 per bbl. He paid for cartage $56.50, storage $37.50, charged $2\frac{1}{2}\%$ commission, and remitted the proceeds by a bank draft bought at $\frac{1}{10}\%$ premium. What was the face of the draft?

17. After taking out his commission of 3% and $50.80 for other charges, an agent remits, by sight draft, to his employer $2081.12 the amount due him on wheat sold at $1.10 a bushel. Find the number of bushels of wheat sold, if the bank draft cost $\frac{1}{10}\%$ premium.

18. What is the cost of a draft on San Francisco for $2345.50, payable in 30 days, exchange being at $\frac{1}{10}\%$ premium and interest 5%?

19. A Chicago merchant bought a draft on New York, paying exchange $1.23 at $\frac{1}{20}\%$. What was the cost of the draft?

20. J. B. Ford & Co., having several out-of-town accounts due and unpaid, gives the same to the American Express Co. for collection. If the collection charges were $2.50 per $1000, how much did Ford & Co. receive from the express company, the accounts being: $276.50; $98.40; and $895?

NOTE. Express companies, like banks, have arrangements by which they undertake the collection of notes, drafts, and accounts. If collections cannot be made, notice is given with reasons for refusal.

21. A of Denver buys horses for C of Detroit to the amount of $2560. How large a draft should be drawn on C so that when sold at a discount of 80¢ ($\frac{2}{25}\%$), the proceeds would be sufficient to pay the bill?

A Clearing House

584. **A clearing house** is a place where associated banks meet to effect exchanges and facilitate the settlement of claims against one another.

At the present time all large cities have clearing houses, and nearly all the banks in these cities are members. Each clearing house elects its own officers and appoints its own committees.

Each bank sends to the clearing house daily, at least one settling clerk. Before going to the clearing house the clerk makes out a credit ticket, showing the total of exchanges for the clearing. When he enters the clearing house this ticket is placed on the manager's desk, and a clerk immediately enters the amount to the credit of the bank bringing the ticket. As soon as the clearings are completed balances are struck between the credits and debits of each bank against all others, then the manager certifies the amount which each bank owes to the associated banks or is entitled to receive from them. The banks whose debits exceed their credits pay in the balance to the clearing house, which issues clearing-house certificates to the bank whose credits exceed their debits. When all the exchanges have been completed, the clearing house will have paid out the same amount that it has received.

In the New York Clearing House, which is the largest in America, of the billions of dollars of clearings which are made annually, only about one-twentieth of the balances are paid in money.

585. The National Banking Laws declare that clearing-house certificates, representing specie or lawful money specially deposited for the purpose of any clearing-house association, shall also be deemed to be lawful money in the possession of an association belonging to such clearing house, holding and owning such certificate.

Foreign Exchange

586. Many of the larger banks make a practice of selling *bills of exchange*, or drafts, on the principal financial centers of England, France, Germany, and other countries. These drafts are generally used for making remittances, though travelers sometimes purchase them instead of letters of credit.

The method of handling foreign drafts does not differ from that employed for domestic drafts. The bank which draws the draft must keep a deposit with the bank on which it draws. As the drafts are paid they are charged the drawer the same as checks.

587. In view of the variety of currencies in use, each country having a different monetary system, we have to compare the different systems and arrive at the value of a foreign coin expressed in terms of our own coin. The value so formed is called the **parity of exchange.**

The *parity*, or *par value*, between our gold dollar and the pound sterling, mark, franc, and other foreign currencies is found by computing the quantity of pure gold contained in the coins of the various countries. One pound sterling, according to the mint laws of Great Britain, contains 123.274 grains of gold, $\frac{11}{12}$ fine and $\frac{1}{12}$ alloy, while the American gold dollar, as fixed by law, contains 25.8 grains of gold, $\frac{9}{10}$ fine and $\frac{1}{10}$ alloy. (See 102.) On this basis the gold contained in one pound sterling, expressed in our money, is equal to $4.8665, which is called the *parity of exchange* between the dollar and the pound sterling.

The rate of $4.8665 is arrived at as follows: $1 = 25.8 grains. 25.8 less $\frac{1}{10}$ alloy = 23.22 grains pure gold. £1 = 123.274 grains. 123.274 less $\frac{1}{12}$ alloy = 113 grains pure gold. Then 113 ÷ 23.22 = $4.8665, the value of £1. The parity between dollars and marks, francs, and other currencies is established by the same method.

588. The **exchange on Paris** and other countries using the French monetary system is usually quoted at so many francs to the dollar, sometimes at so many cents to the franc. The *par of exchange* is about 5.18$\frac{1}{8}$ francs to the dollar, or 19.3 cents to the franc. **Exchange on Germany** is quoted at so many cents on 4 marks. The *par of exchange* is 95.2; quoted in cents, per mark, it is 23.8.

589. The following table shows the comparative values between our dollar and foreign currencies, all based on gold:

Value of Foreign Money

Countries	Name of Units and Fractions	Value of For. Gold in U. S. Cur.		Value of $1.00 in For. Gold
Algeria,	1 French Franc = 100 Centimes,	$ .19,3	Gold,	Fr. 5.18
Austria-Hung.,	1 Krone = 100 Heller,	.20,9	"	Kr. 4.92
Belgium,	1 Franc = 100 Centimes,	.19,3	"	Fr. 5.18
Denmark,	1 Krone = 100 Öre,	.26,8	"	K. 3.73
Egypt,	1 Pound = 100 Piaster,	4.94,3	"	Pr. 0.20
England,	1 Pound Stg. = 20 shillings,	4.86,6½	"	£0, 4*s*, 1¼*d*
Finland,	1 Markka = 100 Pennia,	.19,3	"	FM. 5.18
France,	1 Franc = 100 Centimes,	.19,3	"	Fr. 5.18

Countries.	Name of Units and Fractions	Value of For. Gold in U. S. Cur.		Value of $1.00 in For. Gold
Germany,	1 Mark = 100 Pfennige,	.23,8	Gold	M. 4.20
Great Britain,	1 Pound Stg. = 20 shillings,	4.86,6½	"	£0, 4*s*, 1¼*d*
Greece,	1 Drachma = 100 Lepta,	.19,3	"	D. 5.18
Holland,	1 Gulden = 100 Cents,	.40,2	"	G. 2.48
Hungary,	1 Krone = 100 Heller,	.20,3	"	K. 4.92
Italy,	1 Lira = 100 Centesimi,	.19,3	"	L. 5.18
Luxemburg,	1 Franc = 100 Centimes,	.19,3	"	Fr. 5.18
Norway,	1 Krone = 100 Öre,	.26,8	"	K. 3.73
Portugal,	1 Milreis = 1000 Reis,	1.08,0	"	MR. 0.926
Russia,	1 Ruble = 100 Kopecks,	.51,5	"	R. 1.94
Spain,	1 Peseta = 100 Centimos,	.19,3	"	P. 5.18
Sweden,	1 Krone = 100 Öre,	.26,8	"	Kr. 3.73
Switzerland,	1 Franc = 100 Centimes,	.19,3	"	Fr. 5.18
Turkey,	1 Piaster = 40 Para at 3 Asper,	.04,4	"	Pr. 22.29

ORAL EXERCISE

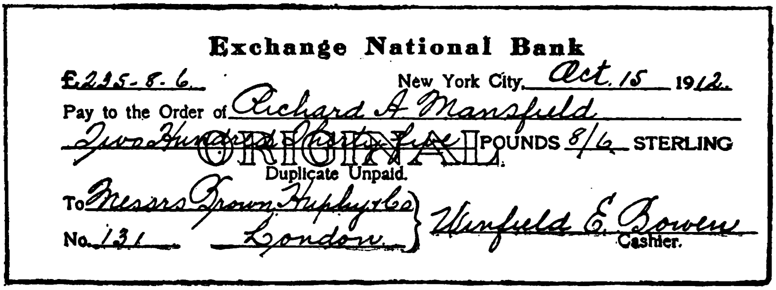
1. Express $500 as English money; about how many marks in German money; how many francs?

2. Express 1000 fr. in United States money. $193 in French money. 238 marks in United States money. 1000 Gulden in United States money.

3. Express $48,665 in English money. What countries make use of the franc? What is the cost of a draft on Paris for 518 francs, exchange 5.18?

Form of Foreign Bill of Exchange

590. A foreign **bill of exchange** is a draft drawn in one country and payable in another. Foreign bills are generally written in duplicate, called a **set of exchange**; that is, in sets of two of like tenor and amount. The following is a form:



Exchange National Bank

£235-8-6 New York City Oct. 15 1912

Pay to the Order of Richard A Mansfield

Two Hundred Thirty-five POUNDS 8/6 STERLING

ORIGINAL

Duplicate Unpaid.

To Messrs Brown Shipley & Co
No. 131 London

Winfield E. Bowen
Cashier.

Duplicate bills of exchange are sometimes sent by different mails, so that if one is lost the other may be presented; but more frequently the duplicate is kept by the purchaser, the same to be used in case of necessity. Foreign bills, drawn in sets, are so worded that the payment of one of them cancels the other.

Bankers frequently issue a *sole bill of exchange* which usually reads: "On demand pay this *sole bill of exchange*, to the order of, etc." This is preferred by many, inasmuch as it may be more easily negotiated by the payee.

591. Bills of exchange are usually drawn at sight, 3, 30, 60, 90 days and from four to six months after sight. Sight bills are payable on presentation, while time drafts, after acceptance or certification by the drawees, are payable at maturity.

The recipient of a time bill of exchange may, after acceptance, either hold the same until maturity, or offer it for discount to his banker, who, provided the bill bears responsible signatures, will advance its amount, less discount. This discount is the banker's charge of interest and his profit for making the advance.

WRITTEN EXERCISE

1. Find the cost of a sight draft on London for £250 when exchange is quoted at 4.865.
2. I wish to send $100 to a friend in Paris. At 19.4¢ to the franc, how large an international money order may I buy?
3. I have a debt of £720 8*s*. to pay in London; what will it cost me in New York to pay it, the rate for exchange being $4.87?
4. What is the cost of a draft on Paris for 1200 francs, exchange 5.16?
5. What is the cost of a draft on Leipzig for 524 marks, exchange 95.4?
6. Reduce to United States money: £25 8*s*.; 250 fr.; 640 M.; 320 R.; 200 P.
7. I sold through a broker a bill of exchange on Paris, and realized $613.62 as net proceeds. What was the face of the bill, exchange being 5.20$\frac{5}{8}$, and brokerage $\frac{1}{8}$?

Kinds of Bills

592. Bills of exchange may be divided into several classes, out of which the following three may be cited:

1. **Bankers' bills.** These are bills drawn by American bankers on foreign banking institutions.

2. **Documentary bills.** These are drawn by merchants on purchases of their merchandise, with shipping documents attached to the draft.

3. **Clean bills.** These are bills in which the bill of lading and the insurance certificate are not attached.

Documentary bills are drawn at sight, and on time. If drawn at sight, the shipping documents are surrendered against payment, and if drawn on time, against payment or acceptance, according to agreement. Clean and documentary bills of exchange are in most cases purchased by bankers, but it also happens that they are taken for collection only, in which event settlement is made as soon as the drawer's banker has been notified of the bills' payment.

Bankers are enabled to purchase these bills because of their connections with foreign banking institutions. They remit the same in payment of obligations incurred in the country on which the drafts are drawn. These obligations arise through a variety of financial operations.

593. *Exchange* is quoted *at par*, if the indebtedness of one country towards the United States, and contrariwise, does not differ in amount in favor of one country as against the other; circumstances will, however, daily arise, causing a rise above, or a fall below, par, because with uninterrupted shipments of merchandise and other operations, exchange can really never stay at any fixed point.

The rate of exchange will fall below par if the United States, through heavy export shipments or other transactions, is able to create a balance in its favor. Shippers in this country expect payment for their merchandise when goods leave the country.

Should the total of the country's exchange offerings exceed the foreign bankers' requirements for covering incurred indebtedness abroad, the law of supply and demand will become operative and exercise a depressing tendency on the price which purchasers are willing to pay for the available exchange supply. The downward movement of exchange rates will be checked as soon as a point has been reached at which it will be profitable for the American banker to take settlement in gold, as we have seen in the case of domestic exchange.

Supposing that the cost for importing gold from England amounts to about $\frac{3}{8}$%, or about $1.85 per £100, the rate at which gold could be imported at a profit would have to be less than $4.848 per pound sterling ($4.8665 less $1.85). If sterling exchange commands $4.848 or more, it would obviously be unprofitable to require settlement in gold.

Rates of Interest

594. The **rates of interest** ruling in the respective countries play an important factor in fixing the value of exchange. High money rates in this country attract capital from other countries, where interest is lower; they tend to depress exchange rates and thus work towards the importation of gold. On the other hand, the same cause, if existent in Europe, combined with other circumstances and conditions, will facilitate gold exports from this country.

The precious metal will be exported when exchange on England, for instance, owing to brisk competitive demand, has reached a point equaling the value of gold contained in one pound sterling ($4.8665), plus the expense of shipping the metal abroad, or about $\frac{3}{8}$%, or in total about $4.885 pound sterling.

595. Gold, because of its high value in relation to bulk and weight, has been adopted as the monetary standard by the principal trading nations.

In settling trade balances by the remittance of gold, coin is not always used, but resort is preferably had to bullion on account of the lesser labor involved in handling bullion, and owing to the abrasion to which coin is subject in a larger degree than gold bars. In either case, the actual quantity of the pure gold contained in either coin or bullion represents the value of the shipment.

596. In calculating the cost per pound sterling in New York for an export shipment of gold to England, two examples are given below, one referring to a coin and the other to a bullion shipment:

Example 1. Find the proceeds of a shipment of gold bars to London.

Note. One ounce of gold, $\frac{11}{12}$ fine, can be sold to the Bank of England at 77 shillings, 9 pence, while one ounce of gold costs, at the sub-treasury of the U. S. in New York, $20.67183, plus 4¢ per $100, charged as premium by the Treasury of the U. S. $4.8665 = 1£, and 1£ = 240 pence; then,

933d. = 1 ounce standard, $\frac{11}{12}$ fine.
(77*s.* 9*d.*, the price paid in London).
12 standard ounces = 11 ounces fine.
1 ounce fine = $20.67183.

Solution:		1£ =	\$4.87438
	Plus charges:	$\frac{5}{32}$% freight	.0075
		$\frac{1}{20}$% insurance	.0024
		Cartage and petties	.0005
		$\frac{1}{25}$% premium	.0020
		*$\frac{1}{8}$% interest	.0060
		Total cost	\$4.89278 per £ Stg.

Example 2. Find the cost per pound sterling for a shipment of eagles to London.

NOTE. If one ounce of eagles, being $\frac{900}{1000}$ fine, can be sold in London at 76*s.* 4*d.*, then 916*d.* = 1 ounce. And

1 ounce = 480 grains.
\$10 = 258 "
($\frac{9}{10}$ fine).

Solution:		1£ =	\$4.8746
	Plus charges:	$\frac{5}{32}$% freight	.0075
		$\frac{1}{20}$% insurance	.0024
		Cartage and petties	.0010
		$\frac{1}{16}$% abrasion on coin	.0030
		$\frac{1}{8}$% interest (about)	.0060
		Total cost	\$4.8945 per £ Stg.

Full price must be paid in New York for each gold piece, but on arrival the price is paid by weight, and coin usually loses weight by abrasion.

If as shown in the first example, exchange can be bought in New York below \$4.89278 per pound sterling, it would be unprofitable to ship gold, but if the cost of exchange in New York exceeds \$4.8945 per pound sterling, it would be advantageous to ship gold.

Arbitrage of Exchange

597. Trade balances between the various countries are not always settled by direct remittances, but ways and means have been devised which, in a measure, resemble the settlements effected under our clearing-house system. The United States may be a debtor to Great Britain, while having funds to its credit in France. In order to pay off our debt to Great Britain, it is necessary to send pounds sterling to that country. The

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* Special transactions occur sometimes, which permit the omission of the interest charge; such is the case if European Central Banks allow interest for the time gold is in transit.

remittance can be made direct from here or via France. The latter course is chosen, if the pound sterling can be secured in Paris at a cheaper rate than in New York. In determining the cost, the foreign exchange banker has to take into consideration the following factors:

1. The rate for pounds sterling in New York.
2. The rate for pounds sterling in France.
3. The rate for francs in New York.

A transaction of this kind is called an **arbitrage operation** and is resorted to for most advantageously liquidating an indebtedness, if large amounts are involved and other conditions are favorable.

WRITTEN EXERCISE

Find the cost of the following bills of exchange:

	Face	*Quotation*		*Face*	*Quotation*
1.	£240	4.86½	6.	1250 marks	95⅕
2.	£215	4.85	7.	940 guilders	41½
3.	£92 2*s.* 6*d.*	4.86	8.	1450 rubles	52¾
4.	2564.5 fr.	5.19½	9.	2540 liras	20½
5.	4320 fr.	5.18½	10.	850 milreis	1.08½

WRITTEN EXERCISE

1. A merchant in Boston paid a demand in Paris of 2800 fr., by remitting to Berlin at the rate of 23.9 per mark, and thence to Paris at the rate of 1.26 fr., per mark. How much Federal money was required?

Solution. $\$ .239 \times \frac{2800}{1.26} = \$531.11$ required.

Note. If, in the above example, direct exchange on Paris should cost 19.3¢ per franc, the cost of 2800 fr. would = $540.40. Hence, the saving by remitting through Berlin would be $540.40 − $531.11 = $9.29, the amount gained by *arbitrage* or *indirect exchange.*

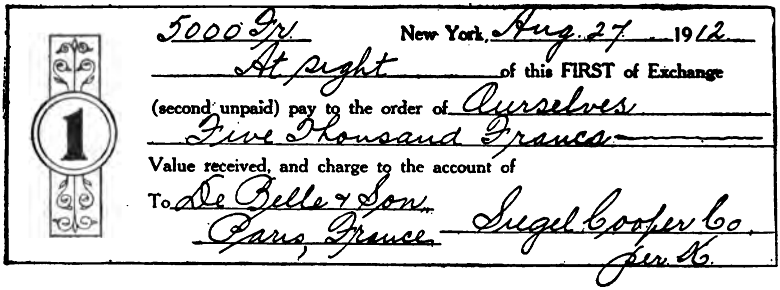
2. A banker in New York remits $5000 to Liverpool by indirect exchange, through Paris, Hamburg, and Amsterdam, the rates being as follows: in New York on Paris 5.18 fr. to the dollar, in Paris on Hamburg 1.22 fr. to the mark, in Hamburg

on Amsterdam 1.70 marks to the guilder and in Amsterdam 11.83 guilders to the pound sterling. How much sterling will he have in bank at Liverpool, and how much does he gain by indirect exchange, sterling being worth in New York 4.83½?

3. If in London exchange on Paris is 25.71, and in New York on Paris it is 5.15½, what is the arbitrated course of exchange between New York and London?

4. John Wanamaker & Co., New York, are quoted 4.8945 on London for a sight bill of £5400. They export the gold instead of buying the exchange, paying $\frac{1}{10}$% insurance, ⅛% freight, $\frac{1}{16}$% abrasion, ⅛% interest. How much did they save by shipping the gold?

5. Siegel-Cooper Co. drew the following *clean bill of exchange* and sold it to Knauth, Nachod & Kuhne, bankers, at 5.20. How much was received for it?



5000 Fr. New York, Aug. 27 1912

At sight of this FIRST of Exchange

(second unpaid) pay to the order of Ourselves

Five Thousand Francs

Value received, and charge to the account of

To De Belle & Son,
Paris, France

Siegel Cooper Co.
per K.

6. Gimbel Bro. sold to Knauth, Nachod & Kuhne, bankers, at 5.21¼ − $\frac{1}{16}$ a sight draft for 15,642 fr. What are the proceeds?

Note. The fraction in the above means, the fraction of 1%. A — fraction always makes the rate lower, and a + fraction, higher.

7. The dividends of the New York Central R. R. Co. are paid in London at the rate of 49⅛ pence to the dollar. What is the equivalent rate of exchange?

8. What are the proceeds of a draft on Paris for 15,360 fr., at 5.19½, brokerage on exchange ¼%?

9. The John A. Roebling's Sons Co., Trenton, N. J., sell to the First National Bank a documentary bill of exchange on a Paris firm for 13,600 fr., drawn 60 days after sight. What are the

proceeds, the rate of exchange being 5.17½; rate of interest 4%; commission, ⅛%; and cost of collecting and sending funds to Paris, ¼%?

10. A commission merchant in Cadiz having sent to his correspondent in Boston an invoice of sherry, valued at 9600 pesetas, draws on him for the cost, exchange being 1 peseta = 20.1¢; what would have been the advantage to the consignee of remitting a draft on Cadiz, 1 peseta being worth 19.3¢ in Boston?

11. Find the value in U. S. money of 18,400 bu. corn at 3*s.* 5*d.* per bushel, exchange 4.86½.

12. A hardware dealer bought an invoice of cutlery from a Sheffield firm on 60 days' credit. Having decided to settle the account at once, he purchased and remitted a 60-day bill of exchange. If the commercial par of exchange was 4.85½, and the amount of the bill was £240 12*s.*, what was the cost of the bill of exchange, money being worth 4% per annum?

NOTE. Drafts on Great Britain have no *days of grace*, but *in* Great Britain 3 days of grace are allowed on all bills drawn payable after sight. Generally no days of grace are allowed on drafts drawn on any of the European countries.

13. Wm. J. Speer sends for a bill of goods for $2000, enclosing a draft on the Chemical National Bank of New York for $950 and directing you to draw on him at sight for the balance. He is at present at Berlin, on which city exchange is quoted at 94½. What is the face of the draft?

14. J. K. Holstetter, Amsterdam, wishes to purchase 500 bbl. flour at $5.25. He requests you to name your 30-day credit price in guilders. Exchange is quoted at .405. What is your price?

ORAL EXERCISE

1. What would you do in case you wished to purchase a bill of exchange on London?

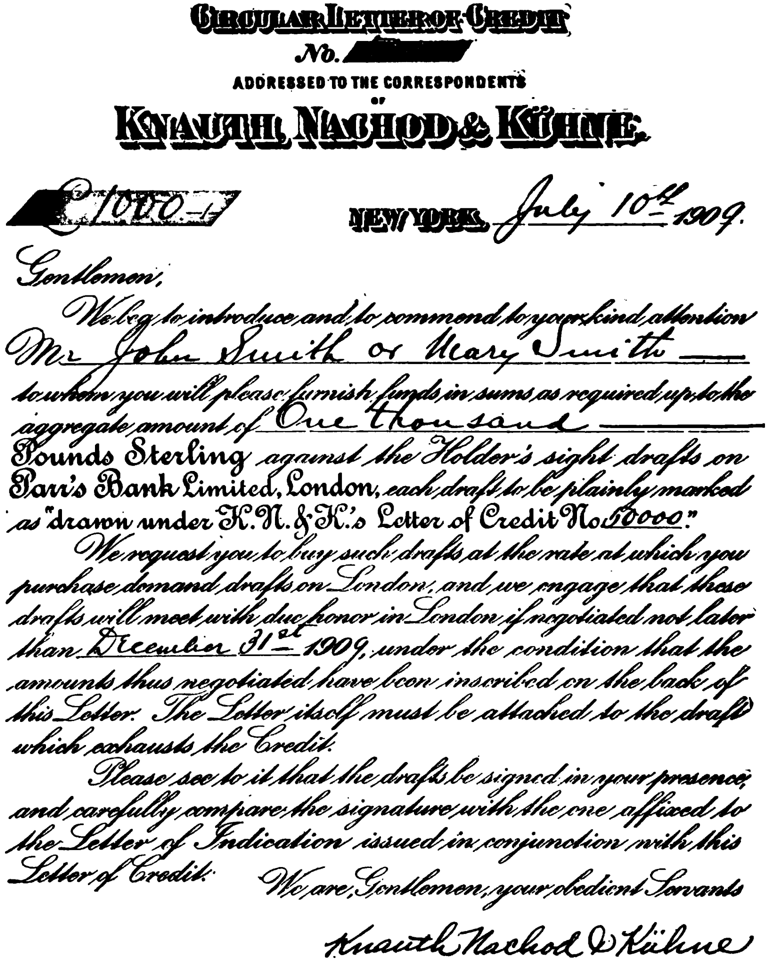
2. On what is brokerage reckoned?

3. What methods of transferring money abroad do you know, besides the bill of exchange?

4. What effect has time on the rate of foreign bills of exchange?

598. The more common forms of *foreign exchange* are the following: a draft, check, bill of exchange, money order, circular letter of credit, traveler's cheque, and cable transfer.

599. **A cable transfer** is a telegraphic order to pay a given sum of money to a certain person in a foreign country.



CIRCULAR LETTER OF CREDIT

No. [redacted]

ADDRESSED TO THE CORRESPONDENTS
OF
KNAUTH, NACHOD & KÜHNE.

£1000-

NEW YORK, July 10th 1909.

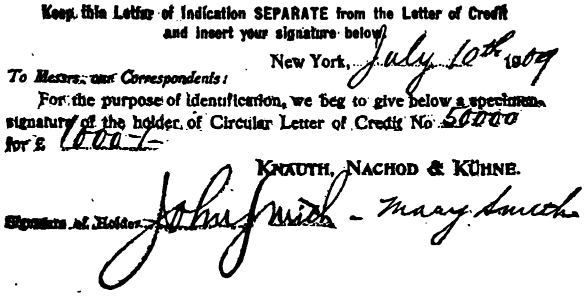
Gentlemen;

We beg to introduce and to commend to your kind attention Mr John Smith or Mary Smith to whom you will please furnish funds in sums as required up to the aggregate amount of One thousand Pounds Sterling against the Holder's sight drafts on Parr's Bank Limited, London, each draft to be plainly marked as "drawn under K.N.&K.'s Letter of Credit No 50000."

We request you to buy such drafts at the rate at which you purchase demand drafts on London; and we engage that these drafts will meet with due honor in London if negotiated not later than December 31st 1909, under the condition that the amounts thus negotiated have been inscribed on the back of this Letter. The Letter itself must be attached to the draft which exhausts the Credit.

Please see to it that the drafts be signed in your presence, and carefully compare the signature with the one affixed to the Letter of Indication issued in conjunction with this Letter of Credit. We are, Gentlemen, your obedient Servants

Knauth Nachod & Kühne



Keep this Letter of Indication SEPARATE from the Letter of Credit
and insert your signature below.

New York, July 10th 1909

To Messrs. our Correspondents:

For the purpose of identification, we beg to give below a specimen signature of the holder of Circular Letter of Credit No. 50000 for £ 1000.–

KNAUTH, NACHOD & KÜHNE.

Signature of Holder John Smith – Mary Smith

The principle of *cable transfer* is the same as the transfer of money by telegraph from one part of the country to another. The transfers are always sent in cipher, and a charge is made for each word in the address of the one to whom it is sent. The rate charged is the quoted or market rate of exchange, plus a commission.

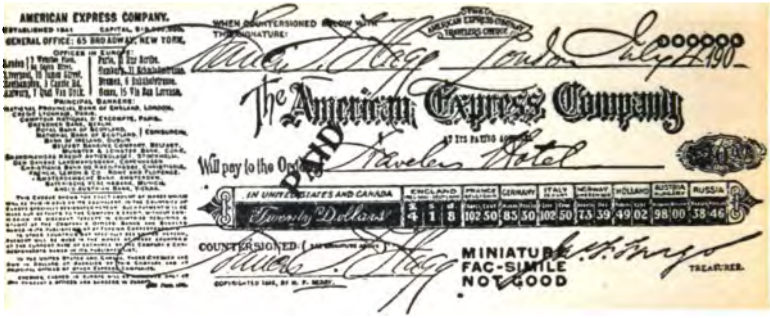
600. A **traveler's letter of credit** is a draft made by a banker in one country, addressed to foreign bankers, by which the holder may draw funds at different places in any amount not exceeding the limits of the letter of credit.

Travelers generally prefer letters of credit to bills of exchange, because they can draw at any time and at different places such sums as their convenience may require.

The holder of a letter of credit is required to subscribe his name at the bottom which provides a simple, yet effective, means of identification. Whenever he wishes money he has only to present his letter to any of the banks named in the list furnished him, and this bank will fill out a draft on the London banker to whom the letter is directed. The traveler signs the draft, and if the signature corresponds to that shown at the bottom of the letter, the amount desired is promptly paid and indorsed on the back of the letter. The bank cashing the draft which exhausts the credit is required to remit it to the London bank. Letters of credit are usually drawn in pounds sterling, but are paid in the current money of the country in which they are negotiated. Banks usually charge 1% commission for issuing a letter of credit, with a minimum rate of $5.00.

601. **Travelers' cheques** are a substitute for letters of credit and bills of exchange. They are issued in denominations of $10, $20, $50, $100, and $200. The charges are ½% for foreign cheques and ¼% for cheques payable in this country.

Form of Traveler's Cheque



AMERICAN EXPRESS COMPANY.
GENERAL OFFICE: 65 BROADWAY, NEW YORK.

WHEN COUNTERSIGNED BELOW WITH THIS SIGNATURE:

London July 4 190_ 000000

The American Express Company

AT ITS PAYING AGENCIES

Will pay to the Order of Travelers Hotel

PAID

IN UNITED STATES AND CANADA	ENGLAND	FRANCE	GERMANY	ITALY	NORWAY	HOLLAND	AUSTRIA	RUSSIA
Twenty Dollars	£4 1s. 8d.	102 50	85 30	102 50	73 39	49 02	98 00	38 46

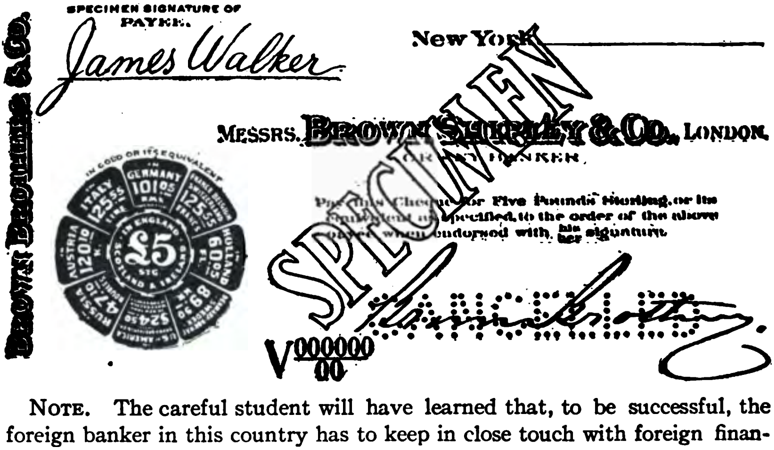
COUNTERSIGNED:

MINIATURE FAC-SIMILE NOT GOOD

TREASURER.

The express companies and several foreign-exchange banks and bankers issue traveler's cheques which are payable to one's own order by a large number of correspondents throughout the world. At the time of purchase, the buyer signs his name in the upper left-hand corner. When he wishes funds, he presents his cheque to the correspondent of the express company or bank and signs his name in the lower left-hand corner in the presence of the one to make payment. If the two signatures agree, the money is given him without question.

The following form of traveler's cheque is preferred by bankers as the holder is required to sign his name on the back, and therefore considered better because it is more difficult to forge as there is no signature near from which to copy.



BROWN BROTHERS & CO.

SPECIMEN SIGNATURE OF PAYEE.

James Walker

New York

MESSRS. BROWN SHIPLEY & CO. LONDON.

Pay this Cheque for Five Pounds Sterling, or its equivalent as specified, to the order of the above payee when endorsed with his/her signature.

SPECIMEN

V 000000/00

IN GOLD OR ITS EQUIVALENT
IN ENGLAND SCOTLAND & IRELAND £5 STG
GERMANY 101.05 RMK
FRANCE BELGIUM SWITZERLAND 125.55 FRANCS
HOLLAND 60.05 FL.
NORWAY SWEDEN DENMARK 89.30 KR
U.S. AMERICA $24.50
RUSSIA 47.10 ROUBLES
AUSTRIA 120.10 K.
ITALY 125.55 LIRE

NOTE. The careful student will have learned that, to be successful, the foreign banker in this country has to keep in close touch with foreign financial centers; he must receive daily reports by cable about money market conditions abroad, quotations for exchange of one country on another, and information about the interest rates at which foreign bankers are able to receive time bills for discount. It also comes within his province to finance the American traveler's needs abroad, whose expenditures are estimated to reach the enormous total of $100,000,000 a year. Foreign bankers also furnish to importers of merchandise Documentary Credits drawn in favor of shippers and available against delivery of shipping documents, and many and diversified are their functions as a go-between in the country's international money transactions.

WRITTEN EXERCISE

1. A traveler drew £75 6*s.* on a letter of credit. At 4.88 per pound sterling, how much was he charged for this item on settlement with the bank at which he obtained the letter, four months after this date, interest at 6%?

2. A tourist purchased a letter of credit and drew £50 at London, 800 marks at Berlin, 1200 francs at Paris. How much did the letter cost him if exchange is ¾ % premium at London, ¼% premium at Berlin, and ½% premium at Paris?

3. At 30¢. per word and 1% of the amount, find the cost of an eighteen-word cable money order from Philadelphia to Paris for 12,500 francs when exchange is quoted at 5.18½?

4. You sell F. Geurard, Paris, a bill of American groceries listed at $1324. You draw on him at sight at the rate of 5.20 and immediately sell it to your banker at 5.12, brokerage ⅛%. Do you gain or lose by the sale to the banker, and how much?

WRITTEN REVIEW

1. How large a draft on Amsterdam can you purchase on 1% commission for $202?

2. What must be paid in New York for a draft on Boston at 30 days for $5400, exchange being at ½% premium?

3. Find the face of a draft that cost $434.70, at ⅝% premium.

4. I hold a 70-day draft on Chicago for $2750. I sell the draft at ¼% premium and with discount off at 6% per annum; what do I receive?

5. A broker sold for me a bill on Liverpool, England, at

4.85¼ and charged ⅛% brokerage. What was the face of the bill, if the proceeds were $5040.29?

6. A man in Detroit wishes to draw on New Orleans for a bank stock dividend of $100. Exchange direct on New Orleans is at 1¼% discount. How much will he gain or lose by drawing on his agent in New York at 1½% premium, allowing his agent to draw on New Orleans at 1% discount, brokerage ½%?

7. John Wanamaker's agent in Brussels, Belgium, purchased for him carpets amounting to 50,000 francs on a commission of 3%. Wanamaker remitted him a 3-day sight draft to cover the cost of the carpets and the commission for buying. If exchange was 5.17½, and he paid for the draft by check, what was the amount of the check?

8. How many dollars are equal to £1 sterling, when $10 in United States gold coin, which is $\frac{9}{10}$ pure, weigh 258 grains and 1869 sovereigns, $\frac{11}{12}$ pure, weigh 40 pounds?

9. A miller sends 4000 bbl. flour to a commission merchant with instructions to sell the flour and remit the net proceeds by draft. The consignee pays $462.40 for freight and other expenses, sells the flour at $6.75 per bbl., charges 3% commission, and pays ¼% premium for the draft. Find the amount of the draft.

10. In either Ex. 5 or 7 is the balance of trade in favor of the United States?

STOCKS AND BONDS

Stocks

602. A **corporation** is a collection of individuals united by authority of law into one body under a special name and acting in many respects as an individual.

The instrument which defines the rights and duties of a corporation is called a *charter*. It is managed by officers elected by a board of directors who are chosen by the stockholders. The *capital stock* is divided into a certain number of *shares*, the *par value* of which is determined by the number of shares into which the stock is divided. They are commonly $25, $50, or $100. Stockholders may own any number of shares and participate in the profits according to the number of shares they own.

603. **Stocks** is a general term applied to shares of capital stock of all kinds, such as: railroad, bank, insurance, and other incorporated or joint-stock companies.

604. **Certificates** of **stock** are the written or printed documents issued by the officers of the corporation to the stockholders certifying the value of each share and the number of shares such certificate represents. The **capital stock** of a company is the total amount of all the shares that may be issued at their par value.

If a company is prosperous and makes more than its expenses, a *dividend* is usually paid to the stockholders. Dividends are declared at a certain per cent on the par value of the capital stock of the company, either quarterly, semiannually, or annually. An *assessment* is sometimes levied upon stockholders to make up losses or deficiencies.

The division of profits of a corporation is seldom the full amount of the gain, as it is desirable to create a *surplus fund* which may be drawn upon as required, for it is evident that in dull and unprosperous periods of business there may be no profits to divide. It is customary, therefore, to leave in the surplus sufficient funds to provide for a dividend or to meet other contingencies which may arise. Certain corporations, notably national banks, are required by law to carry a portion of their net profits to a surplus fund, before declaring dividends, until such surplus shall amount to a certain per cent of the capital stock.

605. Companies frequently issue two kinds of stock, *pre-*

ferred and *common*. Preferred stock gives the holder rights and privileges not enjoyed by the holders of the common stock. These rights usually include a prior claim for dividends.

Six per cent preferred stock would entitle the holder to an annual dividend of 6% before any dividend could be declared on the common stock. Upon dissolution the preferred stock is generally paid before the common. It is sometimes issued to take up the floating indebtedness of a corporation or to protect it from loss or disaster. Preferred stock is usually bought for investment and common stock for speculation.

606. Stock is sometimes issued to the stockholders of a corporation without a corresponding increase in the value of the property. Such stock is called **watered stock.**

The issuing of watered stock is usually for the purpose of either inflating the value of the stock of a corporation or for reducing the high rate per cent of profit which in some states is forbidden by law.

The rate of dividend which a company is able to pay, usually determines the market value of the stock. If the rate of dividend is higher than the current rate of interest, there usually will be a demand for the stock and it will sell at a *premium*. If the rate of dividend is lower, the demand will be slight and the stock will sell at a *discount*.

607. The business of buying and selling stocks is usually transacted through a **stock broker,** since it is difficult for individuals to buy and sell them personally. For this service the broker charges a certain rate per cent commission, called **brokerage,** on the par value of the stocks dealt in.

Brokerage is usually ⅛% of the par value of the stock bought or sold; a charge is also made for both buying and selling. The stock broker generally belongs to an organization called a *stock exchange* of which the most important in America is the New York Stock Exchange.

When stocks are quoted, 95, 105, 125½, etc., it means that one share, the par value of which is $100, can be bought for $95, $105, $125.50, etc. *Stocks bought* through a broker at 95 will cost the purchaser $95.00 + 0.12½ = $95.12½; *stocks sold* through a broker at 105½ will yield the seller $105.50 − 0.12½, or $105.37½. The *fractional quotations* in stock prices are always eighths, fourths, or halves, though in some of the cheaper stocks sixteenths are sometimes quoted. Usually brokers do not trade in less than 10-share lots.

608. The following taken from the *New York Times* is an abbreviated form of the stock quotations for a certain day on the

New York Stock Exchange. The list shows the opening, highest, lowest, and closing quotations, and the net changes from closing prices of the previous day.

THE STOCK MARKET

SALES		FIRST	HIGH	LOW	LAST	NET CHANGE
100	Allis-Chalmers	10	10	10	10	+¾
12,000	Amalgamated Cop.	74⅜	75⅛	74⅛	75	−¼
230	Am. Beet Sugar pf.	93¼	93¼	93¼	93¼	+1
200	Am. Car & Foundry	62½	63	62½	63	−¼
200	Am. Hide & Leather	6¾	6¾	6¾	6¾	..
100	Am. Ice Securities	26	26	26	26	..
6,200	Am. Smelt. & Ref.	81	81½	80⅞	80⅞	+⅝
550	Am. Smelt. & Ref. pf.	106½	106½	106⅛	106⅛	+2
200	Am. Sugar Refining	123	123	123	123	+2¼
4,675	Am. Tel. & Tel.	136½	136½	134	134¾	+1
600	Anaconda Cop. Min.	46½	46½	46⅛	46½	+⅜
4,550	Atch., Top. & S. F.	112	112⅜	111¾	112⅛	+½
200	Baltimore & Ohio	110¾	110¾	110¾	110¾	+⅜
1,460	Brooklyn Rapid Tran.	77¼	77¼	76¾	76¾	−¼
600	Canadian Pacific	181	181¼	181	181¼	+2½
1,300	Chi., Mil. & St. Paul	140½	141	140¼	140½	+⅝
100	Chi. & Northwestern	150⅜	150⅜	150⅜	150⅜	+⅞
500	Erie	29⅞	29⅞	29⅝	29⅝	−⅛
200	Erie 1st pf.	48	48	48	48	..
700	Great Northern pf.	134⅝	134⅞	134½	134½	+¼
500	Mo., Kansas & Texas	40¾	41	40½	40½	+⅞
100	Nat. Lead Co.	80¾	80¾	80¾	80¾	..
110	Nat. Lead Co. pf.	108	108	108	108	..
1,300	N. Y. Central	122	122¼	121¼	121¼	−⅝
400	N. Y., Ont. & West.	45	45¼	45	45¼	+⅜
1,100	Norfolk & Western	103½	103½	103	103	+½
100	North American	76½	76½	76½	76½	−¾
1,300	Northern Pacific	133½	134⅛	133½	134	+¼
5,600	Pennsylvania R. R.	135⅜	135⅞	135⅜	135⅝	−¼
200	Penn. sub. cts., 2d pd.	134¼	134¼	134⅛	134⅛	−⅛
200	P., C., C. & St. Louis	102½	102½	102½	102½	..
215	Pressed Steel Car pf.	101½	101½	101½	101½	..
100	Pullman Company	191	191	191	191	+½
5,100	Southern Pacific	124⅜	124¾	124	124⅛	−⅝
700	Southern R'way ext.	27¼	28¼	27¼	27⅞	+⅛
400	Texas & Pacific	31½	31¾	31½	31½	+¼
400	Texas Pac. Land Tr.	88½	88½	88	88	+¼
15,300	Union Pacific	183½	184⅛	183⅜	183⅞	−⅜
600	Union Pacific pf.	97½	97½	96½	96½	..
200	U. S. Rubber	43⅛	43¼	43⅛	43¼	..
200	U. S. Rubber 1st pf.	114½	114½	114½	114½	+⅛
100	U. S. Rubber 2d pf.	79¼	79¼	79¼	79¼	..
23,800	U. S. Steel	82¾	83¼	82⅝	82⅞	+1⅝
2,200	U. S. Steel pf.	119½	119½	119	119	+2

The plus sign in the above indicates an advance in the market of so many points, or fractions thereof, over the previous day's closing; the minus sign, a decline.

ORAL EXERCISE

1. Where are stocks and bonds bought and sold? Can any one go there and take part in the buying and selling? Is there any place in the city in which you live where dealing in stocks and bonds is carried on? What do you mean by a curb broker?

2. What does the following quotation mean: Penna. R. R. 135$\frac{5}{8}$? What is the difference between common and preferred stock?

3. What is capital stock? How do persons become members of a corporation? From whom does a company obtain the right to carry on business as a corporation? What is preferred stock?

4. Determine the cost at the closing price in the table, of 10 shares of each of the following stocks, adding $\frac{1}{8}$% for brokerage; Amalgamated Copper; Am. Sugar Refining; Baltimore & Ohio; Erie; N. Y. Central; Pennsylvania R. R.; Southern Pacific; U. S. Steel, com.

5. A company with $500,000 capital declares a 10% dividend. What does the holder of 50 shares receive?

WRITTEN EXERCISE

1. Show the gain on 28 shares of stock bought at 97$\frac{1}{2}$ and sold at 103$\frac{1}{4}$.

2. Find the quoted price of bank stock when the cost of 150 shares, including brokerage at $\frac{1}{8}$%, is $16,875.

3. A semiannual dividend of 3% on Southern Pacific R. R. stock bought at 124$\frac{3}{8}$ is equivalent to what per cent of annual interest on the investment?

4. The capital stock of the Acme Manufacturing Company is $120,000. During a year their gross earnings are $25,450 and the expenses $14,460. What even per cent of dividend may be

declared and what would be the amount of undivided profits if 10% of the net earnings is first set aside as a surplus fund?

5. A real-estate broker sold a section of land at $9.50 an acre, and invested the proceeds in railroad stock, receiving 1½% for selling the land and ¼% for buying the stock. What was his brokerage?

6. A man who bought stock at 2% above par, and sold it at 7% below par, lost what per cent of his investment?

7. A railroad company declared a dividend of 1¼% for the quarter ending Sept. 30, 1912. If the stock was quoted at 105, what was the rate of income per annum on an investment in the stock of that company?

8. Anaconda Copper Mining Company levied an assessment of 5% upon its stock for development purposes, 3% for current expenses, and 4% for machinery. What would be the total assessment on 500 shares of the stock, owned by A?

9. During a certain year the gross earnings of a railroad company which was capitalized for $200,000,000 were $116,123,450, and the operating expenses $92,965,445. How much was divided among the stockholders if the company declared an annual dividend of 8%, and how much remained in the undivided profits account?

10. How many even shares of Baltimore & Ohio R. R. stock can be purchased for $2688.65, brokerage ⅛%, and what would be the unexpended balance if they were bought at the opening quotation on the list, p. 341?

11. What sum must be invested in stocks bearing 6½% at 105 to produce an income of $1300 per annum?

12. What annual income will be derived from an investment of $5150 in 4½% stock at 102⅞, brokerage ⅛%?

13. According to the list p. 341, American Car & Foundry stock opened at 62½ and closed at 63. What was the per cent of advance for the day?

14. If 48 shares of 3% stocks be sold at 92⅝, and the proceeds be invested in 5% stocks at 110⅞, brokerage ⅛% for buying and selling, what additional income will be obtained?

Stock Speculation

609. Speculators usually buy and sell stocks on a **margin,** that is, they pay only a part of the purchase price and borrow the rest by depositing the certificate of stock as a collateral.

A person who buys and sells stocks on a margin is usually required to pay down 10% of the par value of the stock dealt in. The broker must, of course, pay for the stock in full.

The margin deposited with the broker is simply to protect the broker against losing any money should the stock move in the wrong direction. In case of its so doing, the margin must be made good by the deposit of an additional amount, otherwise the broker will sell the stock to protect himself from losing any of the money he has advanced.

A *bull* is a buyer of stocks which he hopes to sell at a profit. He acts on the belief that prices will go *up*, and is said to be "long" of stock. A *bear* is a seller of stocks which he does not possess, but borrows on the belief that prices will go *down*. Thus, if stock is quoted at 88, a *bear*, thinking it will go down to 80, may sell at 88, and deliver borrowed stock to the purchaser. If the stock goes down to 80, he will purchase it and return it to the owner, thus realizing a profit of 8 *points*. This is called selling stocks "short," and he is said to be "short" of stock.

Stock Exchanges

610. **A stock exchange** is an unincorporated body of brokers, organized for the purpose of creating a regulated market for the buying and selling of stocks and bonds. The most important in America is the *New York Stock Exchange*, the membership of which is confined to 1100.

There are smaller stock exchanges located in Philadelphia, Pittsburg, Boston, and a few other cities, but trading at these exchanges is confined to local stocks. A membership on a stock exchange is called a "seat." Seats on the New York Stock Exchange vary from $50,000 to $90,000, depending somewhat upon the general prosperity of the country. A seat on a local exchange is considerably less.

The floor of the New York Stock Exchange is open for business from 10 A.M. to 3 P.M., and all transactions of buying and selling are confined to these hours. As has been seen, the uniform charge for buying and selling securities dealt in at the Stock Exchange is usually one-eighth of one per cent ($\frac{1}{8}$%) on par value, or $12.50 on 100 shares of stock at the par value of $100 each.

611. Stocks are generally bought or sold on the Stock Exchange either "cash," "regular way," or "optional" purchases and sales. A stock sold "cash" is deliverable the day sold, and must be so stated at the time of the sale; a stock sold "regular way" is deliverable the next day, or bought "regular way" is to be paid for the next day. Where nothing else is specified, "regular way" is always understood. "Buyer's option," that is, deliverable at the option of the buyer at any time within the option period, which is generally not more than 30 days; "seller's option," that is, deliverable at the option of the seller any time within the option period.

Nearly all sales made are "regular." Transactions for "cash," or "regular" carry no interest; but on options over three days, interest at the legal rate on the selling value of the stock is paid by buyer to seller. There is no interest charged on "short sales."

612. Collaterals. Stocks, bonds, notes, or other value given in pledge as security, when money is borrowed.

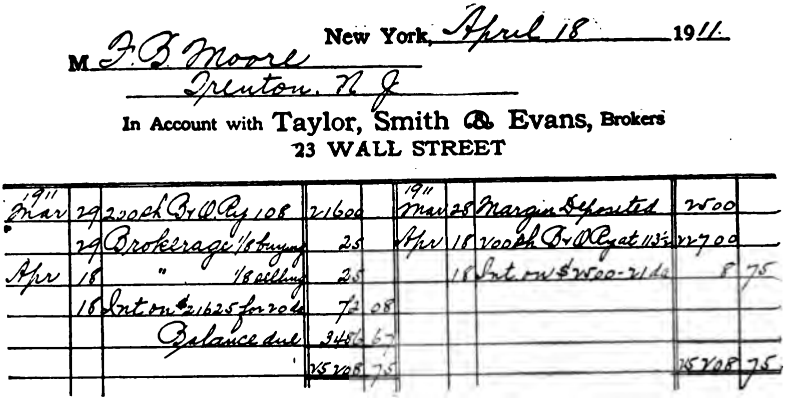
613. Stock sold **ex.-dividend** means that the stock does not carry to the buyer a recently declared dividend.

614. Explanation of words and phrases used in Wall Street.

1. *Buying In.* The act of purchasing stock in order to meet a "short" contract, or to enable one to return stock which has been borrowed.
2. *Buying on a Scale.* Buying at regular intervals on a declining market. "Short sales" are frequently made in the same way on an advancing market.
3. *Investment Buying.* This phrase is generally used in contra-distinction to buying for speculation, or for a quick turn in the market. It is understood to mean buying to hold for a considerable time.
4. *Pyramiding.* Enlarging one's operations by the use of profits which one has made.
5. *Stop Orders.* An operator may give his broker orders to close out his deals when quotations reach a certain point, and in that case, he is said to have given a "stop order."

615. The following example illustrates the method of finding the profit or loss when a person buys and sells stocks on a *margin:*

Example. March 28, F. B. Moore deposited with Taylor, Smith & Evans, his brokers, $2500 with instructions to purchase for him 200 shares of Baltimore & Ohio R. R. stock whenever they could do so at 108. The next day they succeeded in making the purchase; twenty days later, in accordance with instructions from Mr. Moore, they sold the stock at 113½ and sent him a check for the amount due him. What was the amount of the check?



New York, April 18 1911

M F. B. Moore
Trenton, N. J.

In Account with **Taylor, Smith & Evans**, Brokers
23 WALL STREET

1911 Mar	29	200 sh B & O Ry 108	21600		1911 Mar	28	Margin Deposited	2500	
	29	Brokerage 1/8 buying	25		Apr	18	200 sh B & O Ry at 113½	22700	
Apr	18	" 1/8 selling	25			18	Int on $2500-21 da	8	75
	18	Int on $21625 for 20 da	72	08					
		Balance due	3486	67					
			25208	75				25208	75

As may be seen from the above, the amount of the check is $3486.67. F. B. Moore's gain, $3486.67 − $2500 = $986.67.

Written Exercise

1. On a certain date the common stock of U. S. Steel was quoted at 82¾. Find the cost on that date of 24 shares, brokerage ⅛%.

2. A speculator purchased 100 shares of Adams Express stock at 150, brokerage ⅛%, held the shares until a quarterly dividend of 2% was declared and received, and then sold at 149⅛, brokerage ⅛%. How much was his gain by the transaction?

NOTE. Speculators often buy stocks and hold them for a time for the dividend they yield.

3. C owns 300 shares of Erie 1st pref., which cost him $12,000. He realizes annually 5% on his investment. What rate of dividend was declared?

4. May 20, a broker purchased for the account of a customer 200 shares of Brooklyn Rapid Transit stock at the opening

quotation on the list, p. 341. He deposited as a margin $3000. On June 2, the stock was sold at 81½. What was the gain, usual interest and brokerage?

5. Bought stock at 197⅝ and sold it at 194⅞, having meanwhile received a dividend of 6% on it. My net gain by the transaction, after paying ⅛% brokerage each way, is $336. How many shares, par value $40, did I buy?

6. A speculator bought Southern Pacific R.R. stock at 125, kept it a year, during which time a dividend of 5% was paid, and then sold the stock at 128¾. Find the per cent of gain per annum on his investment. No brokerage.

7. A man invests $4484 in C. M. & St. Paul R. R. stock at 140, brokerage ⅛, and receives semiannual dividends of 3½%. Find the annual rate of income on his investment.

8. A man bought 200 shares of Pennsylvania R. R. stock at 122⅜. The market price declined until it reached 118⅝ and then rallied to 121⅛. Believing that another decline was coming, he sold 500 shares at 121⅛, thus making him 300 shares "short" of the market. The price continued to rally, however, and he "covered" by buying 300 shares at 122. What was the net loss on the whole transaction, making no allowance for interest, but allowing ⅛% brokerage for each purchase and sale?

9. A man received $495 as dividend at 4½% on his bank stock, par value $100. He sold 40 shares at 143¼ and the remainder at 144½, paying ⅛% brokerage. What was the net proceeds of the sale?

10. Which is the better investment, and how much per cent, Atch. Top. & S. F. R. R. stock yielding 6% dividend, bought at 112½ or U. S. Steel, com. stock yielding 5% and bought at 90?

11. A speculator buys 500 shares of stock, par value $10, for $7.50 a share, paying 5¢ a share brokerage. Six months later, after having received a 5% dividend, he sells for $9.50 a share. How much does he make, money being worth 6%?

Bonds

616. A stock company often finds that the funds paid in by the stockholders are insufficient to meet the requirements of the company, and in order to meet this need it generally borrows money and issues **bonds,** secured by a mortgage on the property of the company. These bonds are written agreements to pay a certain sum of money within a stated time and at a fixed rate of interest.

Allis-Chal. sf 5s	
3000	79¾
1000	80
3000	79⅞
Am. Hide & L. 6s	
2000	100½
Am. T.& T. cv 4s	
60000	103
Am. Tobacco 4s	
13000	79⅞
10000	79¾
1000	79⅞
Atch.T&SF cv 5s	
10000	114½
Balt. & O.gold 4s	
1000	98⅜
5000	98½
10000	98⅝
Balt. & Ohio 3½s	
10000	90¾
Balt. & O SW 3½s	
500	90⅛
Beth Steel ex 5s	
5000	88
Bkn Rap Tr cv 4s	
10000	86⅛
1000	86
11000	85¾
Chi. B&Q. Ill 3½s	
5000	87½
Chi. & East Ill 4s	
1000	82⅞
ChiMil&St.P.3½s	
8000	86½
Chi. RI&PRR 4s	
52000	80
Chi. & W. Ind. 4s	
1000	92
Cin In StL & C 4s	
7000	98½
South Pacific 4s	
1000	90¾
So. Pacific ref 4s	
1000	94¾
5000 s20	94¾
16000	94¾
2000	95
Southern Ry 5s	
5000	106
So Rwy gm 4s	
4000	78⅝
Term R R Assn. of St. L cn 5s	
2000	115
Third Avenue 4s	
2000	65
Union Pacific 4s	
9000	100¼
12000	100⅜
U Pacific ref 4s	
2000	97⅛
Union Pac. cv 4s	
22000	108¾
5000	108⅝
U S of Mexico 5s	
2000	99¾
U S of Mexico 4s	
1000	96¼
U S Realty & I 5s	
9000	90⅞
10000	91
5000	91⅜
18000	91
U S Rubber 6s	
5000	103¼
U S Steel sf 5s	
3000	104¾
6000	104⅝
4000	104¾
11000	104⅝
5000	104⅛
7000	104⅝
10000	104½

The above list shows part of one day's *bond sales* on the floor of the N. Y. Stock Exchange as taken from the *New York Sun*, Wall Street Edition.

Bonds of business corporations are generally secured by a *mortgage* upon the property of the company; national governments, states, counties, and cities often issue bonds, without mortgages, the credit of such organization being considered good.

617. Government bonds are bonds issued by the Federal Government. Their names are usually derived from the interest they bear and the time when due.

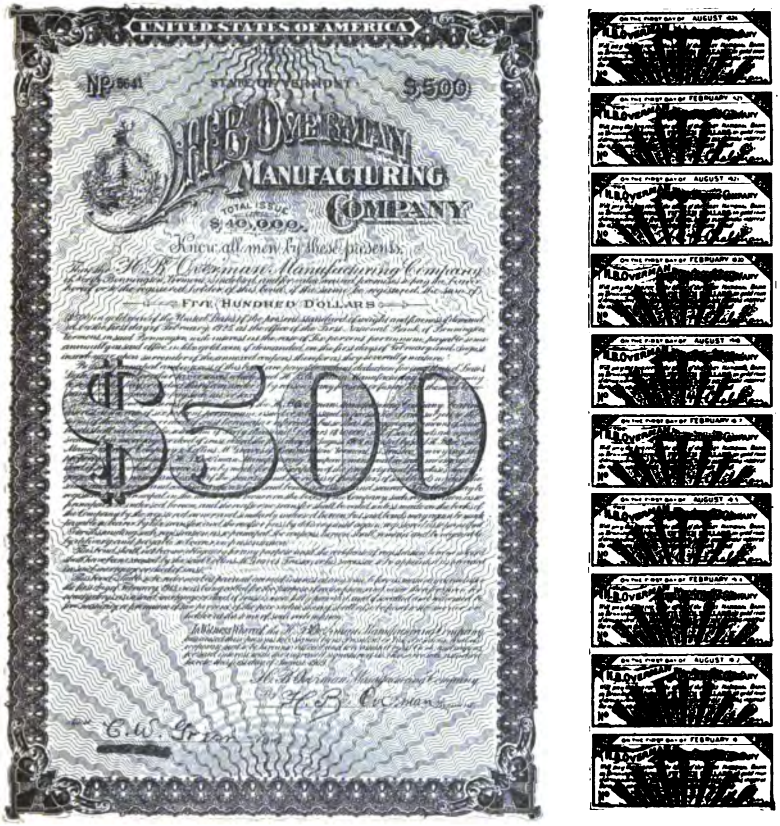
Thus, United States bonds bearing 4% interest, and due in 1912, are quoted "U. S. 4's, 1912"; U. S. bonds bearing interest at 2% and maturing in 1930, "U. S. 2's, 1930," etc.

The bonds of a business corporation with reference to their security are known as *first mortgage, second mortgage,* etc., according to their priority of lien, the class of property upon which they are secured, or other characteristics.

618. Bonds are of two kinds, *Registered* and *Coupon.*

Registered bonds are issued in the name of the owner, and are made payable to him or his assignee. Interest when due is sent direct to the owner as it has no separate contract for the payment of interest. Registered bonds must be recorded on the books of the corporation and can be transferred only by assignment and registry.

Coupon bonds are payable to bearer, and have small interest coupons attached, which are cut off when due, and the amount of interest is collected either personally or through a bank. There is a coupon for each period, and if these are not paid when due, they bear interest at the legal rate.



UNITED STATES OF AMERICA

No. 5641 STATE OF VERMONT $500

H.B. OVERMAN MANUFACTURING COMPANY

TOTAL ISSUE $40,000.

Know all men by these presents,

That the H.B. Overman Manufacturing Company

FIVE HUNDRED DOLLARS

$500

Bonds are discussed with stocks because they are bought and sold at the Stock Exchange, though there is little intrinsically common to stocks and bonds. *Bonds* pay regular interest at fixed rates; the income from *stocks* is variable.

619. The **par value** of bonds, like stocks, is their face value; the **market value** is the sum for which they can be sold in the market. Bonds are said to be **above par,** or **at a premium,** when they are worth more than their face value; **below par,** or **at a discount,** when they are worth less than their face value.

The broker's commission for buying and selling bonds is the same as for buying and selling stocks.

ORAL EXERCISE

1. Tell two ways in which bonds differ from stocks. How are bonds secured?

2. How are stock dividends and bond incomes always computed? If you should buy $5000 worth of 5% bonds quoted at 113½, and $5000 4% bonds quoted at 90½, what rate of income would you receive from both investments?

3. What do the expressions "bulls" and "bears" mean as used in the stock exchange?

4. What kind of bond is shown in the illustration, p. 349? How are registered bonds transferred? Coupon bonds?

5. What do you understand by Republic of Cuba 5's, 1915? What amount invested in Republic of Cuba 5's, 1915, at 105⅞, will secure an annual income of $500, brokerage as usual?

6. What name is given to bonds issued by the United States; by a state; by a city? An investor bought a 5% interest-bearing bond at 119⅞. Not considering the question of the maturity of the bond, what rate of income did he receive on his investment?

WRITTEN EXERCISE

1. A speculator bought through a broker 100 shares of Penna. R. R. stock when it was quoted at 114⅜. He held it a year and received 5% dividend. He then sold it at 118⅛. What did he gain? Would it have been better to have loaned the money on first mortgage at 5%?

NOTE. Hereafter it will be understood, unless otherwise stated, that all sales and purchases are made through a broker whose commission is ⅛% both for buying and for selling.

2. How much money must be invested in U. S. 4's of 1925 at 120 to produce an annual income of $600 from the bonds? No brokerage.

3. A gentleman wishes to invest in 4½% bonds, selling at 102, so as to provide for a permanent income of $1620. How much should he invest?

4. When New York Central 3½% bonds are bought for 90⅜, what per cent is realized on the investment?

5. A man invests $4275 in 4% bonds at 94⅞, and at the end

of the year sells out at 90⅛ and invests half the proceeds in 3½'s at 80⅞, and the other half in 6's at 112⅜. Find the change in his yearly income.

6. Jan. 1, 1912, a certain township borrowed $40,000 with which to build a high school, and issued 5% 10-year coupon bonds as security. The entire issue was sold to a broker at 108¾; how much did the township receive? If B bought four $1000 bonds at 110½ from the broker, what did they cost him and what was his rate of income?

7. A company with a capital of $200,000 paid 8% dividend to its shareholders. Afterwards new stock was issued and with the same amount of gain the company paid only 5%. What was the amount of the new stock issued?

8. What rate can you afford to pay for stock paying regular annual dividends of 8%, in order to realize 6% on the investment?

Suggestion. $8.00 ÷ .06 = 133⅓.

9. What must be paid for 5% bonds, that the investment may yield 8%?

10. How much premium may be paid on stock that pays 10% dividends, so as to realize 6% on the investment?

11. A man owns a house which rents for $1350, and the tax on which is 2½% on a valuation of $9000. He sells for $13,505, and invests in stock at 92⅜ that pays 7% dividends. Is his yearly income increased or diminished, and how much?

12. If a broker invested on his own account in Chi. R. I. & P. R. R. stock 4's as quoted in the list, p. 341, what per cent income would he receive?

13. At what price must 6% bonds, maturing in 5 years, be purchased to enable the holder to realize 5% on his investment, simple interest?

Solution. $100 + ($6 × 5) = $130, am't of $100 bond at maturity.
$1 + ($.05 × 5) = $1.25 am't of $1 for given time at 5%.
$130 ÷ $1.25 = $104, sum to be paid.

14. If 3% bonds, maturing in 20 years from date of investment, are purchased at 89⅞ through a broker, what rate per cent will be realized on the sum invested, simple interest?

Solution. \$100+(\$3×20)=\$160, am't of \$100 bond at maturity.
\$160−(89⅞+⅛)=\$70, income from investment in 20 yr.
\$90×.01×20=\$18, int. on sum invested at 1%.
\$70÷\$18=$3\frac{8}{9}$%, rate of income.

15. What must I pay for 5% bonds, maturing in 15 years, that my investment may yield 4%?

16. What rate of interest do I receive on my investment, if I buy 6% bonds maturing in 20 years at 125?

Produce Exchanges

620. There are a number of different exchanges located in many of the large cities throughout the country which deal in grain, cotton, provisions, etc., and which supply a regulated market for such products. These are variously called: *Board of Trade, Produce Exchange, Chamber of Commerce,* etc.

The most important of these exchanges is the *Chicago Board of Trade.* On this exchange are bought and sold a large part of the cereals and meat products of the United States, and it practically determines the price of these commodities, not only for this country but for the world.

Trading in grain or other commodities differs but little from stock operations. Grain, seeds, cotton, or other commodities are bought and sold for present or for future delivery. "Cash contracts" are contracts for present delivery; "futures," for future delivery. Most of the speculative trading in cereals and cotton is in "futures."

In Board of Trade parlance parties who have sold for future delivery are called "shorts" and those who have bought for future delivery are called "longs." Hence the term frequently used on an advancing market, "shorts" are covering or buying in, and on a declining market "longs" are unloading, selling out or realizing.

Not less than 5000 bu. of grain, 250 bbl. pork, 250 tier. of lard, and 50,000 lb. of short rib sides and other meats are traded in as a rule.

The accompanying table shows

	Opening	*Highest*	*Lowest*	*Closing*
Wheat—				
May.........	1 07½	1 07½	1 05¾	1 06⅝
July..........	1 01¾	1 01¾	1 00⅜	1 01⅛
September....	1 00	1 00	98⅞	99½
Corn—				
May.........	57⅞	57¾	56½	57¼
July..........	60⅞	60⅞	60	60½
September ...	62⅝	62¼	61½	62
Oats—				
May.........	42⅝	42⅞	41½	42⅛
July..........	40¼	40⅛	29⅜	40
September....	38	38¼	37¾	38¼
Mess Pork, per bbl.—				
May.........	21 10	21 20	20 65	20 60
July..........	21 10	21 47½	20 90	20 90
September....	20 95	21 40	20 90	20 90
Lard, per 100 lbs.—				
May.........	12 00	12 32½	11 80	11 85
July..........	12 00	12 22½	11 80	11 85
September....	11 95	12 25	11 75	11 85
Short Ribs, per 100 lbs.—				
May.........	11 80	12 10	11 80	11 87½
July..........	11 80	12 00	11 70	11 75
September....	11 80	12 07½	11 70	11 70

the opening, highest, lowest, and closing prices of certain provisions for a certain day on the *Chicago Board of Trade.*

May wheat, signifies wheat to be delivered in May; *July wheat*, to be delivered in July; etc. May, July, September, and December are usually the months for future delivery.

The following margins are usually required from customers to buy or sell the various commodities; on *wheat*, $50 for each 1000 bu.; on *corn* and *oats*, $30 for each 1000 bu.; on *pork*, $250 on each 250 bbl.; on *lard*, $500 for each 250 tier.; on *short ribs* or other *meats*, $250 for each 50,000 lb.

During excited times when prices change rapidly and vary widely, larger deposits are often required, and when the market fluctuates slightly smaller deposits are often accepted.

Broker's commission for buying and selling grain, ⅛¢ per bu.; for pork, 5¢ per bbl.; for lard, 6¢ per tierce; for short ribs, 25¢ per 1000 lb.

NOTE. In the following problems it is assumed that the transactions are effected through a broker who charges the usual commission.

WRITTEN EXERCISE

1. Give the net cost for buying 5000 bu. corn at the closing price in the table, July option.

2. A speculator deposited with his broker $500 as a margin, and directed him to purchase for his account 10,000 bu. corn at the opening price in the accompanying table, July option. Three days later he sold the corn for 63½¢, what was his net gain?

3. Calculate the net loss from buying 15,000 bu. May wheat at the opening price in the accompanying table, and selling the same at the closing price. Usual brokerage.

4. An operator, supposing that May oats would advance in value, ordered his broker to buy 10,000 bu. at the opening price in the table, and at the same time deposited with him $300 as a margin. The market, however, declined and the operator ordered him to sell at the closing price for the day. How much was his loss?

5. April 20, a broker received from a customer a remittance of $500 as a margin with orders to sell short for his account 500 bbl. mess pork, May option, at the opening price in the table. One week later, the customer ordered him to "cover" his short sale as the market had declined. He did so at $19.75 per bbl. What was the speculator's net profit by the transaction?

6. C sold 20,000 bu. of July corn at the highest price and "covered" his short sale at the lowest price in the table. Did he gain or lose, and how much?

REVIEW EXERCISE

1. What shall I pay for 5% bonds to realize 7% on my investment?

2. How many shares of Western Union Telegraph stock can be purchased for $9338, at 83¼, usual brokerage?

3. A cement company divides $75,000 among its stockholders. What is the rate of dividend, the capital stock being $1,000,000? How much is paid to a person who owns 140 shares of $25 each?

4. What must be invested in Atchison 5's at 114½ to secure an annual income of $1500?

5. A bond and mortgage for $800 given for three years at 6% per annum was sold the day it was made for such a sum as would net the purchaser 5% per annum on his investment. For how much was it sold?

6. A speculator bought 80 shares of B. R. & P. R. R. stock at 16¾, and sold the same at 17⅝, brokerage ⅛. How much was his gain by the transaction?

7. The preferred stock of a railroad company is $50,000,000, and the common stock $100,000,000. The company declares a dividend of 6% on the preferred stock and 2% on the common stock. What is the surplus, if the net earnings are $5,476,345.

8. The gross receipts of a railway company in a certain year are apportioned thus: 40% to pay working expenses, 54% to give shareholders a dividend at the rate of 3½% on their shares, and the remainder, $42,525 is reserved. What was the paid-up capital of their company?

9. The present income of a railway company would justify a dividend of 3¾% if there were no preferred stock, but as $1,200,000 of the stock consists of such shares which are guaranteed 5% per annum, the ordinary stockholders receive only 3%. What is the whole amount of stock?

10. My broker, after selling for me 150 shares of N. Y. Cen-

tral stock, remitted to me $18,375. At what price did he sell the stock?

11. The capital stock of a company is $2,000,000, ¼ of which is preferred, entitled to a 6% dividend, and the rest common. If $82,500 is distributed in dividends, what rate of dividend is paid on the common stock?

12. On Mar. 30, H deposited $800 as a margin with his broker, who bought for him 5000 bu. Sept. wheat at 97⅜¢, and 10,000 bu. Sept. corn at 60⅛¢. One week later the broker sold the grain at the highest quotation in the list, p. 352. How much money should the broker return H in settlement?

13. The directors of a steel company decide to increase their stock, by declaring a stock dividend of 6%; that is, the stock of the company was "watered" to the extent of 6% of its holdings. If the market value of the old stock was 132½, what should be the value of the new stock?

14. An operator, supposing C. M. & St. P. R. R. stock would decline in value, ordered his broker to sell short 100 shares at 141, and at the same time deposited with him as a margin $1000. The broker on receiving the order sold for his account 100 shares at 141. When the market price declined to 134½, he ordered the broker to "cover his short sale." What was the speculator's gain?

15. I have $12,750 to invest. I can buy 3% bonds at 81, and 5% bonds at 120. How much must I invest in each kind so that my income may be the same from each?

16. A speculator holding $11,771.25, 6% stock, finds that by selling out and investing the proceeds in 7% stock at 107½, he can increase his annual income by $6.57; before he can effect the transfer each stock has increased in value 2 points. By how much would his annual income be now increased?

17. A capitalist had $20,000 to invest. He purchased $8700 Union Pacific 4% bonds at 103, and $7300 Canadian 3½% bonds at 93½, and invested the balance as far as he could in bank stock, par $100, at 149¼, paying half-yearly dividends of 4%. What was the gross amount of his investment, he paying usual brokerage for buying each class of securities. What was his annual

income from these investments? What average rate per cent per annum did he receive?

EXAMINATION TEST

(Selected from State Papers)

1. A man bought stocks at 25% discount and sold them at 20% above par; what per cent did he make?

2. I bought a 6% mortgage for $2500 at 5% discount, with two years to run. What interest do I get on the money invested if the mortgage is satisfied at maturity?

3. You desire to purchase 100 lb. nutmeg. The Rankin Spice Co. offers goods at 65¢, less discounts of 10%, and 5%. The Ceylon Spice Co., at 70¢, less discounts of 5%, 5%, and 10%. You accept the lower offer, and pay by 30-day draft purchased at 25¢ per $100 discount. What did the goods cost you?

4. Sept. 13, a broker bought for a customer 500 shares of railroad stock at 78¾, and the customer deposited with the broker $5000 as a "margin" for the latter's security against loss by a decline in the price of the stock. October 7, the stock was sold at 75½. How much of the margin remained after the sale? Usual brokerage, and interest at 6%.

5. A merchant in Winnipeg owes $4000 in New York. Exchange on New York is ¼% premium, but exchange on Chicago is ½% discount, and from Chicago on New York ¾% premium. Compare the cost of a bill on New York direct, with that of one through Chicago, which would pay the debt.

6. A person buys 6% city bonds, the interest on which is paid yearly, and which are to be paid off at par 3 yr. after time of purchase. If money be worth 5%, what price should be given for the bonds?

INSURANCE

621. **Insurance** is a contract whereby for a certain consideration one party undertakes to indemnify the other, or his representatives, against loss by certain risks.

Insurance is classified as *personal* and *property insurance.* Insurance on the *person* is in the form of life, endowment, accident, and health insurance, and insurance on *property* is in the form of fire, marine, tornado, transit, live stock, plate glass, boiler insurance, title insurance, insurance against bad debts, etc.

Property Insurance

622. **Property insurance** is that which indemnifies the owner against loss by fire, wind, or other casualty.

Fire insurance contracts include loss as the result of fire in close proximity, the heat from which damages the property insured; also, loss or damage by water from the fire engine or from the exposure or theft of the goods during their removal to a place of safety at the time of a fire, and for the destruction of buildings to prevent fire from spreading.

623. A **policy** is a written contract between the insured and the insurer, and the sum to be paid by the company in case of loss, the **face** of the policy. The **insurer,** sometimes called the **under-writer,** is the one who agrees to indemnify, and the **insured** is the person protected against loss or damage. The **premium** is the sum paid by the insured for the insurance.

The premium is usually computed as a certain per cent of the face of the policy, or as a certain sum on each $100 of insurance. It depends upon the nature of the risk and the length of time for which the policy is issued. In either case it is called the *rate of insurance.*

624. The **term of insurance** is the time for which the property or person is insured. Insurance for less than a year is called a **short rate.**

The term of fire insurance is usually from 1 to 5 years; marine insurance, is usually for a voyage or from port to port. The term of life insurance is usually the life of the insured or for a period of years.

If an insurance policy is terminated at the request of the policy holder, the company retains the customary "short rates" for the time the policy has been in force; if terminated at the option of the company, a ratable proportion of the premium is refunded for the unexpired term of the policy.

625. **Insurance companies** may be classified according to the way in which they are organized; as *Stock Companies*, *Mutual Companies*, and *Mixed Companies*.

A *Stock Insurance Company* is one in which the capital is owned by the stockholders, who share all the gains and are liable for all the losses. A *Mutual Insurance Company* is one in which the parties insured share the gains and bear the losses. A *Mixed Insurance Company* is one in which the profits, above a stipulated sum paid to the stockholders as a dividend, are divided among the insured.

626. Policies of insurance may be classified as **open** and **valued.** A *Valued Policy* is one that specifies the value to be paid to the insured in case of loss. An *Open Policy* is one that specifies the maximum of value to be paid in case of loss or damage. In this form of policy additional insurance may be entered at any time at rates and under conditions agreed upon, and no fixed value is given to the thing insured.

The amount which may be recovered under an *open policy* is not fixed in advance by the policy, but is left to be ascertained after loss. Whether a policy is to be regarded as open or valued depends upon the intention of the parties. The words "valued at," following the description of the property, are usually conclusive in rendering the policy a valued one.

627. The standard policy contains a *pro rata clause*, under which the insured can not recover more than the amount of his loss in the property insured, where there is more than one policy on the same property. This rule does not apply to the case of several persons with different interests in the same property, but to the case of any insurer who, if he recovered the full amount on all policies, would be getting double insurance upon the loss.

Thus a man may have his house insured in three companies, as follows: in number one for $2000, in number two for $3000, and in number three for

$4000. There is a loss by fire to the amount of $4500. The insured can recover only this amount, and the companies will be compelled to pay their *pro rata* portions.

628. Among some of the special features of fire insurance the following may be mentioned:

1. One who has insured another has himself an insurable interest which will enable him to validly insure himself, in turn, against loss of the risk insured. This is called *re-insurance.*
2. The insured must have an insurable interest in the risk or property insured.
3. The policy must contain a description of the property insured.
4. A fire insurance policy is not assignable.

ORAL EXERCISE

1. What is the cost of $4000 insurance at $\frac{5}{8}\%$ premium?
2. What is meant by *insurable interest?* What is *re-insurance?* What is meant by *short rate?*
3. What will it cost to insure my house against loss by fire for 3 yr., if my property is worth $5000 and the annual premium is $\frac{1}{5}\%$?
4. A insures his store in the Queen for $2000; Hartford, $3000; Continental, $3000; Manchester, $4000; and in the Standard for $3000. It suffers damage to the extent of $5000. All these policies contain the usual provisions as to other insurance which all of them permit. What companies are liable for the loss and for what sums?
5. Goods worth $5000 are insured for $\frac{4}{5}$ value. If the annual premium is $50, what is the rate?
6. If B takes out a fire insurance policy of $1000 on a house worth $2000 and the house burns, what amount can he recover? If the insurance policy is for $3000 and the house is worth $2000, how much can he recover?

629. The computations in insurance are based on the principles of percentage. The corresponding terms are as follows: The *amount insured* is the *base;* the *rate of premium* is the *rate;* and the *premium* is the *percentage.*

WRITTEN EXERCISE

1. A stock of goods valued at $18,500 is insured for $\frac{3}{5}$ of its value, at $1\frac{3}{8}\%$. Find the premium.

2. A house that cost $8250, rents for $750 a year, the insurance is $\frac{3}{5}\%$, and repairs $\frac{1}{2}\%$ every year. What rate of interest does it pay?

3. Mr. A insured his house, worth $48,000, for one year for $\frac{5}{8}$ of its value, at $\frac{3}{4}\%$. What would the insurance company lose if the house should burn?

4. The premium for insuring $\frac{2}{3}$ of a house at 4% was $32. What was the value of the house?

5. At $1\frac{1}{8}\%$, the premium on a policy covering $\frac{3}{4}$ of the value of the stock of merchandise amounts to $\$189.84\frac{3}{8}$. Find the value of the entire stock.

6. A town paid $9 for insuring the school furniture, books, etc., of a public school, the insurance being effected on $\frac{2}{3}$ the value of the property at the rate of $\frac{3}{4}\%$. What was the estimated cost of the articles insured?

7. I took a risk at $1\frac{1}{2}\%$; re-insured $\frac{2}{5}$ of it at 2%, and $\frac{1}{4}$ of it at $1\frac{5}{8}\%$. What rate of insurance do I get on what is left?

8. A company insured a block of buildings for $150,000, at $\frac{3}{4}\%$; but the risk being too great; it re-insured $40,000 in another company, at $\frac{7}{8}\%$, and $35,000 in another company at $\frac{4}{5}\%$. How much premium did it receive more than it paid?

9. A flouring mill valued at $150,000 was insured Jan. 1, for one year as follows: in the Des Moines for $\frac{1}{3}$ of its value at $\frac{3}{5}\%$; in the Globe for $\frac{1}{4}$ its value at 1%; and in the Standard for $\frac{1}{5}$ its value at $\frac{2}{3}\%$. Before the year was up, the mill was totally destroyed by fire. What was the owner's total loss?

10. The contents of a store, valued at $8500, was insured at the rate of 75¢ per $100. For what must the policy be written in order to cover loss and premium in case of total destruction?

11. I insured my house and furniture at an expense of $33.60, paying $\frac{3}{5}\%$ premium for insuring the house and $1\frac{3}{5}\%$ for the furniture. The amount of insurance on the house was $1\frac{1}{2}$ times that on the furniture, what was the amount of insurance on both?

12. A building which cost $2000 was fraudulently insured

at 3%, so that when it burned the owner gained $425. What was the amount of the policy?

13. In a town where the regular police force consists of 20 or more patrolmen a company will insure a bank against burglary for 1 year at 50¢ per $100 up to $3000, and 25¢ per $100 above that amount. How much will it cost to insure a bank for $25,000 against burglary in such a town?

14. A cargo of goods valued at $14,500, is insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$ premium. In case of the total destruction of the goods, how much will the owner lose?

15. A warehouse with its contents was insured for $175,000, and was damaged by fire and water to the extent of $115,000. Of the risk $50,000 was taken by company A, $30,000 by company B, $25,000 by company C, and the remainder in equal parts by companies X and Y. What was the net loss of each company if the rate of premium paid was $1\frac{1}{8}\%$? But the agents were allowed to retain 10% of the premiums.

16. The Acme Rubber Co.'s factory and its contents are insured as follows:

Building No. 1, value $6,000, insured for $3,000 in Co. M.
Building No. 2, value 12,000, insured for 6,000 in Co. N.
Building No. 3, value 8,000, insured for 7,000 in Co. O.
Building No. 4, value 14,000, insured for 12,000 in Co. P.
Building No. 5, value 26,000, insured for 20,000 in Co. Q.

Premium in each case 85¢ per $100; cost of policy and survey $4.50 each. In a fire, buildings and contents of 1, 2, and 4 were totally destroyed, while building and contents of 5 were damaged to the extent of $17,500, and No. 3 to the extent of $1000. In building No. 2, $700 worth of contents was removed to safety; in No. 5, the insurance company sold the goods, damaged and otherwise, for $5464.

Estimate, the cost of insurance; amount of loss each company will pay; loss uncovered by insurance; and the gross loss covering the cost of the policies.

Marine Insurance

630. **Marine insurance** is insurance against loss or damage

to a vessel or its cargo, by storms, or other perils incident to navigation.

631. Salvage is a compensation allowed to those who voluntarily assist in saving a ship or her cargo from marine disasters.

Marine policies, and some fire-insurance policies, usually contain what is called an "average clause," which provides that, in the adjustment of a *partial loss* of property that was *partially insured*, the sum allowed by the company will be such a part of the loss as the *insured value* is of the *whole value* of the property. Hence, if a vessel is insured for ⅔ its value, and is damaged to the extent of $6000, the owners will be entitled to only $4000. However, in case of total loss, the owners can collect the full value of the insurance.

WRITTEN EXERCISE

1. A cargo of goods worth $8670 was insured for ¾ of its value at 1½% premium. In case of shipwreck, what would be the actual loss of the owners?

2. What premium must be paid to insure a cargo of 4880 bu. wheat, valued at $1.04 per bushel, at 1¼%, the policy being for only ⅝ of the value?

3. A cargo of goods valued at $60,000 was insured for $50,000 under marine insurance, average clause policy. How much would the company be obliged to pay if the merchandise was damaged $36,000?

4. A stock of goods was insured March 1, for 2 years for $8000, at $1.50. The policy was canceled 1 year later at the request of the insured. How much was the return premium, the "short rate" for 6 months being 65¢? How much would have been returned by the company, if the policy had been canceled at its request?

5. A merchant owns ⅔ of a steamship, and insures ⅗ of his interest at ⅝%, paying $337.50 premium. What was the value of his interest in the steamer? If during the continuance of the policy the vessel be damaged in a collision to the extent of $35,000, what sum will the merchant be entitled to receive from the insurance company, the policy containing the "average clause"?

6. For what sum must a vessel worth $18,000 and a cargo worth $24,000 be insured at $\frac{1}{4}\%$ and $\frac{1}{2}\%$ respectively to cover the total value and the premium paid?

Personal Insurance

632. Personal insurance may be divided into three classes: *life*, *accident*, and *health* insurance.

1. *Life Insurance* is a contract between an insurance company and the insured to pay a certain sum of money, upon the death of the insured, to a specified person called the *beneficiary*, to his heirs, or to himself if he survive a certain number of years.
2. *Accident Insurance* is an indemnity for misfortunes occasioned by accident.
3. *Health Insurance* is an indemnity for loss in case of sickness from specified diseases.

633. Policies issued by life insurance companies are usually of four kinds, *Life*, *Limited Payment Life*, *Endowment*, and *Term Policies*.

1. *Life policies* are payable to the beneficiaries at the death of the insured.
2. *Limited Payment Life* policies are whole life policies on which the premium payments are limited to a certain number of years after which the policy is fully paid.
3. *Endowment* policies are payable to the person insured if living at the end of a definite period of years, or to his heirs in case of death within that period.
4. *Term* insurance is temporary insurance payable only in the event that death occurs within a specified term.

Life-insurance companies are known as "*Old Line*," and *Assessment*, or *Fraternal* companies. Old-Line companies adopt the *level premium* plan which has a fixed amount during the life of the policy, as distinguished from the charges of Assessment-insurance organizations, which are subject to increase. The latter depend upon monthly assessments or "calls" to pay death claims.

634. The **expectation of life** means "average after life time."

Table of Expectation of Life

AGE	EXPECTANCY	AGE	EXPECTANCY
15	45.50	49	21.63
16	44.85	50	20.91
17	44.19	51	20.20
18	43.53	52	19.49
19	42.87	53	18.79
20	42.20	54	18.09
21	41.53	55	17.40
22	40.85	56	16.72
23	40.17	57	16.05
24	39.49	58	15.39
25	38.81	59	14.74
26	38.12	60	14.10
27	37.43	61	13.47
28	36.73	62	12.86
29	36.03	63	12.26
30	35.33	64	11.67
31	34.63	65	11.10
32	33.92	66	10.54
33	33.21	67	10.00
34	32.50	68	9.47
35	31.78	69	8.97
36	31.07	70	8.48
37	30.35	71	8.00
38	29.62	72	7.55
39	28.90	73	7.11
40	28.18	74	6.68
41	27.45	75	6.27
42	26.72	76	5.88
43	26.00	77	5.49
44	25.27	78	5.11
45	24.54	79	4.74
46	23.81	80	4.39
47	23.08	81	4.48
48	22.36	82	4.18

The total after life time in years of those living at any age divided by the number alive at that age.

The "expectation of life" can not be made the basis of an exact calculation of monetary values that depend upon the contingency of death. The term is used to designate the average number of years of life remaining after having attained a given age.

635. The ***premiums on life insurance*** are graded according to the age of the risk. The person insured must undergo a physical examination, as only healthy persons are insured. The amounts of the premiums are determined by average results computed upon the length of life of a large number of persons carefully arranged and tabulated. These results so arranged are called ***mortuary tables***.

The elements which make-up the premium rates are: 1st, *Reserve*, or that part of the premiums of a policy required by law to be set aside as a fund for the purpose of meeting obligations as they fall due; 2nd, *Surplus*, or a fund in excess of all liabilities. Generally this fund may be used for such purposes as the company deem best; 3rd, *Loading*, or commonly a percentage of the net premium to meet expenses and contingencies.

636. A **dividend** is the unused portion of premium returned to the policy holder. It exists solely because, by skill and economy, the management has carried on the business at less cost than that assumed in computing the premiums.

Dividends may be used: (1) to decrease the cost of insurance by taking the dividend in cash; (2) to purchase additional insurance by buying full-paid additions: (3) to shorten the time to run.

637. The following table shows the **premium rates*** of a leading life-insurance company:

Annual Premiums for a Policy of $1000.

WHOLE LIFE POLICIES Payable at Death only					ENDOWMENT POLICIES Payable as Indicated, or at Death, if Prior.						
Age.	Payments for life.	20 Payments.	15 Payments.	10 Payments.	Age.	Due in 10 Years.	Due in 15 Years.	Due in 20 Years.	Due in 25 Years.	Due in 30 Years.	Due in 35 Years.
20	......	28 29	34 21	46 29	20	$101 57	65 93	48 36	37 93	31 25	26 73
21	$18 76	28 73	34 73	46 98	21	101 78	66 11	48 45	38 03	31 36	26 86
22	19 18	29 18	35 26	47 69	22	102 01	66 29	48 54	38 14	31 48	27 01
23	19 61	29 65	35 82	48 43	23	102 24	66 47	48 64	38 25	31 62	27 17
24	20 07	30 13	36 39	49 19	24	102 48	66 59	48 75	38 37	31 76	27 35
25	20 55	30 63	36 98	49 97	25	102 73	66 69	48 86	38 50	31 92	27 54
26	21 05	31 15	37 60	50 78	26	102 99	66 80	48 98	38 65	32 09	27 75
27	21 57	31 69	38 23	51 62	27	103 26	66 92	49 12	38 80	32 27	27 98
28	22 12	32 25	38 89	52 49	28	103 47	67 04	49 26	38 96	32 48	28 23
29	22 70	32 83	39 57	53 39	29	103 58	67 17	49 41	39 15	32 70	28 50
30	23 31	33 44	40 28	54 31	30	103 71	67 32	49 57	39 34	32 94	28 80
31	23 95	34 07	41 01	55 27	31	103 84	67 47	49 75	39 56	33 20	29 13
32	24 63	34 72	41 77	56 27	32	103 98	67 63	49 94	39 79	33 50	29 50
33	25 34	35 40	42 56	57 30	33	104 14	67 81	50 15	40 05	33 82	29 90
34	26 09	36 11	43 38	58 36	34	104 30	68 00	50 38	40 33	34 17	30 33
35	26 88	36 85	44 23	59 46	35	104 48	68 21	50 64	40 64	34 56	30 81
36	27 71	37 63	45 12	60 60	36	104 67	68 44	50 91	40 99	34 99	31 34
37	28 60	38 44	46 04	61 78	37	104 88	68 68	51 22	41 37	35 46	31 92
38	29 53	39 28	46 99	63 01	38	105 10	68 96	51 55	41 79	35 98	32 56
39	30 52	40 17	47 99	64 27	39	105 35	69 25	51 92	42 25	36 55	33 26
40	31 56	41 10	49 03	65 58	40	105 62	69 58	52 33	42 76	37 18	34 03
41	32 67	42 08	50 11	66 94	41	105 91	69 95	52 78	43 32	37 88	34 86
42	33 84	43 11	51 25	68 36	42	106 23	70 35	53 29	43 95	38 65	35 78
43	35 09	44 20	52 44	69 82	43	106 59	70 79	53 84	44 64	39 49	36 78
44	36 41	45 35	53 68	71 35	44	106 98	71 29	54 46	45 40	40 43	37 88
45	37 82	46 57	54 99	72 94	45	107 42	71 84	55 15	46 25	41 45	39 07
46	39 32	47 85	56 36	74 60	46	107 91	72 45	55 91	47 19	42 58	.....
47	40 92	49 22	57 81	76 32	47	108 46	73 14	56 75	48 23	43 82	.....
48	42 62	50 67	59 33	78 12	48	109 06	73 89	57 69	49 37	45 18	.....
49	44 43	52 22	60 93	80 00	49	109 74	74 73	58 73	50 63	46 66	.....

The premium rates as exhibited by the foregoing table are all participating; that is, the holder of a policy is entitled to share in the *profits*, or *dividends* of the company. Premiums on life policies are to be paid during the entire life of the insured, or during the period indicated in the table. The endowment policy provides for the payment of the face of the policy in 10, 15, 20, 25, 30, or 35 years from the date of issue, or at the death of the insured if it occurs before the close of the stated period.

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* Northwestern Mutual Life Insurance Co.

Premium rates of *old line* companies are made higher than the above considerations render necessary in order to secure safety to the policy holder. The portion of the premium unused at the end of any year may be returned to the policy holder in the form of an annual dividend as indicated in 636, or it may be allowed to accumulate for a term of years, called the *accumulation period.*

638. Some states require all insurance companies organized under its laws to place a **cash value** upon every policy issued so that in case the insured should surrender his insurance he may secure an *equitable return* for the reserve accumulated. Many companies, though not required to do so by law, have incorporated the cash surrender values in their policies.

20-Payment Life, $1,000

End of Year	Age 35		PREMIUM: A. $36.85 S. A. 18.79 Q. 9.49		
	Loan	Cash Value	Paid up Policy	Extended Insurance	
				Years	Days
2	$ 32	$ 34	$ 80	3	360
3	55	58	131	6	255
4	77	82	183	9	144
5	101	107	234	11	340
6	127	134	287	14	110
7	153	162	340	16	127
8	180	190	393	18	36
9	209	220	445	19	215
10	237	250	497	20	319
11	267	282	549	21	363
12	298	314	600	22	361
13	330	348	652	23	329
14	362	382	703	24	284
15	397	418	753	25	248
16	431	454	802	26	219
17	466	491	851	27	245
18	502	529	901	29	8
19	539	568	950	31	24
20	578	609	Policy full-paid		
21	589	621			
22	600	632			

In case the insured ceases to pay premiums before the maturity of the policy, ordinarily he may choose one of several options as illustrated by the accompanying table. These options are generally endorsed on the policy and therefore become a part of its contract. They are as follows:

1. The company will *loan* up to the then cash surrender value, less 5%, provided extended term insurance be not in force. 2. Receive a certain amount of *cash* at once. 3. Secure *participating paid-up insurance* to be paid in case of death at any time. 4. *Extended insurance* for the face of the policy for as many years and days as its reserve value will purchase. These privileges are granted in consideration of the premiums already paid.

639. Other forms of insurance somewhat similar in character to personal insurance are:

1. *Employer's Liability Insurance* is protection afforded to the employers of men engaged in manufacturing or other business, against liability for damages for personal injuries caused by the negligence of the employer or his servants. The occasion for this class of insurance has arisen because of the fact that as soon as an employee in a factory is killed by reason of

some faulty machinery his survivors sue the company for damages. The insurance company in which the employer has insured this risk defends the case and if the proprietor is defeated, the insurance company pays the loss.

2. *Fidelity* or *Guaranty Insurance* is a contract by hich an employer is insured against loss by the fraud or dishonesty of his employees. It is, in fact, a guarantee of the honesty of an employee. These insurance companies also issue bonds guaranteeing the faithful performance of contracts as well; such as paving, tunneling, etc.

20-Year Endowment, $1,000

Age 35 PREMIUM: A. $50,64 S. A. 25.83 Q. 13.04

End of Year	Loan	Cash Value	Paid up Policy	Extended Insurance Yrs.	Extended Insurance Days
2	$ 57	$ 60	$ 98	7	46
3	92	97	154	11	185
4	128	135	209	15	203
5	166	175	264	15	*$ 75*
6	206	217	319	14	*157*
7	247	261	373	13	*235*
8	290	306	427	12	*311*
9	335	353	480	11	*383*
10	381	402	531	10	*452*
11	429	452	582	9	*519*
12	479	505	633	8	*583*
13	531	559	682	7	*644*
14	584	615	730	6	*703*
15	640	674	778	5	*759*
16	696	733	824	4	*811*
17	756	796	869	3	*862*
18	817	861	913	2	*910*
19	881	928	957	1	*956*
20	....	1000	....	...	....

Pure Endowment

640. *Insurable Interest.* Every person has an insurable interest in his own life and the life of any one whose death would naturally cause him a pecuniary loss or disadvantage; as, a partner in the life of his co-partner, a creditor of a partnership in the life of each partner, a woman in the life of a prospective husband. However, mere relationship is not enough to give an insurable interest, there must be the eleme t of dependency.

WRITTEN EXERCISE

1. From the table find the annual premium required for a life policy for $3500, age 22; for a 15-payment life policy for $5000, age 34.

2. From the table find the annual premium required for a 20-year endowment policy for $8000, age 30; for a 35-year endowment policy for $6000, age 25.

3. Mr. A, aged 35, insured his life on the 20-payment life plan, for $5000, and after making five annual payments decided to discontinue paying premiums. How much paid-up insurance, exclusive of the surplus, might he receive? How much did he pay in annual premiums?

4. A person is insured for $5000 at an annual premium of $28.90 per $1000 and dies after making nine payments, how much more will his heirs get than has been paid in premiums?

5. B at the age of 32 took out a $3000 life policy and at the age 35 a $2000 25-year endowment policy and a $2500 20-year

endowment policy. How much does the insurance exceed the premiums paid if he dies at the age of 44?

6. The Northwestern Life Insurance Company issues a 20-payment life policy for $20,000 to a person who lived long enough to pay 16 annual premiums amounting to $11,555.20. What was the age of the insured at the time of his death?

7. A life insurance company offers straight insurance to a man 29 years of age, for an annual premium as per the table p. 365; to the same man they offer a 10-year endowment policy, the premiums in each case being paid at the beginning of the year. The man takes out a straight life policy for $1000, and deposits in a savings bank the difference between the premiums of the policies, which pays 4% a year, compounded annually. The man dies at the end of five years. How much better off are his heirs than if he had taken an endowment policy?

8. A person aged 27 years takes out a 10-year endowment policy for $5000; the dividends reduce his annual premiums 15% on the average. Computing annual interest at 6% on his premiums, does he gain or lose, and how much?

9. A man aged 36 years took out a life policy for $12,000, on the 15-payment plan, and died 4 yr. 6 mo. afterward. What was gained to his estate by insuring, calculating compound interest on his payments at 4%, also adding three dividends of $95 each?

10. A insured his life at the age of 37 for $4000 in a 20-payment life policy. What would be the actual protection at the end of 5 years, if money was worth 4% compound interest?

11. Mr. D takes out a $5000 15-payment life policy at the age of 42. At the end of 10 years, instead of continuing the insurance, he surrenders for a cash value of $2376.25, which includes the accumulated dividends. Allowing $15 per annum per $1000 for protection afforded, what rate of interest has his money earned in the 10 years?

12. B and C are partners, aged 35 and 45 years respectively. Each insures his life in favor of the other by a 20-payment life policy for $10,000. At the end of 8 years, C died, and now B receives the face of C's policy and also surrenders his own for

the then cash value. How much more than the premiums paid in by both men does B get?

13. Taking the expectancy of life as the basis, per the table p. 364, what should be the annual premium rate on a $1000 policy for a man aged 57, allowing a 3% compound interest rate for reserve, 20% of the premium as a surplus for annual losses, and 10% for loading?

14. Which is the better, and how much, a 20-year endowment policy at the age of 22 for $5000 or an equal amount of the premiums deposited in a bank each year, interest compounded annually at 4%, allowing $40 each year for dividend, to reduce the premium?

15. In Ex. 12, if the amount of the two premiums had been placed at compound interest at 4%, compounded annually, how much less than the sum received by B would they have amounted to?

REVIEW EXERCISE

1. A man pays $27.50 premium for having his house insured for five years at 1¼% on ⅔ of its value. What is the value of the house?

2. A gentleman wishes to invest in 4½% bonds, selling at 102, so as to provide for a permanent income of $1620. How much should he invest?

3. A boy is 17 years of age. How much must be invested for him at 5%, compound interest, that when he is 21 he may have $2000?

4. A man 35 years of age took out a life policy for $5000 for the benefit of his wife, paying the entire premium at the rate of $480.52 on $1000 in one payment. He died in 5 years after securing the policy; how much less would the company have received if he had paid the premium at the annual rate?

5. Mr. D age 42, has $9024 at interest at 6%, which he intends to leave to his family. How much will he leave them if he takes out a life policy and pays the premiums with the interest on his investment of $9024?

STATE AND LOCAL TAXES

ORAL EXERCISE

1. Assessed value, $4000; rate, ⅜%. Required the tax.
2. Assessed value, $5000; tax, $25. Required the rate.
3. Tax, $30; rate, 1½%. Required the assessed value.
4. If a man pays annually for public purposes 1% of the value of his property, estimated at $15,000, what is the amount of his tax?

641. The *city* and *village* must have money for public improvements, schools, fire protection, police, etc. The *county* needs it for bridges, public buildings, educational purposes, salaries, etc. The *state* must have it for the care of the blind, deaf and dumb, insane, criminals; for salaries of state officials, educational purposes, etc. These expenses are met by **taxes.**

The expenses for maintaining the National Government are met chiefly by duties on imports, internal revenue tax, sales of public lands, etc.

642. There are two kinds of taxes: **direct taxes,** which are taxes levied on the property, income, or business of a person; **indirect taxes,** which are taxes levied by the General Government on imported goods, and on liquors, tobacco, etc.

Taxes levied by the state and local government are mostly *direct taxes,* assessed upon real and personal property. In addition to the property tax most states collect a *capitation* or *poll tax* of from $1 to $3 from each male citizen over 21 years of age and under 50, which is levied without regard to the property he owns.

The laws in most states exempt the property belonging to religious and charitable institutions from taxation.

643. An **inheritance tax** is an assessment imposed on estates received by inheritance.

Inheritance taxes have been instituted in many of the states as a source of domestic revenue. The justice and utility of such a tax have been testified to by experience and the decision of the law courts.

644. The method of levying and collecting taxes is practically the same in all the states. An **Assessor** is elected in every city and town whose duty it is to estimate the value of each piece of property within his district. He prepares an *Assessment Roll*, or *schedule*, containing the names of all persons liable to taxation within his district, and the valuation of each one's taxable property.

In many states a *County Board of Equalization* reviews and passes upon the original assessments before passing an abstract of them to a State Board of Equalization. The county board corrects in cases where assessed valuations are regarded as being too high or too low, and all appeals must be made to this board. The taxes are then equitably apportioned among the different counties, cities, and towns. Each town, city, county, and school district also levies taxes for its own local expenses.

645. A **tax receiver** is a public officer appointed to receive and collect taxes. The **tax rate** is a certain per cent of the taxable value of the property assessed. The rate is usually expressed as so many mills on the dollar or so many dollars on a hundred or a thousand.

The taxable value of property is its estimated worth, but it is frequently assessed at from 20% to 33⅓% less than its market value.

Taxes are generally assessed and made payable in money, but road taxes are sometimes made payable in days' work. Collectors receive a commission on the amount collected or a fixed salary.

ORAL EXERCISE

1. What are the two general divisions of taxes and by whom are they levied?

2. What is an assessment roll and by whom is it prepared? What is an inheritance tax, and do you have such a tax in your state?

3. A person's tax is $20. If he pays for 4 polls at $1.25 each and his property is valued at $1000, what is the rate of taxation?

NOTE. If a poll tax is included, it must be deducted from the total tax to find the property tax.

4. If the district school tax is 4 mills, the county tax 2 mills, and the state tax ⅖%, what should you pay on a property assessed at $5000?

5. If the rate of taxation is 75¢ on each $100, how much tax should B pay on his property, the assessed valuation of which is $8000?

WRITTEN EXERCISE

1. A tax of $327.50 is levied on a school district having a valuation of $113,600. Find B's tax on an assessment of $4250.

2. A school house is to be built at a cost of $8000. The property of the town is assessed at $1,000,000. What is the rate of tax, and what tax will a man pay whose property is assessed at $16,000?

3. If the tax rate in a village is 4½ mills on the dollar, and the total tax is $8641.35, what is the assessed valuation?

4. If the State House tax was ⅛ of a mill per $100, what was the valuation of a man's property who paid $5 State House tax?

5. The one-mill tax in a certain school district is $620; the rate of the general tax is 1¼%. What is the amount of the general tax assessed upon the district?

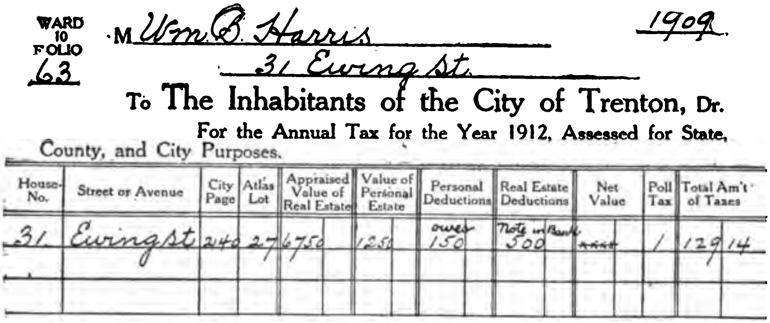
6. The valuation of a certain village is $225,600; the number of polls 241, at 75¢ each. A's property is assessed at $1560, and he pays 2 polls. If his tax amounts to $24.90, what is the entire tax to be raised?

Collecting Taxes

646. The methods of **collecting taxes** varies in different states. The one which seems most satisfactory is for one collector in each city or town to collect all the taxes due the city, town, and county at one time, when the money is distributed to the various purposes for which it was raised.

If taxes are not paid, the property taxed may be sold to satisfy the claims of the state. In some states if the poll tax is not paid, the person taxed may be committed to jail.

647. The following illustrates a form of **tax notice** as sent out by a tax receiver:



WARD
10
FOLIO
63

M Wm B. Harris 1909

31 Ewing St.

To The Inhabitants of the City of Trenton, Dr.

For the Annual Tax for the Year 1912, Assessed for State, County, and City Purposes.

House-No.	Street or Avenue	City Page	Atlas Lot	Appraised Value of Real Estate	Value of Personal Estate	Personal Deductions	Real Estate Deductions	Net Value	Poll Tax	Total Am't of Taxes
31	Ewing St	240	27	6750	1250	owes 150	Note in Bank 500		1	129 14

Terms of Payment. If this bill is paid on or before Dec. 1 a discount of 2% will be allowed. If paid after Dec. 1, and on or before Dec. 20, a discount of 1% will be given. On bills not paid until after Dec. 20, interest will be charged at the rate 7% per annum, dating from Dec. 20.

WRITTEN EXERCISE

1. What will be the amount of a man's tax on an assessed valuation of $5000, the rate of taxation being 2½% and the fee for collecting 2%?

2. A farmer paid $45 taxes. He paid 1½% school tax, 1% town and county, and 5 mills state tax. He owned 80 acres of land. What was it worth an acre, if the assessment was ⅓ of the market value?

3. From the illustrative tax notice above, how much is levied on the property 31 Ewing St., and what is the rate of taxation if the one poll is valued at $1.25? If Wm. B. Harris had failed to pay his tax until Dec. 30, how much would his tax have been?

4. The cost of maintaining the public schools of a city for a certain year was $49,579.40, and the assessed value of real estate was $5.460,250, and of personal property $2,167,350. How many mills on a dollar must be assessed for school purposes? If 5% of the tax cannot be collected, how many mills on a dollar must then be assessed?

5. The expense of constructing a bridge was $2749.95, which was defrayed by a tax upon the property of the township. The rate of taxation was 2¼ mills on a dollar, and the commission

for collecting 3%; what was the assessed valuation of the property of the township?

6. Taxes were levied in a certain city for the following purposes: public schools $30,600; support of poor $2500; interest on borrowed money $2375.50; auditing accounts $2500; printing $650; salaries $8500; public library $3954.75; public highways $9460; general expenses $6500; sinking fund $10,000; county tax $4210.50; state tax $3275.85. What was the rate of taxation if the total value of real and personal property, as made by the assessors, was $6,233,050; 2624 polls were assessed $1.25 each; and $6450 was collected from license fees? What was the tax of Mr. A, whose valuation was $7650 and who pays for 3 polls?

7. A has a lot and building 50 feet frontage worth $12,500, B has a building and lot 40 feet frontage worth $7500, and C has 2 vacant lots 80 feet frontage worth $5000. Divide a tax of $345 for street improvements among them.

8. The statutes of New York provide that an inheritance tax of 1% shall be paid on all property values of over $10,000 to lineal heirs, and 5% on all amounts or values over $500 to collateral heirs. M, who died possessed of property in the state, willed $16,275 to his daughter, and a sum to a friend sufficiently large so that the inheritance tax for both amounted to $442.75. What was the value of the estate?

UNITED STATES CUSTOMS DUTIES

ORAL EXERCISE

1. From what source does the United States Government derive its revenues?

2. Does any of the money raised by taxation in your county or state go for the support of the government?

3. What is a port of entry? Name the ports nearest you.

4. Name at least five articles on the *free list.* What is an internal revenue tax? On what goods is it levied?

648. Duties, or **customs** are taxes levied by the government upon imported goods for the purpose of revenue and the protection of home industries. **Customs revenue** is a tariff or duty on imports collected at *custom-houses* established by the government at ports of entry. **Internal revenue** is a tax on spirits, tobacco, etc.

649. A **custom-house** is an office established by the National Government for the transaction of business relating to duties or customs.

The waters and shores of the United States are divided into collection districts in each of which there is a *port of entry.* In each of these districts there may exist also one or more *ports of delivery* at which vessels may discharge their cargoes if they have previously entered and cleared at a port of entry. All ports of entry are also ports of delivery.

650. A **tariff** is a schedule of goods and the rates of duties imposed by Congress on the same. A **free list** is a schedule of such articles as are exempt from duty.

651. Duties are of two kinds: ad valorem and specific. An **ad valorem duty** is a certain per cent levied upon the net cost of the goods in the country from which they are exported. A **specific duty** is a fixed sum upon the weight or quantity of the goods imported without regard to their value.

Some imported goods are admitted *duty free,* others pay both *ad valorem* and *specific duty.* Cut glass, laces, and machinery pay an ad valorem duty. Tin plates, horses, and wheat pay a specific duty; while pencils and cigars, pay both duties. Books published in foreign languages are admitted duty free.

In all computations of ad valorem duties, the fractional part of a dollar equaling or exceeding ½ is regarded as $1; and fractional parts of a dollar less than ½ are rejected. In computing specific duties, the long ton of 2240 lb. is employed in cases of weight.

652. **Allowances** are made for the tare, leakage, breakage, etc., before the duties are computed. **Tare** is an allowance made for the box, crate, or covering containing the goods. **Leakage** is an allowance made for waste of liquids imported in casks, barrels, etc. **Breakage** is an allowance made on account of waste of goods shipped in glass or other breakable material.

653. The principal officer of every collection district is the *collector,* who is assisted by deputy-collectors, surveyors, appraisers, weighers, gaugers, inspectors, etc. There is also in

the leading ports of entry, a "naval officer," whose department is a check upon that of the collector.

The *collector of the port* is the officer who collects duties for the government. An *appraiser* is an officer who examines imported merchandise and determines its dutiable value. The *surveyor* takes charge of the vessel and cargo, ascertains the weight and quantity subject to duty, and delivers the merchandise to the importer.

One package of every invoice and one package, at least, out of every ten similar packages is sent to the public store for examination, but bulky and heavy articles are examined at the wharf where they are unloaded.

654. Every vessel from a foreign port must have a **manifest** of her cargo, a document signed by the master, stating the port of departure, the consignee, the port of delivery, and the items of cargo.

All invoices must be made out in the weights and measures of the country from which the importation is made, and all merchandise subject to a duty *ad valorem* must be invoiced in the currency of the country or place into which it is imported.

The *value of foreign coins* is periodically proclaimed by the Secretary of the Treasury and these values are taken in estimating duties. In cases when the currency of a country from which goods are imported has a depreciated value, a consular certificate is attached to the invoice, to show the amount of such depreciation. See 589 for value of foreign coins.

655. A **bonded warehouse** is a storehouse in which goods on which the duties are unpaid are stored under bond and in the joint custody of the importer, or his agent, and the customs officer.

By this plan an importer may store his goods in a government warehouse, by giving a bond for double the amount due to pay the duties when the goods are withdrawn from the warehouse. Often foreign goods are bought to advantage, earlier than they can be placed on the market, and stored in the government warehouse until they are seasonable, the only expense being storage, and labor charges.

Warehoused goods may be withdrawn at any time within a year in quantities not less than an entire package, or in bulk, if not less than one ton, on payment of duties; and on payment of 10% extra the time may be extended to three years. Beyond this time they shall be regarded as abandoned to the government, and sold under certain regulations, and the proceeds paid into the Treasury. If warehoused goods are re-exported, duties are not required; if goods on which duties have been paid are exported, the duties are paid back, except 1%. The amount refunded is called a ***drawback***. Similarly, on goods subject to internal revenue duties, a drawback is paid by the government when they are exported.

656. **Tonnage** is a tax levied upon a vessel, independent of its cargo, for the privilege of coming into a port of entry.

657. **Internal revenue,** or excise duties are taxes levied on certain goods produced and consumed in the United States.

Half of the internal revenue now comes from taxes on distilled spirits, about ⅕ from taxes on fermented liqucrs, about ⅕ from taxes on tobacco, and the remainder from the tax on oleomargarine and some minor items. Ordinarily the internal revenue tax will not only assess but collect itself. The manufacturer must purchase stamps, which for beer, tobacco, and cigars are destroyed when the packages are opened.

658. The illegal introduction of goods into a country other wise than through the regular ports of entry is called **smuggling.**

This is done either by not entering them at a Custom House, or by showing less than their real value in the invoice. It is a crime, for the prosecution and punishment of which stringent laws are enacted.

UNITED STATES CUSTOMS DUTIES ON CERTAIN IMPORTS

ARTICLES AND DESCRIPTION	DUTY	
	Specific	Ad Valorem
Boots and Shoes		15%
Cement	8¢ per 100 lb.	
Cigars and Cigarettes	$4.50 per lb.	25%
Carpets	10¢ per sq.yd.	40%
Coal, bituminous	45¢ per ton.	
Coke		20%
Cotton Cloth	1¢ to 8¢ per sq.yd.	
Furs, dressed		20%
Hay	$4 per ton	
Honey	20¢ per gal.	
Iron Ore	15¢ per ton	
Lumber, boards,planks not planed	50¢ per M- ft.	
Marble, in block	65¢ per cu. ft.	
Musical Instruments		45%
Nails, horseshoe	1-1/2¢ per lb.	
Oil, castor	35¢ per gal.	
Oil, cod liver	15¢ per gal.	
Opium, not adulterated	$1.50 per lb.	
Paintings and Statuary		15%
Plate glass	8¢ per sq. ft.	
Pins, not jewelry		35%
Shingles	50¢ per M-	
Silk, yarns	45¢ to 60¢ per lb.	
Timber	1/2¢ per cu. ft.	
Tobacco, unmanufactured	$1.85 per lb.	
Wines, champagne, quarts	$9.60 per doz.	

ORAL EXERCISE

1. What name is given to the schedule of rates or duties?

2. By whom is the value of foreign goods certified? Under what conditions is merchandise forfeited to the United States?

3. What are bonded warehouses? How does the tariff affect goods in bond that are reshipped abroad? What is a drawback?

4. Calculate the duty on a piece of marble 5′×4′×2′ at 60¢ per cu. ft.

5. Calculate the duty on 22,400 lb. of hay at $4 per ton.

6. Calculate the duty on 100 doz. penknives, valued at $1 per doz., at 25¢ per doz., and 25% ad valorem.

Finding a Specific Duty

WRITTEN EXERCISE

1. Find the duty on 234,500 ft. of lumber at 50¢ per M.

Solution. 234,500 ÷ 1000 = 234.5 M.
50¢ × 234.5 = $117.25.

2. Find the duty on 50 hhd. of sugar, each containing 500 lb., at 95¢ per 100 lb.

3. Using the table on page 377, find the total duty on: 25 blocks of marble, each 6′×3′×2′; 1,232,000 lb. bituminous coal; and 620 M. shingles.

4. A wine merchant in Philadelphia imported from Havre 200 doz. quart bottles of champagne, and 25 casks of sherry wine, each containing 30 gal. What is the duty, the rate on the champagne being $9.60 per dozen, and on the sherry 60¢ per gal.?

5. What is the duty on 600 lb. of cast hollow ware, at 2¢ per lb., breakage and tare 5%?

6. I imported 324 pieces of colored cotton goods, 32 in. wide, having 38 yd. in each piece; 64 pieces of woolen bunting, each 38 yd. long, 42 in. wide. What is the total duty, the rate on the cotton goods being 5½¢ a sq. yd., and on the woolen bunting 20¢ per sq. yd.?

7. From the table find the duty on an importation of 654 lb. of unadulterated opium.

Finding an Ad Valorem Duty

WRITTEN EXERCISE

1. From the table on page 377 find the duty on 860 yd. of carpet, invoiced at 6*s*. 6*d*. per yard.

Solution. 6½*s*.×860=$1360.19 original cost.
$1360×40%=$544 ad valorem duty.
10¢×860=$86 specific duty.
$544+$86=$630 total duty.

2. From the table find the duty on $5425 worth of boots and shoes.

3. If the duty on linen collars and cuffs is 40¢ per doz. and 20%, what is the duty on 20 doz. collars at 75¢ a dozen and 40 pr. of cuffs at 25¢ a pair?

4. An art collector imported from Paris paintings valued at 125,400 francs upon which there was a duty of 20%. What was the total cost of the paintings, American money?

5. A grocer imported 150 cases of port wine, 24 bottles in each case. After 5% had been allowed for breakage, he paid an ad valorem duty of 20%. The freight and cartage expenses were $100, and the whole cost was $4384; what was the invoice price per bottle?

6. Calculate the duty on 40 pieces of plate glass, each piece 24″×30″, valued at 1 mark per sq. ft., and imported under a duty of 8¢ per sq. ft. and 10% ad valorem.

7. Find the gross cost of a purchase of 1000 lb. of figs from a French wholesaler at ⅕ fr. per pound. Exchange is quoted at 5.1, and a duty of 20% ad valorem.

8. The duty on wine containing 26% or less of spirits is 25¢ per gallon, and 30% ad valorem; for every degree above 26%, 3¢ more per gallon. A man wishes to import 100 gallons of wine 30% strong, bought at $2 per gallon, and orders enough water to be mixed with it to reduce it to 26% strong. Does he gain or lose in the amount of duty paid, the total value of the wine being reduced 10% by the mixing? Does he gain or lose on the whole?

9. Find the duty on an importation of 2450 bu. of potatoes,

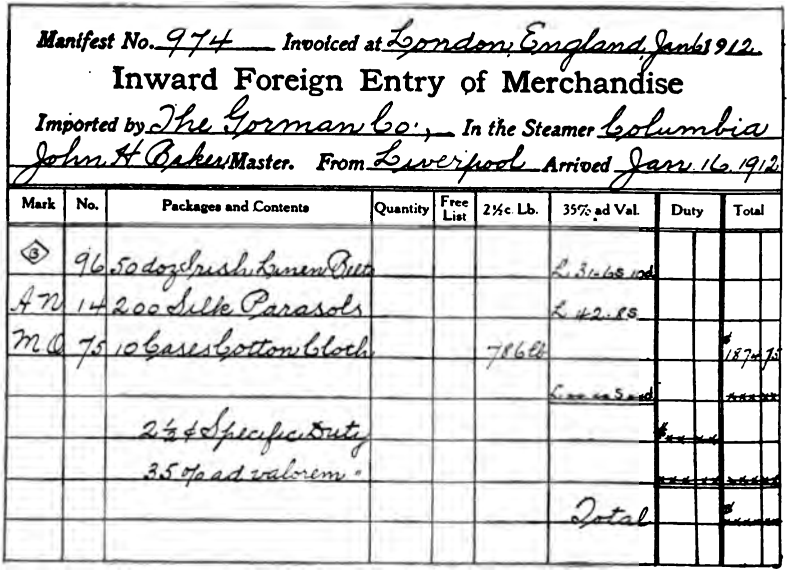
invoiced at 48¢ per bushel, and 28,000 lb. of hay, invoiced at $12.50 per ton, the duty on potatoes being 25¢ per bushel, and on hay as per the table.

10. B imported from Geneva 24 watches invoiced at $40 each, and 12 clocks, at $8.50. What was the duty, the rate being on clocks 35%, and on watches, 40% ad valorem?

11. An American importer bought in Belgium 1000 meters Brussels carpet, 27 inches wide, at 4½ francs per meter. He paid a specific duty of 44¢ per square yard, and an ad valorem of 40%. Calculate the gross cost per yard, estimating one meter as equivalent to 1.0936 yards.

12. An importer on making his entry at the custom house paid duty on 9478 lb. of unmanufactured tobacco according to the table on page 377. According to the weigher's return, the net weight was only 9264 lb. How much of the duty should be returned to the importer?

13. Compute the duty and find the total cost on the following entry of merchandise:



Manifest No. 974 **Invoiced at** London, England, Jan 6 1912

Inward Foreign Entry of Merchandise

Imported by The Gorman Co., **In the Steamer** Columbia

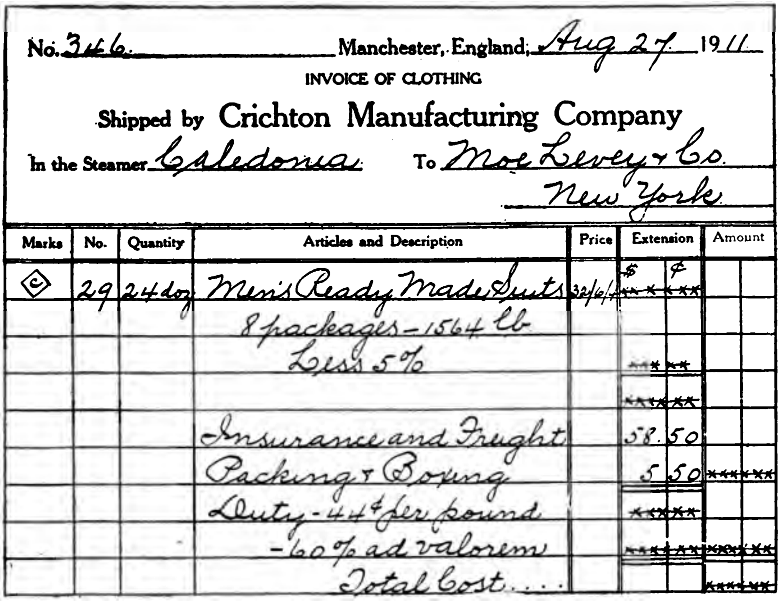
John H Baker **Master. From** Liverpool **Arrived** Jan. 16. 1912

Mark	No.	Packages and Contents	Quantity	Free List	2½c. Lb.	35% ad Val.	Duty	Total
③	96	50 doz Irish Linen Bu				£ 31-6s 10d		
A N	14	200 Silk Parasols				£ 42.8s		
M O	75	10 Cases Cotton Cloth			786 lb			$1874 75
						£ xx xx s xx d		xxxx xx
		2½¢ Specific Duty					$ xx xx	
		35% ad valorem					xxxx xx	xxxx xx
						Total		$ xxxx

14. A imported from England 6 cases of cloths and cassimeres, net weigh 864 lb., value as per invoice, £420 10*s*. What was the duty, the rate being 44¢ per lb. and 40% ad valorem?

15. I bought an invoice of German musical instruments, paying 9650 marks for them in Berlin. What was the total cost of the instruments, including the duty?

16. At what price per suit must the clothing in the following invoice be sold in order to realize a gain of 25%?



No. 346 Manchester, England, Aug 27 1911

INVOICE OF CLOTHING

Shipped by Crichton Manufacturing Company

In the Steamer Caledonia To Moe Levey & Co.
New York

Marks	No.	Quantity	Articles and Description	Price	Extension		Amount	
					$	¢		
◇C	29	24 doz	Men's Ready Made Suits	32/6/-	xx x	xxx		
			8 packages – 1564 lb					
			Less 5%		xxx	xx		
					xxxx	xx		
			Insurance and Freight		58	50		
			Packing & Boxing		5	50	xxx	xx
			Duty – 44¢ per pound		xxx	xx		
			– 60% ad valorem		xxx	xx	xxx	xx
			Total Cost . . .				xxx	xx

17. C. G. Euler, New York, purchased 2000 quarts of olives in Greece at 3 dr. per quart, and paid a duty of 20% ad valorem, with an allowance of 5% for waste. On receipt of the goods, they were put up in pint bottles at a cost of 3¢ each for bottling. Mark them so as to gain 20% on the gross cost.

18. If the duty on an importation of 40 gross bottles of porter, at 60¢ per dozen, amounted to $273.60, how many bottles were broken?

19. What is the duty at 30% ad valorem on a bale of burlap

containing 52 yds. 16 in. long and 27 in. wide, valued at 30¢ per sq. yd.?

20. Find the total duty on 4 cases of machinery, total value £24 8*s.* 6*d.* at 35%; 8 cwt. 0 qr. 24 lb. of castings at 1¼¢ per pound; 5 cwt. 0 qr. 40 lb. chain at 2¢ per pound; 4 cwt. 2 qr. 12 lb. chain at 1½¢ per pound.

21. Find the duty on an invoice of leather goods from the Netherlands, dutiable value 8456 florins, at 20%. [1fl. = $.402.]

STORAGE

ORAL EXERCISE

1. Calculate the storage due on 500 bu. wheat for 5 months at 2¢ per month.

2. Calculate the storage due on 400 bbl. flour, from May 2 to June 6, at 1¢ per week.

3. To what extent is a storage company liable for damage by fire and water to furniture or goods stored in its warehouse? To damage of grain through water, heating, or fire?

659. **Storage** is a charge for keeping goods or merchandise in a warehouse until they are required for use, sale, or transportation. The **rate of storage** may be fixed by agreement between the contracting parties, sometimes by boards of trade, associations of warehousemen, etc., and is usually calculated at a certain rate per barrel, bale, bushel, box, or other unit for a certain time.

660. A **storage term** is the time for which the rate is charged. It does not usually exceed one month, and is often less.

In estimating storage charges in some cities, a part of a term is counted the same as a full term, while in other cities, a fraction of a term, less than one-half, is regarded one-half; and a fraction exceeding one-half is counted a full term.

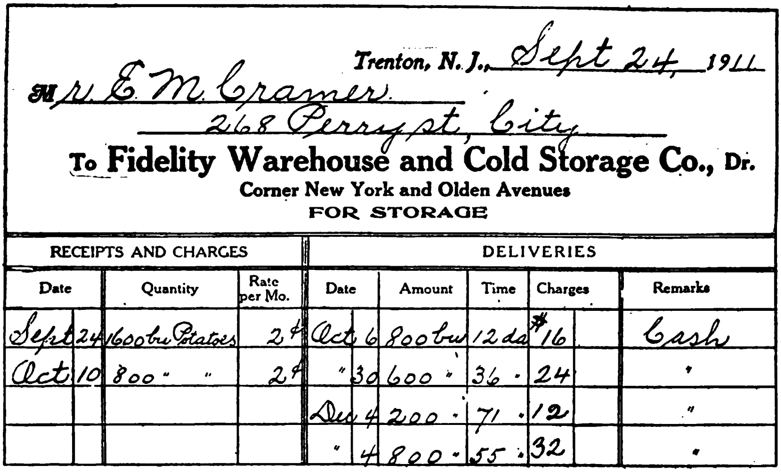
Cash Storage

661. **Cash storage** is storage computed on each delivery at the time of the withdrawal of the goods from the warehouse.

In cash storage all goods delivered are deducted from the oldest receipt on hand. Certain warehouses render bills at the end of each month for all goods taken out during the month, others render bills monthly for all storage dues, whether the goods have been withdrawn or not.

WRITTEN EXERCISE

1. How much should be paid for the following storage of potatoes, storage being charged on each delivery?



Trenton, N. J., Sept 24, 1911

Mr. E M. Cramer.
268 Perry st, City

To **Fidelity Warehouse and Cold Storage Co.,** Dr.
Corner New York and Olden Avenues
FOR STORAGE

RECEIPTS AND CHARGES				DELIVERIES						
Date		Quantity	Rate per Mo.	Date		Amount	Time	Charges		Remarks
Sept	24	1600 bu Potatoes	2¢	Oct	6	800 bu	12 da	$16		Cash
Oct	10	800 " "	2¢	"	30	600 "	36 "	24		"
				Dec	4	200 "	71 "	12		"
				"	4	800 "	55 "	32		"

Solution

			Term	Rate	Storage
Sept. 24, received	1600 bu.				
Oct. 6, delivered	800 bu.,	which were in storage	12 da.	2¢	$16.
	800 bu.,	balance in storage.			
Oct. 10, received	800 bu.				
	1600 bu.,	balance in storage.			
Oct. 30, delivered	600 bu.,	which were in storage	36 da.	4¢	24.
	1000 bu.,	balance in storage.			
Dec. 4, delivered	200 bu.,	which were in storage	71 da.	6¢	12.
and	800 bu.,	which were in storage	55 da.	4¢	32.
			Total storage,		$84.

2. What was paid for the following storage at 5¢ per barrel per month or part of a month, storage being calculated at each delivery? Received Aug. 1, 1200 bbl.; Sept. 16, 600 bbl.; Nov. 1, 800 bbl. Delivered Aug. 24, 900 bbl.; Sept. 30, 500 bbl.; Dec. 20, 1200 bbl.

3. I pay cash in full settlement to date of a flour storage bill, at the rate of 5¢ per barrel per 15 days, or fraction thereof.

Receipts	*Deliveries*
June 1....300 bbl.	June 16....100 bbl.
June 28....100 bbl.	July 7.....200 bbl.
July 14....250 bbl.	Aug. 12....350 bbl.

4. The West End Storage House received and delivered on account of Wm. J. Bixby of New Orleans sundry bales of cotton as follows: received Jan. 1, 2200 bales; Jan. 24, 150 bales; Feb. 16, 500 bales. Delivered Feb. 20, 1200 bales; Mar. 10, 400 bales; Apr. 1, 450 bales; Apr. 16, 500 bales. Balance the account to May 15, and find the storage due at 15¢ a bale per month, or fraction thereof.

5. Find the storage on the following account, the term being one month, and the rate 6¢ for the first term or part thereof, and 4¢ for each subsequent term or part.

Received	*Delivered*
March 1....600 bbl.	March 24....200 bbl.
March 16....320 bbl.	March 30....500 bbl.
April 4....500 bbl.	April 20....640 bbl.
April 16....840 bbl.	May 31....920 bbl.

6. Mr. B, a produce speculator, bought on Oct. 15, 2000 bbl. apples at $1.75 a barrel and placed them in cold storage. Dec. 10, he withdrew them from the warehouse and had them re-sorted, finding a waste of 1%. He sold the remainder for $2.60 per barrel. If the storage rate was 5½¢ a barrel per month, or fraction thereof, and the charges for repacking 10¢ per barrel, did he gain or lose, and how much?

Average Storage

662. In computing the storage on grain, flour, etc., when there are frequent receipts and deliveries, it is customary in some warehouses to average the time and charge a certain rate per month of 30 days. The process is called **average storage.**

It is the custom for some storage houses to rent certain space or rooms for storage purposes, charging a uniform rent from month to month for the same regardless of the quantity of goods stored.

WRITTEN EXERCISE

1. The John V. Manning's Sons Storage Co. received and delivered on account of H. K. Baldwin sundry bushels of grain as follows: received July 1, 2400 bu. wheat; July 20, 1800 bu. oats; Aug. 14, 4000 bu. corn. Delivered Aug. 6, 2000 bu.; Sept. 8, 3000 bu.; Oct. 1, 3200 bu. If the charges were 1½¢ per bushel per term of 30 days, average storage, what was the amount of the bill to render at the time of the last delivery?

Solution

The simplest way in which to solve such storage problems is to calculate the storage upon each receipt from the time it was received until the time of settlement as though no deliveries had been made, and then subtract the deliveries, as follows:

Receipts

July 1 to Oct. 1 = 92 da., 2400 bu. for 92 da. = 220,800 da. for 1 bu.
" 20 to " 1 = 73 da., 1800 bu. for 73 da. = 131,400 da. for 1 bu.
Aug. 14 to " 1 = 48 da., 4000 bu. for 48 da. = 192,000 da. for 1 bu.
8200 544,200

Deliveries

Aug. 6 to Oct. 1 = 56 da., 2000 bu. for 56 da. = 112,000 da. for 1 bu.
Sept. 8 to " 1 = 23 da., 3000 bu. for 23 da. = 69,000 da. for 1 bu.
Oct. 1 to " 1 = 00 da., 3200 bu. for 00 da. = 00,000 da. for 1 bu.
8200 181,000 da. for 1 bu.

544,200 da. − 181,000 da. = 363,200 da. for 1 bu.
363,200 da. ÷ 30 da. = 12,107 terms of 30 da. each.
1½¢ × 12,107 = $181.61, the total storage bill.

2. The Fidelity Warehouse and Cold Storage Co. received and delivered for L. E. Adams the following:

Receipts	*Deliveries*
Aug. 20....240 cases eggs	Sept. 10....200 cases eggs
Sept. 1....160 " "	Oct. 8....500 " "
" 20....360 " "	" 28....300 " "
Oct. 10....400 " "	Nov. 11....160 " "

Compute the storage charges, at 3¢ a case for a term of 30 days, average storage.

3. A farmer agrees to pasture cattle at 60¢ a head per week. Find the total cost of the following, average storage: received May 1, 12 head; May 14, 13 head; May 30, 20 head; June 6, 32 head; June 18, 24 head; June 28, 15 head. Delivered May 24, 15 head; June 10, 16 head; June 20, 25 head; July 3, 14 head; July 27, 24 head; Aug. 4, 22 head.

4. The West End Storage House received and delivered the following on account of C. V. Richards: received Oct. 1, 20 bbl. potatoes; Oct. 10, 60 bbl. potatoes; Oct. 16, 160 bbl. potatoes; Oct. 30, 140 bbl. potatoes. Delivered Nov. 26, 200 bbl. potatoes; Dec. 8, 140 bbl. potatoes; Dec. 20, 40 bbl. potatoes. What was the amount of the bill, at 5¢ per barrel for a term of 30 days, average storage?

PARTITIVE PROPORTION, PARTNERSHIP, BANKRUPTCY, AND BALANCE SHEETS

ORAL EXERCISE

1. Two persons form a partnership, one furnishing $\frac{2}{5}$ of the capital, and the other $\frac{3}{5}$ of it, and the gain is $1500; what should be the gain of each?

2. Three men form a partnership and furnish money in the proportion of $1500, $3000, and $4500 respectively. What fractional part of the gain should each receive?

3. A mercantile firm failed with liabilities amounting to $9000, and assets worth $3000. If it owes A $600, B $1500, and C $1800, how much should each receive?

4. The profits of the Acme Manufacturing Co. were $900 for a certain time. What share of the profits should each partner receive if the capital contributed by them was $600, $900, and $1200 respectively?

663. Partitive proportion is the process by which a number is divided into parts, proportional to other given numbers.

Fractions have the ratios of their numerators when their denominators are the same.

WRITTEN EXERCISE

1. Divide $2150 into parts proportional to $\frac{3}{5}$, $\frac{3}{10}$, and $\frac{8}{15}$.

NOTE. See suggestions under sec. 214.

2. So divide $19,000 among M, N, O, and P, that their share shall be to each other as $2\frac{1}{2}$, $3\frac{1}{3}$, 4, and 6.

3. A, B, and C are to share $100,000 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, respectively; but C dying, it is required to divide the

whole sum proportionally between the other two; how much is each one's share?

4. Divide $500 among X, Y, and Z, so that X may have $50 more than Y and twice as much as Z.

5. Three persons, A, B, and C, traded in company. A put in $75, B $40. They gained $64, of which C took $18 for his share. What did C put in?

6. A, B, and C engage in business, agreeing to share the profit or loss in the ratio of 1, 1¼, 1½. If they gain $600, what is A's share?

7. A rents a house for $187.20 for one year. At the end of 4 months he takes in B as a co-tenant, and later admits C in like manner for the last 2½ months of the year. What portion of the rent should each pay?

8. Three persons contract to do a certain piece of work. A can do it in 34 days, B in 42½ days, and C in 51 days. If $125.80 be paid for the work and all join till it is completed, how much should each receive?

9. A man agreed to work a year for $216 and a suit of clothes. He left at the end of 10 months, receiving $175 and the suit of clothes. What was the value of the suit?

10. A, B, and C pasture an equal number of cattle upon a field of which A and B are the owners; A of 9 acres and B of 15 acres. If C pays $24 for his pasturage, how much should A and B each receive?

11. Divide $144 among A, B, and C, giving A ¾ of B's share and B ⅘ of C's share.

12. A and B were employed to do a piece of work for $60, and were to be paid in proportion to their ability to work, which was as 4:5 and to the time each worked, which was as 3:4. How much did each receive?

Partnership

664. Partnership is the relationship resulting from an agreement between two or more persons to place their money, effects, labor, and skill, or some or all of them, in some enterprise or business, and divide the profits and bear the losses in certain proportions.

A partnership may exist for a single transaction or enterprise as well as for a continuous business. The relation is a contractual one and may be *written*, *oral*, or *implied*. It is advisable, and probably most advantageous to all concerned, that the agreement be reduced to writing.

665. The members of a partnership are called partners, of which there are four classes: ***ostensible***, ***nominal***, ***dormant***, and ***limited***.

An ***ostensible*** or ***real partner*** is one of the active and known parties in a partnership; a ***nominal partner*** is one who is held out to the world as a partner, but who in reality is not such; a ***dormant*** or ***silent partner*** is one whose name is not known to third parties but who in reality partakes of the benefits of the business and thereby becomes a partner; a ***limited partner*** is one whose liability is limited to a certain amount.

The partners who conduct the business are called ***general partners***. Their private property is liable for the debts of the partnership, and a concealed partner is responsible for the debts of a firm should his connection become known, while the limited partner is responsible only for the amount named in the partnership agreement.

In all the states statutes have been passed providing for the creation of limited partnerships. The law generally provides that at least one member of a firm must be a general partner and who is the manager of the business.

666. The **capital** or **capital stock** is the money, or its equivalent, which is invested in a business.

Treasury stock is the unsubscribed capital stock of a company, or the ***unsold*** stock. Capital is frequently real estate, personal property, skill, time, etc.

667. The entire property of a firm or a debt owing to it, having a financial value, is a **resource** or **asset**. A **liability** is a debt owing by the firm. The **net capital** or the ***net worth*** of a firm is the excess of the resources over the liabilities. The **net insolvency** is the excess of the liabilities over the resources.

A firm being unable to pay its debts in full, it is said to be ***insolvent;*** when the resources exceed the liabilities, the business is said to be ***solvent***.

668. The **net investment** of a person is his investment in the business less his withdrawals; and the average sum in use during a given time, when investments and withdrawals have occurred at different times, is called his **average investment**.

669. The **net gain** is the excess of total gains over total

losses for any given period of time. The **net loss** is the excess of total losses over total gains for any given period of time.

The ***net gain*** of a firm may be found by subtracting the net worth at the beginning from the net worth at closing; the ***net loss*** is found by subtracting the net worth at closing from the net worth at beginning.

ORAL EXERCISE

1. Capital at beginning, $6000; capital at closing, $4500; state gain or loss, no withdrawals having been made.
2. Capital at beginning, $8000; capital at closing, $9000; state gain or loss, no withdrawals having been made.
3. Capital at beginning, $7000; insolvency at closing, $2000; state gain or loss, no withdrawals.
4. Insolvency at beginning, $3000; capital at closing, $1000; state gain or loss, no withdrawals.
5. Insolvency at beginning, $5000; capital at closing, $8000; state gain or loss, no withdrawals.
6. Insolvency at beginning, $4000; insolvency at closing, $2500; state gain or loss.
7. A's capital at the beginning of the year was $4000; his gain during the year, $1500; state his capital at closing.
8. B's insolvency at the beginning of the year was $2500; his gain during the year, $3000; state his capital or insolvency at closing.
9. C's insolvency at the beginning of the year was $2400; his loss during the year, $1200; state his capital or insolvency at closing.
10. Jan. 1, 1911, D's resources were $4000; his liabilities, $1500. Jan. 1, 1912, his resources were $6500; his liabilities, $2000. What was his gain or loss during the year?
11. Jan. 1, 1911, E's resources were $5000; his liabilities, $6000. Jan. 1, 1912, his resources were $8000; his liabilites, $5000. What was his gain or loss during the year?

Find the capital or insolvency at beginning.

12. Capital at closing $6000; gain during the year, $3000.
13. Capital at closing, $8400; loss during the year, $600.
14. Capital at closing, $6600; loss during the year, $7400,

15. Insolvency at closing, $4000; loss during the year, $1500.
16. Insolvency at closing, $5000; gain during the year, $2000.
17. Insolvency at closing, $1800; loss during the year, $2200.
18. Insolvency at closing, $2500; gain during the year, $3000.

Partnership Settlements

670. Partnership settlements usually occur as often as once a year, or according to the provisions of the contract or articles of agreement. The *gains* or the *losses* are divided according to certain fractions or percentages, or in proportion to investments.

Inequalities of investments are sometimes adjusted by allowing interest upon the same and the partners receiving salaries for their services rendered. In joint stock companies the gains, or *dividends*, and the losses, or *assessments*, are shared in proportion to the investments or the amounts of stock held.

WRITTEN EXERCISE

1. Two partners, X and Y, begin business Jan. 1, each with a capital of $2000. March 1, X adds $500 to his capital and Aug. 1, $500 more; Y adds $800 April 1. During the year the firm gained $3605.25. What was each man's share of the gain if it was divided according to average investment?

Solution

X's *investment*, $2000 for 2 mo. = $ 4,000 for 1 mo.
2500 for 5 mo. = 12,500 for 1 mo.
3000 for 5 mo. = 15,000 for 1 mo.
X's total investment = $31,500 for 1 mo.
Y's *investment*, $2000 for 3 mo. = $ 6,000 for 1 mo.
2800 for 9 mo. = 25,200 for 1 mo.
Y's total investment = $31,200 for 1 mo.
$31,500 + $31,200 = $62,700, the firm's investment for 1 mo.
X's gain = $\frac{31500}{62700}$, or $\frac{315}{627}$ of $3,605.25 = $1,811.25.
Y's gain = $\frac{31200}{62700}$, or $\frac{312}{627}$ of $3,605.25 = $1,794.

2. A, B, and C formed a partnership. A put in $5400 for

3 mo., B $3500 for 6 mo., and C $4000 for 8 mo. What should be the share of each if the gain was $3633?

3. M, N, and O close business at the end of the year in the following conditions:

Resources		*Liabilities*	
Bills Receivable....	$ 1854.25	Bills Payable.....	$7860.70
Cash on hand......	12469.50	Acc'ts Payable....	4346.30
Acc'ts Receivable..	3978.40		
Mdse. unsold.......	10764.10		

If M invested $4000, N $5000, and O $6000 and the gains or losses were to be in proportion to their investment, what was each partner's *net worth* at closing?

4. A has 6 cows in pasture for 3 weeks, B 5 cows for 4 weeks, and C 13 cows for 2 weeks. The total cost for pasture is $12.80. How much should each pay?

5. Three butchers rent a field and pay for 6 months' rent $153.14. The first puts in 20 oxen for 10 weeks and 50 sheep for 8 weeks; the second, 25 oxen for 8 weeks and 30 sheep for 7 weeks; the third, 18 oxen for 10 weeks and 10 sheep for 12 weeks. What share will each have to pay, counting 3 sheep equal to 1 ox?

6. J. H. Smith & Co. fail in business with available resources amounting to $8800; liabilities $22,000. If they owe A $4275, and B $2175.50, what will each of these creditors receive?

7. A, B, and C furnish capital for a business as follows: A puts in $500 for 2 mo., B $400 for 3 mo., and C $200 for 4 mo. If they gain $600, what should be each partner's share?

8. Three merchants enter into partnership. F puts in $960 for 6 mo., G a certain sum for 12 mo., and H $640 for a certain time when the accounts were settled. F received $1200 for his stock and profits, G $2400 for his, and H $1040 for his. What was G's stock and H's time?

9. Three men paid $100 for a pasture. A put in 9 horses, B 12 cows for twice the time, and C some sheep for 2½ times as long as B's cows. If C paid ½ the cost, how many sheep had he

and how much did A and B each pay, provided 6 cows eat as much as 4 horses, and 10 sheep as much as 3 cows?

10. A owed B $900 to be paid in 3 years, but at the expiration of 9 months A agreed to pay $300 if B would wait long enough for the balance to compensate for the advance. How long should B wait after the expiration of the 3 years?

11. I, J, and K form a partnership. I invests his capital for 8 mo. and receives $\frac{1}{2}$ of the profits; J invests his capital for 12 mo.; K invests $4000 for 6 mo. and received $\frac{1}{6}$ of the profits. Find the capital invested by I and J.

12. A, B, and C trade together, having a joint capital of $4700. A's money is in the business 6 mo., B's 8 mo., and C's 10 mo. Each receives $600 as his share of the profit. How much of the capital did each contribute?

13. R and S enter into partnership, R putting in $1200, S $1500. At the end of 6 mo., R draws out $200 and S enough to make his share $\frac{7}{8}$ of R's. Six months later they divide their total capital of $3150. What was each man's share?

14. A and B trading in partnership for two years make each year a profit of $1200. A's capital the first year was $2\frac{1}{2}$ times B's, and the second year it was $1\frac{1}{2}$ times B's. Find each partner's share of the gain.

15. A, B, and C form a partnership, with capitals of $7500, $15,000, and $22,500 respectively. A draws out at the end of each year $750, B $1200, and C $1350. At the end of 5 years, their capital is $42,900. How much of it belongs to B?

Accounts, Statements, and Balance Sheets

671. An **account** is a collection of related debits and credits recording the transactions of a business. They may be divided into **resource and liability**, and **loss and gain** accounts.

Resource and liability accounts comprise the resources or property of the business, and its liabilities or debts; *loss and gain* accounts indicate a gain or loss to the business, such as expense, merchandise, discount, interest, etc.

WRITTEN EXERCISE

1. The following ledger accounts represent postings from a journal. Study carefully the arrangement and balance sheet shown on the following page. The balance sheet is a summary of the ledger accounts.

NOTE. The teacher will instruct that figures representing balances, inventories, and ruled lines indicating the closing of an account should be in red.

W. N. Ferris, Prop.

1912					1912				
					May	16	Invested	4	5000

Mdse

May	17		5	2000	May	18		5	2200
	20		6	4400		28		6	3000
	31	Gain (red)		850		31		6	1300
						31	Inventory (red)		750
				7250					7250

Cash

May	16		4	5000	May	17		5	2000
	20		5	2000		22		6	3520
	28		5	3000		31	Balance (red)		5480
	30		6	1000					
				11000					11000

J. R. Ross

May	18		5	2200	May	20		6	2000
	31		6	1300		30		6	1000
						31	Balance (red)		500
				3500					3500

Bills Pay

May	22		6	3500	May	20		6	4400
	31	Balance (red)		900					
				4400					4400

Interest

May	22		6	20	May	31	Loss (red)		20

Balance Sheet, May 31, 1912

L. F.		Dr.	Cr.	Losses	Gains	Resources	Liabilities
14	W.N.Ferris		5000				
14	Cash	5480				5480	
14	Mdse.		100		850	*750	
14	J.R.Ross	500				500	
15	Bills Payable		900				900
15	Interest	20		20			
		6000	6000	20	850	6730	900
	W.N.Ferris, net gain			830			
				850	850		
	" net credit		5000				
	" " gain		830				
	" present worth						5830
						6730	6730

* Merchandise on hand.

NOTE. As may be seen from the above, the *present worth* of any business may be found by subtracting the liabilities from the resources, or by adding the net gain to the net investment, or if a loss by subtracting the net loss from the net investment.

672. An **inventory** is a list of unsold merchandise and other property on hand at the time the statement, or balance sheet, is rendered.

The word *inventory* is also applied to a class of unpaid items: as interest payable, unpaid rent, etc.

673. A **trial balance** is a schedule showing the debit and the credit footings of the ledger accounts of a business. A **balance sheet** is a statement of actual assets and liabilities at a given date designed to exhibit the financial condition of a business.

674. The **goodwill** of a business is the value of its reputation acquired during its continuance, and which induces confidence and continued patronage so long as the business is conducted in the same place upon the same principles.

The actual value of a ***goodwill*** is its capacity to bestow upon the purchaser more than he could earn by his own unaided exertions. Frequently, however, goodwill is an inflation of the purchase price of property.

2. At the end of the year, Miller, Brown & Co. find that their resources and liabilities are as follows: merchandise on hand, $9650; cash on hand, $5645.50; due from sundry persons, $6244; bills receivable on hand, $2560. The firm owes sundry persons, $3488; outstanding notes, $175. Miller's investment in the business is $6000; Brown's, $5000; and Field's, $4000. Make a statement showing the resources, liabilities, net capital, and net gain of each partner.

Solution.

Resources		
Merchandise on hand	$9650.00	
Cash on hand	5645.50	
Due from sundry persons	6244.00	
Bills receivable on hand	2560.00	
Total resources		$24099.50
Liabilities		
Debts due sundry persons	$3488.00	
Outstanding notes	175.00	
		3663.00
Present worth		$20436.50
Investments. $6000+$5000+$4000		15000.00
Net gain		$ 5436.50

Miller's fractional part, $\frac{6000}{15000}=\frac{2}{5}$ of $5436.50 = $2174.60.

Brown's " " $\frac{5000}{15000}=\frac{1}{3}$ of $5436.50 = $1812.17.

Field's " " $\frac{4000}{15000}=\frac{4}{15}$ of $5436.50 = $1449.73.

WRITTEN EXERCISE

1. A, B, and C engage in a mercantile business, investing jointly $18,000. At the end of a year, A's share of the gain is $1856, B's $2784, and C's $3712. What was the investment of each?

2. X, Y, and Z formed a partnership to carry on the milling business. X owned the site, valued at $1500; Y put up the building, worth $2400, and Z put in the machinery, costing

$3000. They ran the mill for one year on this basis and then agreed to take equal shares in it; but meantime the value of the land had increased 20%, the value of the building had decreased 10%, and the worth of the machinery had depreciated 25%. How should they settle, and how much is due each?

3. A merchant had on hand at the beginning of the year $7246.24 worth of merchandise. He bought during the year $3624.87, and sold during the year $13,617.18. If he had on hand at the close of the year merchandise worth $125.18, find his gain or loss for the year.

4. Evans, Smith, and Jones are partners, each to share ⅓ of the gain or loss, and each investing $9000. At the end of a year they propose to separate, and the business stands as follows: cash on hand, $8673.25; merchandise on hand, per inventory, $9452; bills receivable on hand, $2491.50; personal accounts due the firm, $6835.75; real estate, $10,335; stocks and bonds, $2500. On the same date their liabilities were as follows: owing by the firm to sundry persons, $2456.50; mortgage on the firm's real estate, $4000. Evans has withdrawn during the year $1452.30; Smith, $1510; and Jones, $1632.60. What is each partner's interest in the business at closing?

5. A and B were partners in business with a capital of $1250, of which A furnished $750. At the end of the first year A's share of the profits was $340.65, when B sold his interest to C for $637.50. At the end of the second year C's share of the profits was $247.80, when he bought A's interest for $876.74. How much did A and B each make. How much had C invested more than he realized?

6. A and B went into business, A putting in $1000 and B, $500, but by agreement A was to receive 40% and B 60% of the profits. At the end of the year the cash on hand, debts owing to the business and inventories, less liabilities, showed a net worth of $2596.58. During the year, A had withdrawn $227.53, and B $934.05. What was the worth of each partner at closing?

7. An estate worth $12,620.25 is divided among three heirs so that the first receives ⅙ more than the other two. What does each receive?

8. At the beginning of the year A, B, and C form a partnership, contributing $1200, $1500, and $2000 respectively. A acted as bookkeeper at a salary of $846, and B as manager at half as much again; both salaries to be increased in proportion as the capital increased. After 2 months C added $1000 to his capital. After 4 months B added $500 to his, and in 6 months A added $300 to his. If the gain is $9025, find the share of each.

9. Richards, Walsh, and Rice enter into partnership with proportionate interests as follows: Richards 50%, Walsh 30%, and Rice 20%. Richards invests $20,000 capital, Walsh $7500, and Rice nothing. At the end of two years, and after the gains and losses have been adjusted, it is found that Richards's capital remains unimpaired, Walsh has a credit of only $2200, while Rice has overdrawn his account $6584.40. It is found that Rice is insolvent; how much, therefore, must Walsh pay into the business to adjust his share of the loss?

10. A and B purchase goods amounting to $10,000. A invests $6000, B invests $4000. With the goods purchased they propose to commence a partnership business, but C comes along, and he has money, and money is a desideratum. Therefore A and B offer a $\frac{1}{3}$ interest in their business to C for $5000, and C accepts. Now, the question is, what portion of the $5000 should A and B receive respectively in order to make the interests of A, B, and C equal?

11. Phelps and Fisher enter into partnership, Phelps contributing $5000 more capital than Fisher. At the end of 5 months Phelps withdraws $2500 of his capital, and 2 months later Fisher increases his investment $2500. At the end of their first year of partnership their assets exceed their liabilities by $24,800, and on dividing their net gain in the ratio of their average investments, Phelps's interest in the business is found to exceed that of Fisher by $461.54. Find the amount of the original investment of each.

12. W, X, and Y are partners in a business in which W invests $5000, X $7500, and Y $8500. W and X gained $440 more than Y. What was the whole gain if each one's gain is in proportion to his investment?

13. Three men, Smith, Jones, and Robinson are in partnership, gains and losses to be divided as follows: Smith 35%, and Jones and Robinson each 32½%. At the beginning of the business Robinson advanced $25,233.37; at the close of the partnership the firm had outstanding debts to the amount of $46,795.75 with no resources. How much is due Robinson from each of the other partners, and what is the condition of Robinson's account after settling all liabilities among them?

14. Three companies, A, B, and C, are to be consolidated on the basis of the relative market value of their stock. A's capital is $1,000,000 at 90; B's is $750,000 at 70; C's is $625,000 at 80. The capital of the new company is to be $2,000,000 and in shares of $100 each. What amount of the capital should be allotted to each of the old companies, and to how much stock in the new company should the holder of 100 shares of the stock of each of the old companies be entitled?

15. Kent and Lewis are partners, Kent sharing ⅗ of the gain or loss and Lewis ⅖. Kent invested $12,000 and Lewis $8000. At the close of the first year, their resources and liabilities are as follows: cash on hand, $3425.50; merchandise on hand, $8635; accounts owing the firm, $14,548.40; bills payable, $3500; accounts owing by the firm, $7655.25; bills receivable, $2956; rent paid in advance, $250; mortgages held by the firm, $8425; interest accrued on the same, $224.16; interest accrued on notes held by the firm, $112.20; interest accrued on outstanding notes of the firm, $124.60. Kent has withdrawn during the year, $1956.75; and Lewis, $1866.33. By agreement, each partner is to receive a salary of $2400 per year. What is the separate interest of each partner at the close of the year?

16. Robert Magowan is employed by James Tolcott and Hiram Cook, who are engaged in the shoe manufacturing business. Tolcott and Cook are to furnish the necessary capital, and Magowan, instead of salary, ¼ of the profits. Tolcott and Cook have a retail store connected with the factory. On the books are entered time sales amounting to $1045.25. The leather, etc., for the factory are furnished by Tolcott and Cook. An inventory taken at the beginning of the year shows materials

amounting to $2862.54. Tolcott and Cook bought materials for the factory amounting to $21,562.76. Magowan received $17,762.54 in cash for the cash sales, of which $16,923.62 were turned over to Tolcott and Cook. Another inventory taken at the end of the year shows materials on hand amounting to $10,792.10. Magowan bought of Tolcott and Cook, boots, shoes, etc., amounting to $23.75 and which is included in the above time sales. How much does the firm owe Magowan at the close of the year?

17. George Spink, Walter Weber, and Joseph Coleman formed a copartnership on January 1, 1911. Spink furnished $\frac{1}{5}$ of the capital, Weber $\frac{3}{8}$, and Coleman $\frac{17}{40}$; gains and losses divided in proportion to the required investments. On January 1, Spink invested $12,000, on Feb. 11, withdrew $4000, on Oct. 19, $5000, and on Nov. 3, invested $11,000. On January 1, Weber invested $6000, on March 19, $1000, on Aug. 15, $500, and on Oct. 23, withdrew $4500. On January 1, Coleman invested $10,000, on May 31, $5000, on June 1, he withdrew $8000, and on Oct. 23, invested $5500. If the net gain during the year was $6724.40, and interest at the rate of 6% is to be allowed on excess of each partner's required investment, and to be charged on deficiency of same, what was each partner's interest in the firm Dec. 31, 1911?

18. A, B, and C agree to start in business with a capital of $400,000, of which A is to furnish $200,000, B $100,000, and C $100,000. A is to have $\frac{1}{2}$ interest in the business, and B and C each $\frac{1}{4}$ interest, and interest at the rate of 5% per annum is to be paid on excess capital, if any. A contributed $200,000, B $90,000, and C $80,000. How would the capital accounts stand on the books after adjusting the interest account at the end of one year?

19. A and B form a copartnership, A's capital being to B's as 7:23. At the end of 7 months, B withdraws $\frac{1}{5}$ of his capital, and A $\frac{3}{8}$ of his. At the end of the year, their net gain is $7500. How much of the gain belongs to each?

20. A. M. Tanner, H. M. Sipe, and E. S. Sears form a partnership on January 1, 1911. Sipe is to manage the business at

an annual salary of $2500 and to receive interest on his salary and to pay interest on all withdrawals at the rate of 6% per annum. His salary is to be credited on June 16, 1911. Tanner and Sears are to receive interest at the rate of 6% per annum on all investments, and are required to furnish the necessary capital also. The gain or loss is to be divided equally. Tanner invests $11,000 on January 1; June 6, $5000; Oct. 16, $4150. Sears on January 1, $5675; Aug. 15, $6115; and draws out Nov. 6, $1320. Sipe withdraws on Feb. 15, $565; July 17, $1005; Aug. 19, $515.65; Oct. 23, $672.50. At the end of the year, the gain without taking into account either the salary to be paid to Sipe or the interest on the partners' accounts, is $5987.73. What will be the partners' balances when all the items have been properly entered?

21. John Roe dies, bequeathing his entire property, after payment of all debts and funeral expenses, to the following beneficiaries, viz: $\frac{1}{5}$ to John Roe, Jr.; $\frac{1}{5}$ to George Roe; $\frac{1}{5}$ to William Roe; $\frac{3}{10}$ to Mary Roe; and $\frac{1}{10}$ to Frank Roe. The inventory filed by his executors was as follows: stocks and bonds, $100,000; bonds and mortgages, $150,000; furniture and wearing apparel, $1500; cash in bank, $1000. There are two executors, and their transactions are as follows:

Cash receipts: stocks and bonds sold, $84,525.50; bonds and mortgages paid in, $116,422.73; furniture and wearing apparel sold, $1247.28; dividends collected on stocks and bonds, $45,422.50; interest collected on bonds and mortgages, $62,527.87; interest received on deposits in bank, $1083.48; real estate sold, $25,000; rents collected, $18,000.

Cash disbursements: funeral expenses, $750; expenses of probating will, $5000; general legal services, $10,000; rent of safe in safe deposit company, $450; publishing citations, $37.75; care of cemetery lot, $400; repairs to real estate appurtenances, $5724.68; premium on executor's bond, $1250; stationery, postage, etc., $125.38; accountant's services, $2000; debts of deceased, $12,472.41; taxes, $20,672.48; assessments for improvements to real estate, $471.83; insurance, $250; John Roe, Jr., account of legacy, $55,000; George Roe, account of legacy,

$62,500; William Roe, account of legacy, $55,500; Mary Roe, account of legacy, $77,500; Frank Roe, account of legacy, $24,000.

The inventory at the date on which they wish to account is as follows: stocks and bonds, $25,000; bonds and mortgages, $32,000; cash in bank, $20,124.83. Required, to make up the summary statement of the executors' accounts, showing the balance due each legatee and the amount of the executors' commission if the law allows the executors 5% on the first $2000 of the inventoried valuation of an estate and 3% on all additional sums with a 5% fee on all incomes from rent, interest, and dividends.

22. Morris and Curtis formed a partnership for one year with a combined capital of $18,000. At the end of the year Morris's share of the gain was $2246.20 and Curtis's $1429.40. What was the investment of each if they shared the gains in proportion to their investments?

23. C. F. Doran and E. H. Wilson purchase a rubber manufactory, Doran investing $15,450 and Wilson $16,750, and agreeing to share gains and losses equally. Wilson is to operate the factory, receiving therefor an annual salary of $2500. Doran is to negotiate the sales and is allowed 10% of the profit, exclusive of his regular share. The net gain for the first month is $1725. Estimate the share due each member of the firm.

24. A. B. Smith and C. A. Call offer you $92,000 for the real estate, stock and goodwill, and outstanding accounts of your business. Terms $20,000 cash, $35,000 at the end of one year, and the balance at the end of two years, money worth 6%. You value your business at $86,000 cash. If their offer equals this in cash value you determine to accept it and retire from business. Do you accept, and what is the difference between the offer and your cash value?

25. Brown and Jones began a partnership business Jan. 1, 1911. At the time of closing the books, Dec. 31, 1911, an examination of the accounts revealed the following: Jan. 1, Brown paid in $9000; May 1, $2400; Oct. 1, $800. June 1, Brown drew out $1800; Sept. 1, $2000. Jan. 1, Jones paid in $3000; June

1, $1500; Oct. 1, $3000. March 1, Jones drew out $1600; May 1, $1200.

Their merchandise account was, Dr. $32,000.00, Cr. $27,000.00. Balance of merchandise on hand per inventory, $10,500. Cash on hand, $4900. Bills receivable, $12,400. Chas. Green owes on account $250; F. Draper owes $700; Wm. Clark owes $650; F. Hart owes $850. They owe A. Reed on account $240; C. W. Smith $500; A. M. Clark $100. Their profit and loss account shows before closing entries, Dr. $866, Cr. $1520. Expense account is Dr. $2520. Commission account is Cr. $2760; interest is Dr. $840, Cr. $950. The gain or loss is to be divided in proportion to each partner's capital, and in proportion to the time it was invested.

Prepare a statement of losses and gains, resources and liabilities and show the condition of each partner's account at the time of closing.

26. Messrs. Doe and Roe are partners in business, sharing gains and losses equally. Required a statement showing the gains and losses, and also the net capital of each partner on the basis of the following trial balance of their double-entry ledger at the close of the fiscal year:

John Doe, investment	....	$8,405.26
Richard Roe, investment	....	8,405.28
Cash	$9,017.33	
Merchandise	3,424.89	
Bills receivable	12,000.00	
Bills payable	....	8,350.00
Expense	576.00	
Interest	129.74	
Loss and gain	250.00	
Sundry book accounts receivable	3,566.00	
Sundry book accounts payable	....	3,803.42
	$28,963.96	$28,963.96
Inventory of merchandise on hand		$8,000.00

27. Adams, Wilson, and Green enter into partnership as grain dealers. Green invests $10,000 capital, Wilson, $15,000

and Adams nothing. Wilson and Adams are each allowed $1500 per year salary; no interest on capital, and profits to be divided among the three equally. In the second year Green, being on a holiday trip, sees a chance of buying a quantity of grain, which he secures at a profitable figure with the consent of his partners. He sells about ¼ of the lot on his way home, but after his return the market takes a disastrous turn and the firm is obliged to assign. Their statement of affairs is as follows:

Assets

Office furniture	$ 500.00
Grain in elevator, hypothecated to bank	110,000.00
	$110,500.00

Liabilities

Bills payable to bank (Secured by warehouse receipts on 100,000 bu. wheat at $1.10. Wheat realizes $80,000)		$ 75,000.00
Trade creditors		25,000.00
Capital account—		
Green	$10,000.00	
Wilson	15,000.00	
		25,000.00
		$125,000.00

The partners have sufficient personal assets to pay all creditors. The furniture sells for $500. Adjust the loss, and show the condition of each partner's account.

APPENDIX

The Metric System

675. The **metric system** of weights and measures is based upon the *decimal notation*, and is so called because its primary unit is the *meter*. The *meter* (m) is the base of the system, and is the *one ten-millionth* part of the distance on the earth's surface from the equator to either pole, or 39.37079 inches.

Meter means *measure*, and the three principal units are units of *length*, *volume*, and *weight*.

The *metric system* is used by most of the civilized nations, except the United States and Great Britain and some of her colonies. It is legalized by the United States government and is principally used in scientific treatises.

The *metric system* is founded upon a uniform scale of ten; that is, a decimal scale. Therefore, it makes arithmetical operations and reductions relating to measure much more simple.

676. The *principal units* of the system are the *meter*, for lengths, the *square meter* for small surfaces, the *cubic meter* for solids, the *liter* for capacities, and the *gram* for weights. The names of the higher denominations, or multiples, of the units are formed by prefixing to the several *units* the Greek numerals, as follows: *deka*, *hecto*, *kilo*, and *myria*, which mean respectively, 10, 100, 1000, and 10,000 times the unit. The names of the lower denominations, or divisions, of the unit are formed by prefixing to the several *units* the Latin numerals, as follows: *deci*, *centi*, and *milli*, which mean respectively .1, .01, .001 of the unit.

Measures of Length

10 **millimeters (mm.)**	= 1 **centimeter (cm.)**	= .01	meter
10 **centimeters**	= 1 decimeter (dm.)	= .1	meter
10 decimeters	= 1 **meter (m.)**	= 1.	meter
10 **meters**	= 1 dekameter (Dm.)	= 10.	meters
10 dekameters	= 1 hektometer (Hm.)	= 100.	meters
10 hektometers	= 1 **kilometer (Km.)**	= 1000.	meters
10 **kilometers**	= 1 myriameter (Mm.)	= 10,000.	meters

NOTE. The units in common use are indicated by black-faced type.

Measures of Surface

677. The units of square measure are *squares*, the sides of which are equal to a unit of long measure. The *square meter* is used for small surfaces and the *are* for land measurements.

Table

100 sq. millimeters	= 1 **sq. centimeter (sq. cm.)**	= .0001	sq. m.
100 **sq. centimeters**	= 1 sq. decimeter (sq. dm.)	= .01	sq. m.
100 sq. decimeters	= 1 **sq. meter (sq. m.)**	= 1.	sq. m.
100 **sq. meters**	= 1 sq. dekameter (sq. Dm.)	= 100.	sq. m.
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.)	= 10,000.	sq. m.
100 sq. hektometers	= 1 **sq. kilometer (sq. Km.)**	= 1,000,000.	sq. m.
100 **sq. kilometers**	= 1 sq. myriameter (sq. Mm.)	= 100,000,000.	sq. m.

In measuring land, the square meter is called a *centare* (ca), the square dekameter an *are* (a) and the square hektometer a *hektare* (Ha).

Cubic Measure

678. The unit for measuring ordinary solids is the *cubic meter*, called the *stere* for wood measurements.

Table

1000 cu. millimeters	= 1 **cu. centimeter (cu. cm.)**	= .000001	cu. m.
1000 **cu. centimeters**	= 1 cu. decimeter (cu. dm.)	= .001	cu. m.
1000 cu. decimeters	= 1 **cu. meter (cu. m.)**	= 1.	cu. m.
1000 **cu. meters**	= 1 cu. dekameter (cu. Dm.)	= 1000.	cu. m.
1000 cu. dekameters	= 1 cu. hektometer (cu. Hm.)	= 1,000,000.	cu. m.
1000 cu. hektometers	= 1 **cu. kilometer (cu. Km.)**	= 1,000,000,000.	cu. m.
1000 **cu. kilometers**	= 1 cu. myriameter (cu. Mm.)	= 1,000,000,000,000.	cu. m.

The cubic meter is used in measuring embankments, excavations, etc.; cubic centimeters and cubic millimeters for minute bodies.

Measures of Capacity

679. The *liter* is the unit of capacity, both of liquid and dry measures. It is a cube whose side is 1 decimeter; hence a liter is a cubic decimeter.

Table

10 milliliters (ml.)	= 1 **centiliter (cl.)**	=	.01	liter.
10 **centiliters**	= 1 deciliter (dl.)	=	.1	liter.
10 deciliters	= 1 **liter (l.)**	=	1.	liter.
10 **liters**	= 1 dekaliter (Dl.)	=	10.	liters.
10 dekaliters	= 1 **hektoliter (Hl.)**	=	100.	liters.
10 **hektoliters**	= 1 kiloliter (Kl.)	=	1000.	liters.

Measures of Weight

680. The *gram* is the unit of weight, and is equal to the weight of 1 cubic centimeter of distilled water.

The gram is used for weighing gold, silver, medicines, and all small or costly articles. The *gram*, *kilogram*, or *kilo*, and ton are principally used in weighing groceries and coarse articles.

Table

10 **milligrams (mg.)**	= 1 **centigram (cg.)**	=	.01	gram.
10 centigrams	= 1 decigram (dg.)	=	.1	gram.
10 decigrams	= 1 **gram (g.)**	=	1.	gram.
10 **grams**	= 1 dekagram (Dg.)	=	10.	grams.
10 dekagrams	= 1 hektogram (Hg.)	=	100.	grams.
10 hektograms	= 1 **kilogram (Kg.)**	=	1000.	grams.
10 **kilograms**	= 1 myriagram (Mg.)	=	10,000.	grams.
10 myriagrams	= 1 quintal (Q.)	=	100,000.	grams.
10 quintals	= 1 **tonneau (T.)**	=	1,000,000.	grams.

The tonneau is usually called a *metric ton*.

Table of Approximate Values

1 inch	=	2.54 centimeters.	1 cu. inch	=	16.387 cu. centimeters.
1 foot	=	30.48 centimeters.	1 cu. foot	=	28.317 cu. decimeters.
1 yard	=	.9144 meters.	1 cu. yard	=	.7645 cu. meters.
1 rod	=	5.029 meters.	1 cord	=	3.624 steres.
1 mile	=	1.6093 kilometers.	1 fl. ounce	=	2.958 centiliters.
1 sq. inch	=	6.452 sq. centim.	1 gallon	=	3.786 liters.
1 sq. foot	=	9.2903 sq. decim.	1 bushel	=	.3524 hectoliters.
1 sq. yard	=	.8361 sq. meters.	1 Troy gr.	=	.0648 grams.
1 sq. rod	=	25.293 sq. meters.	1 Troy lb.	=	.3732 kilograms.
1 acre	=	40.47 ares.	1 Avoir. lb.	=	.4536 kilograms.
1 sq. mile	=	259. hectares.	1 ton (short)	=	.9072 tonneau.

WRITTEN EXERCISE

1. If I pay $85 a ton for sugar, what do I pay a kilogram?

2. A farmer sold 540 Hl. of wheat, at $1 per bushel, and invested the proceeds in coal at $7 a ton, how many tons did he buy?

3. If alcohol is .8 as heavy as water, how many pounds avoirdupois will 1250 cu. cm. of alcohol weigh?

4. If $13.50 will buy 4.5 m. of French broadcloth 5 dm. wide, how much money will buy 94.05 m., 6½ dm. wide, at the same rate per square measure?

5. A merchant bought 300 m. of silk in Lyons, at 12.5 francs a meter. If he paid 75¢ a yard for duty and freight, and sold it in New York at $5 a yard, what was his gain?

6. A wheel is 3.6 m. around. How many times will it revolve in rolling a distance of 1.08 Km.?

7. How many meters of carpeting .6 m. wide will cover a floor 8 m. long and 5.7 m. wide?

8. In a pile of wood 18.6 m. long, 2.8 m. high, and 1.5 m. wide, how many steres?

9. What must be the length of a bin 1.5 m. wide, 1 m. deep, to contain 7500 liters of grain?

10. A wine merchant imported 1000 dekaliters of wine, at a cost of 75¢ a liter, delivered. At what price per gallon must he sell the same to clear $2000 on the shipment?

11. If a letter weighs 3.5 g., how many such letters will weigh 1.015 Kg.?

12. What will be the cost of excavating a cellar 18.3 m. long, 10.73 m. wide, and 3.4 m. deep, at 20¢ per stere?

13. The air is composed of oxygen and nitrogen. In 100 volumes of air there are 21 volumes of oxygen and 79 of nitrogen. Reckoning the weight of a liter of oxygen to be 1.4295 g., that of a liter of nitrogen 1.2577 g., find the number of grains of each gas in 100 g. of air.

14. What is the weight of 10 cu. m. of ice, it being .93 as heavy as water?

CUBE ROOT

681. The **cube root** of a number is one of the three equal factors of that number. Thus, the cube root of 64 is 4, since $4\times4\times4=64$. A **perfect cube** is a number which has an exact cube root.

Example. Find the cube root of 1728.

Operation

$$\begin{array}{rr}
 & 1728(10+2=12 \\
10\times10\times10= & \underline{1000} \\
3\times10\times10=300 & 728 \\
3\times10\times\ 2=\ \ 60 & \\
2^2=\underline{\ \ \ 4} & \\
2\times364= & 728
\end{array}$$

Solution. The largest cube that can be taken out of 1728 is 1000, leaving additions amounting to 728. The trial divisor is $3\times10\times10=300$ because three additions of approximately that size are to be made. By dividing we see that 2 is the thickness of the additions. Then they are exactly $3\times10\times12\times2+2\times2\times2=728$. So we find the cube root of 1728 to be 12.

682. Principles. *1. The cube of a number cannot have more than three times as many figures as its root, nor but two less.*

2. If a number is separated into periods of three figures each. beginning at units' place, the number of figures in the cube root will be the same as the number of periods.

WRITTEN EXERCISE

1. What is the cube root of 12,167, or what is the edge of a cube whose solid contents are 12,167 units?

$$\begin{array}{r|l}
 & 12'167(23 \\
 & \underline{8\qquad\quad} \\
20^2\times3=1200 & 4\,167 \\
20\times3\times3=\ \ 180 & \\
3^2=\underline{\ \ \ \ 9} & \\
1389 & \underline{4\,167\quad}
\end{array}$$

Solution. Since 12,167 consists of two periods, its cube root will consist of two figures (Prin. 2). Then the greatest cube in the left-hand period, 12, is 8, and its root is 2, which is written in the quotient. Subtracting the cube from the given number and annexing the next period to the remainder, we form the dividend, 4167. By multiplying the square of the root already found with one cipher annexed by 3, we form the trial divisor, 1200 ($20^2\times3=1200$). The trial divisor is contained in the dividend, 4167, 3 times. Write 3 as the next figure of the root. To the trial divisor, to form the complete divisor, add 3 times the product of the last figure of the root by the first figure considered as tens ($20\times3\times3=180$), and the square of the last figure ($3^2=9$). Multiplying the complete divisor, 1389, by 3, the last figure of the root, and subtracting the product from the dividend ($4167 - 4167 = 0$), there is no remainder. Therefore, 23 is the required cube root.

683. Rule. *Separate the given number into periods of three figures each, beginning at the right.*

Find the greatest cube contained in the left-hand period, and write its root for the first figure of the required root. Subtract the cube of this figure from the left-hand period and to the remainder annex the next period to form a dividend.

Divide this dividend by the partial divisor, which is 3 times the square of the root already found, considered as tens; the quotient is the second figure of the root. To the partial divisor add 3 times the product of the second figure of the root by the first considered as tens, also the square of the second figure, and the result will be the complete divisor. Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend.

If there are more periods to be brought down, proceed as before, using the part of the root already found, the same as the first figure in the previous process.

If there is a remainder after the root of the last period is found, annex periods of ciphers and proceed as before. The figures of the root thus obtained will be *decimals.*

If a cipher occur in the root, annex two ciphers to the trial divisor, and another period to the dividend and proceed as before.

If it is required to find the cube root of a mixed decimal, form periods from the decimal point right and left, and when necessary annex decimal ciphers to make full periods.

Extract the cube root of the following:

2. 17576.
3. 175616.
4. 551368.
5. 1331000.
6. 4492125.
7. .024389.
8. 970.299.
9. 8.365427.
10. .000008.
11. Find the cube root of $\frac{27}{1728}$.

Operation. $\sqrt[3]{\frac{27}{1728}} = \frac{\sqrt[3]{27}}{\sqrt[3]{1728}} = \frac{3}{12} = \frac{1}{4}$

NOTE. In finding the cube root of a fraction, extract the cube root of the numerator and denominator separately.

12. Find the cube root of $\frac{2197}{6859}$; $\frac{1000}{1331}$; $7762\frac{49}{125}$.

13. Find the area of the surface of a cube which contains 59,319 cu. in.

14. Separate the number 7056 into its prime factors and from these factors determine its square root.

15. If a cubical block contains 21,952 cubic inches, how many square feet of paper will be required to cover its entire surface?

16. The entire surface of a cubic block is 384 square feet. How many 1-foot cubes can be cut from the block, allowing nothing for waste?

17. A cubical cistern holds 400 bbl. of water. How deep is it?

18. What are the dimensions of a cube that has the same volume as a box 2 ft. 8 in. long, 2 ft. 3 in. wide, and 1 ft. 4 in. deep?

19. If a grain of gold is beaten out into 54 sq. in. of leaf, what weight of gold will be required to cover a cube $1\frac{1}{2}$ feet on each edge?

20. The capacity of a square reservoir is 2000 barrels, but its depth is only $\frac{1}{3}$ its width. Find its dimensions.

Roots of Higher Degree

684. Any root whose index contains no other factors than 2 or 3 may be extracted by means of the square and cube roots.

Since the square of the cube of a number equals the sixth power, the sixth root of a number equals the square root of the cube root of the number, and the same is true in any other case.

WRITTEN EXERCISE

1. Extract the sixth root of 729.

Solution. To find the 6th root of 729, first extract the square root, which is 27, and then find the cube root of 27, which is 3. Hence the 6th root of 729 is 3.

Required the value of:

2. $\sqrt[4]{625}$
3. $\sqrt[6]{2985984}$
4. $\sqrt[8]{390625}$
5. $\sqrt[6]{1.771561}$
6. $\sqrt[9]{10077696}$
7. $\sqrt{(531441)^{\frac{1}{3}}}$

PROGRESSIONS

685. An **arithmetical progression** is a series of numbers which increase or decrease by a constant common difference.

Thus, 3, 8, 13, 18, 23, etc., is an arithmetical progression of which the common difference is 5.

Example 1. The first term of an arithmetical series is 5 and the common difference is 3, what is the 15th term?

686. Rule. *Any term of an arithmetical progression is equal to the first term, increased or diminished by the common difference multiplied by a number one less than the number of terms.*

Solution. $5+(3\times14)=47$.

2. The first term of an increasing progression is 2, and the common difference 3; what is the 50th term?

3. A man agreed to work for 40 days at 30¢ the first day, and an increase of 5¢ per day. What were his wages the last day?

4. Find the sum of an arithmetical series of which the first term is 3, the common difference 4, and the number of terms 15.

687. Rule. *To find the sum of an arithmetical series: multiply ½ the sum of the extremes by the number of terms.*

NOTE. If the first or last term of an arithmetical progression be not given, find this as shown above before proceeding to find their sum.

5. How many strokes, beginning at 1 o'clock, does the hammer of a clock strike in 12 hours?

6. If a body falls $16\frac{1}{12}$ ft. the first second, three times as far the next second, five times as far the third second, and so on, how far will it fall in 12 seconds?

Geometrical Progression

688. A **geometrical progression** is a succession of numbers, each of which is greater or less than the preceding one in a constant *ratio.*

Thus, 4, 12, 36, 108, 324, etc., is a geometrical progression, of which the multiplier or ratio is 3.

Example 1. The first term of a geometrical series is 6, the ratio 4, the number of terms 6. What is the last term?

689. Rule. *Any term of a geometrical progression is equal to the first term, multiplied by the ratio raised to a power one less than the number of the term.*

Solution. $4^5=1024$, and 6 times $1024=6144$.

2. The first term of a geometrical progression is 8, and the ratio 2. What is the 7th term?

3. A merchant doubles his capital every 4 years. If he begins with $5000, how much has he at the end of 16 years?

4. The first term of a decreasing geometric series is 128, the ratio 2. What is the 10th term?

5. The first term of a geometrical series is 2, the last term 1458, and the ratio 3. What is the sum of all the terms?

690. Rule. *To find the sum of a geometrical series: multiply the last term by the ratio, and divide the difference between the product and the first term by the difference between 1 and the ratio.*

Solution. $1458\times3-2=4372$, and $4372\div(3-1)=2186$.

6. The extremes are $4\frac{4}{5}$ and $\frac{8}{405}$, and the ratio is $\frac{1}{3}$. What is the sum of all the terms?

7. A farmer bought 10 cows, agreeing to pay 75¢ for the first, $1.50 for the second, $3.00 for the third, and so on. What did he pay for the 10 cows?

NOTE. If the first or last term of a geometrical progression be not given, find this as shown above before proceeding to find their sum.

PROOF CHECKING

691. Brief methods are in use for testing the correctness of the work in the fundamental processes of arithmetic. The law of divisibility of nine was first used by the Arabs, but it is not known who discovered it. This is called "proof by casting out the nines."

692. The law is: *a number is divisible by 9 when the sum of its digits is divisible by 9, and the remainder arising from dividing any number by 9 is the same as that arising from dividing the sum of its digits by 9.*

Thus, the remainder arising by dividing 56,893 by 9 is 4, the same as arises by dividing $5+6+8+9+3$ by 9. (See 122, 7.)

693. The above theorem may be demonstrated by taking any number, as 5842, and analyzing it as below. It will be seen that it consists of two parts; the first a *multiple* of 9, and the second part, the *sum of its digits:*

$$5842\left\{\begin{array}{rr} 2= & 2\\ 40=4\times10=4(9+1)=4\times9+ & 4\\ 800=8\times100=8(99+1)=8\times99+ & 8\\ 5000=5\times1000=5(999+1)=5\times999+ & 5\end{array}\right.$$

$$\therefore 5842=(4\times9)+(8\times99)+(5\times999)+2+4+8+5$$

From the above it is seen that the first part is divisible by 9 since it is the sum of a number of products of which 9 is a factor, hence the only remainder that can arise from dividing a number by 9 will be equal to the remainder arising from dividing the sum of its digits by 9. So when the sum of the digits of a number is exactly divisible by 9, it is evident that the number itself is divisible by 9, which is in accordance with the theorem.

Proof of Addition

694. Add 76767, 7654, 50121, 775 and check the work by casting out the 9's.

$$\begin{array}{rl} 76767 = & \text{a number of 9's}+6.\\ 7654 = & \text{a number of 9's}+4.\\ 50121 = & \text{a number of 9's}+0.\\ \underline{775} = & \text{a number of 9's}+\underline{1}.\\ 135317\,..2 & \qquad\qquad\qquad 11\,..2\end{array}$$

Explanation. From the numbers to be added the 9's are cast out. From the sum of the several excesses, which is 11, cast out 9, and the excess is 2. From the sum of the numbers, 135317, cast out the 9's and the excess is 2. As the excess, 2, is the same in each, the work is presumably correct.

NOTE. It should be borne in mind that this is a very useful check, but fails to detect such errors as the addition of 9, the interchange of digits, and all errors not affecting the sum of the digits.

Proof of Subtraction

695. Prove that 79685−30253=49432.

79685 = a number of 9's + 8.
30253 = a number of 9's + 4.
49432..4 4.

Explanation. Cast the 9's out from the minuend and subtrahend, and subtract the excess of the subtrahend, 4, from the excess of the minuend, 8, and the excess is 4. Cast the 9's out of the remainder, 49,432, and the excess is 4. Hence the work is presumably correct.

In case the excess from casting out the 9's in the minuend is less than the excess from casting out the 9's in the subtrahend it is necessary to add a 9 to the excess in the minuend before making the subtraction.

Proof of Multiplication

696. Prove that 368×245=90160.

368 = a number of 9's + 8.
245 = a number of 9's + 2.
90160..7 16..7

Explanation. Casting out the 9's from the multiplicand the remainder is 8. Casting out the 9's from the multiplier, the remainder is 2. The product of these remainders is 16, and casting the 9's from 16 there is a remainder of 7 which is the same as the remainder after the 9's have been cast from the product, 90160. Hence the work is presumably correct.

Proof of Division

697. Prove that 6775÷25=271.

7 7 1
25)6775(271

Explanation. The excess of 9's in each of the terms divisor, dividend, and quotient are respectively 7, 7, and 1. Then we find that the remainder 7 in the divisor multiplied by the remainder 1 in the quotient is equal to the remainder 7 in the dividend. Therefore the work is presumably correct, since the divisor multiplied by the quotient is equal to the dividend.

698. Prove that 698345 ÷ 237 = 2946 rem. 143.

3 8 3 8

237)698345(2946. rem. 143.

Explanation. Division being the inverse of multiplication, the dividend is equal to the product of the divisor and quotient plus the remainder. Therefore the excess of 9's in the divisor times the excess in the quotient plus the excess of 9's in the remainder is equal to 17. Casting the 9 from 17, we have 8 which is equal to 8, the remainder in the dividend. Hence the work is presumably correct.

Casting out Elevens

699. The properties of 11 may be used in proving addition, subtraction, multiplication, and division the same as shown with the 9 above. The method of casting out the 11's is considered the better proof by some as casting out the 9's will not show an error caused by a transposition of figures; but casting out the 11's will show such an error.

APPROXIMATE RESULTS

700. *Approximate results* are often desirable especially in multiplication and division of decimals. It will be sufficient in most cases to carry out the partial products correct to two places more than the required result.

It is frequently necessary to determine the value of a decimal fraction correct to a definite number of decimal places, but the student should bear in mind that it is a waste of time to carry out results to a greater degree of accuracy than the data on which they are founded.

701. Contracted Multiplication

1. *Write the multiplicand, and draw a vertical line after the last decimal place required to be correct.*
2. *Place the multiplier in reversed order underneath the multiplicand, so that the decimal point of the former shall be under the vertical line.*
3. *If a digit of the multiplier has no digit above it place a cipher there.*

4. Multiply by each figure of the multiplier—commencing at the right—the figure above it in the multiplicand and the other figures on the left of that figure, setting down the first figure of the product immediately on the right of the vertical line, and adding in the carrying figure from the previous column.

5. Add the partial products; then the number on the left of the vertical line is the product correct to the required decimal place.

Example. Multiply 4.73217 by 235.3135 correct to four decimal places.

Fully worked out.

4·73217	
235·3135	
23	66085
141	9651
473	217
14196	51
236608	5
1419651	
946434	
111135434	85295

Working contracted.

4·7321	700
. 5313	532
9464340	0
1419651	0
236608	5
14196	5
473	2
141	9
23	6
1113·5434	

702. Contracted Division

1. Find by the ordinary method, or by inspection, the unit of the first figure in the quotient; then from that find the number of figures which there will be in the answer.

2. Retain in the divisor one more than that number of figures, commencing at the left, and striking out the rest.

3. Should there, however, not be enough figures in the divisor for this purpose, proceed by the uncontracted method until the number of figures still to be found increased by one can be marked off from the divisor; then proceed with the rule.

4. Divide the contracted divisor into the dividend, and, in obtaining the first figure of the product of the divisor and the first figure of the quotient, carry to the tens from the preceding figure.

5. To get the second figure of the quotient, diminish the new divisor by the right-hand figure and proceed as before. Continue this process until all the quotient has been obtained.

Example. Divide 3216300 by 2175675 correct to two decimal places.

Fully worked out.

$$\begin{array}{r|l}
2175675)3216 & 300(1.47+ \\
\underline{2175} & \underline{675} \\
1040 & 6250 \\
\underline{870} & \underline{2700} \\
170 & 35500 \\
\underline{152} & \underline{29725} \\
18 & 05775
\end{array}$$

Working contracted.

$$\begin{array}{rl}
2175|675)3216 & 300(1.47+ \\
\underline{2175} & \\
1040 & \\
\underline{870} & \\
170 & \\
\underline{152} & \\
18 &
\end{array}$$

WRITTEN EXERCISE

Find by contracted multiplication correct to 4 places of decimals the product of the following:

1. .003296×5.782.
2. 41.78×.0629.
3. 3.2517×.023.
4. .23257×.243.
5. 37.643×3.86
6. .5678×.9243.

Find by contracted division correct to 3 places of decimals the quotients of the following:

7. 756.98÷76.73612.
8. 47.5782975÷26.175.
9. 3.14159÷2.718.
10. 1.09861÷2.302585.

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